Predicate Exchange
Inference with Declarative Knowledge

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Objective

Motivation: Conditioning on equality-to-data is insufficient to express most facts. Inference support for the broader class of predicates is limited.
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**Objective:** Given a probabilistic simulator $\pi$ and predicate $\ell$ on the output of $\pi$, sample from posterior $p(\pi \mid \ell$ is true).
Priors with constraints
condition on balls not intersecting
Inverse Graphics with constraints
condition on balls not intersecting
Predicate Exchange: An inference procedure which samples from models conditioned on predicates, through two steps:

(i) **Predicate Relaxation** constructs a soft predicate $\hat{\ell}$ from $\ell$. $\hat{\ell}$ maps $x$ to a value in a continuous Boolean algebra: the unit interval $[0, 1]$ with continuous logical connectives $\hat{\land}$, $\hat{\lor}$ and $\neg$.

   (i) Soft equality $x \approx y$: $k_\alpha(\rho(x, y))$
   (ii) Soft inequality $x \triangleright y$: $k_\alpha(\rho(x, [y, \infty]))$
   (iii) Soft conjunction $\hat{\land}$: $\max(x, y)$
   (iv) Soft disjunction $\hat{\lor}$: $\min(x, y)$

Figure 1: log of $x \triangleright y$, $x \triangleleft y$, $x \approx y$, and $\neg(x \approx y)$
Convert predicates into soft predicates
Soft predicate represents degree to which hard predicate is satisfied

\[(x > y) \lor \neg(x^2 = 2) \rightarrow (x \gtrsim y) \lor \neg(x^2 \equiv 2)\]
Approximate Posterior

Assuming a prior density \( p \), the approximate posterior \( f \) is the product:

\[
f(x) = p(x) \cdot \tilde{\ell}(x)
\]

*Example:* if \( X_{1,2} \sim \mathcal{N}(0, 1) \) is conditioned on \( X_1 + X_2 = 0 \), aprx posterior:

\[
f_\alpha(x_1, x_2) = \mathcal{N}_{0,1}(x_1) \cdot \mathcal{N}_{0,1}(x_2) \cdot k_\alpha(\rho(x_1 + x_2, 0))
\]
Temperature trades-off accuracy / convergence

\( \alpha = 0 \)
\( \alpha = 10 \)
\( \alpha = 100 \)
\( \alpha = 1000 \)
(ii) **Replica Exchange** is a MCMC method that simulates several replicas conditioned model at different temperatures.

*Figure 2:* Samples from single site MH, Hamiltonian Monte Carlo, and replica exchange.
Omega.jl: A Causal, Higher-Order PPL

github.com/zenna/Omega.jl

Poster #52

cond
conditional inference
do
causal inference
rcd
higher-order inference