

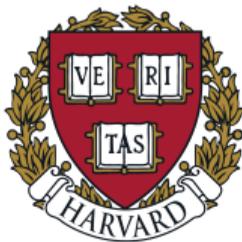
SATNet:

Bridging deep learning and logical reasoning using a differentiable satisfiability solver

Po-Wei Wang ¹ Priya L. Donti ¹ Bryan Wilder ² J. Zico Kolter ^{1,3}



¹ School of Computer Science,
Carnegie Mellon University

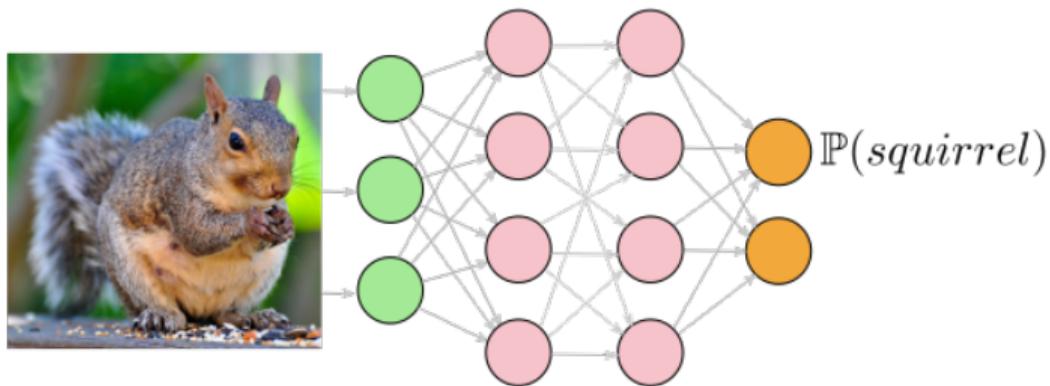


² School of Engineering and Applied Sciences,
Harvard University



³ Bosch Center for Artificial Intelligence

Integrating deep learning and logic



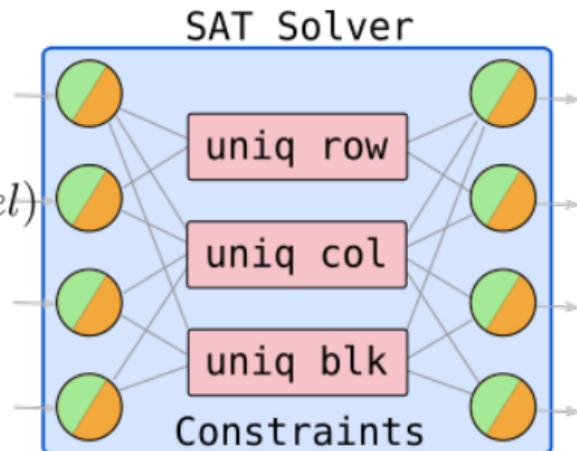
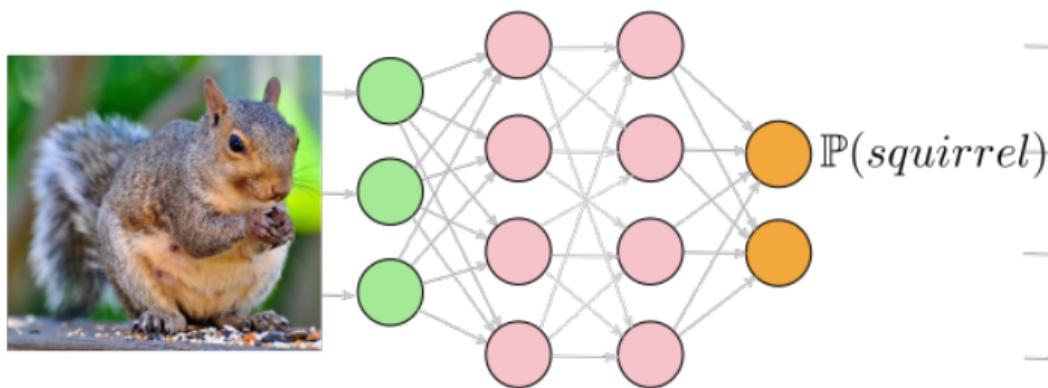
Deep Learning

No constraints on output

Differentiable

Solved via gradient optimizers

Integrating deep learning and logic



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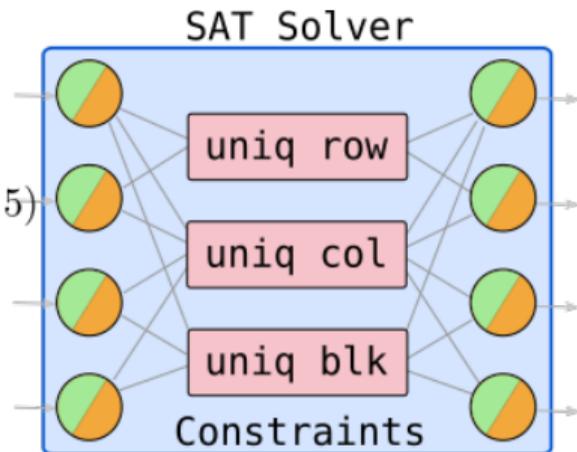
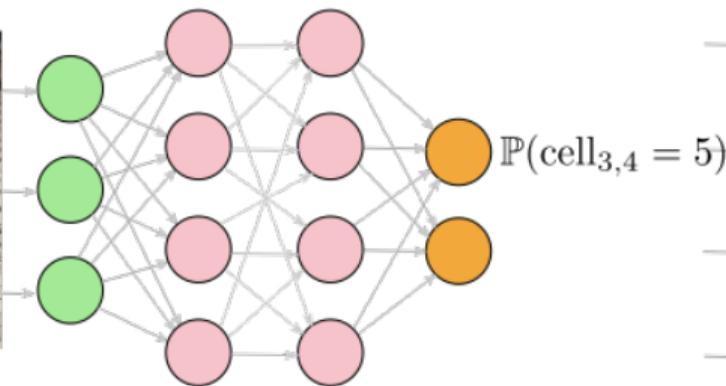
Logical Inference

Rich constraints on output

Discrete input/output

Solved via tree search

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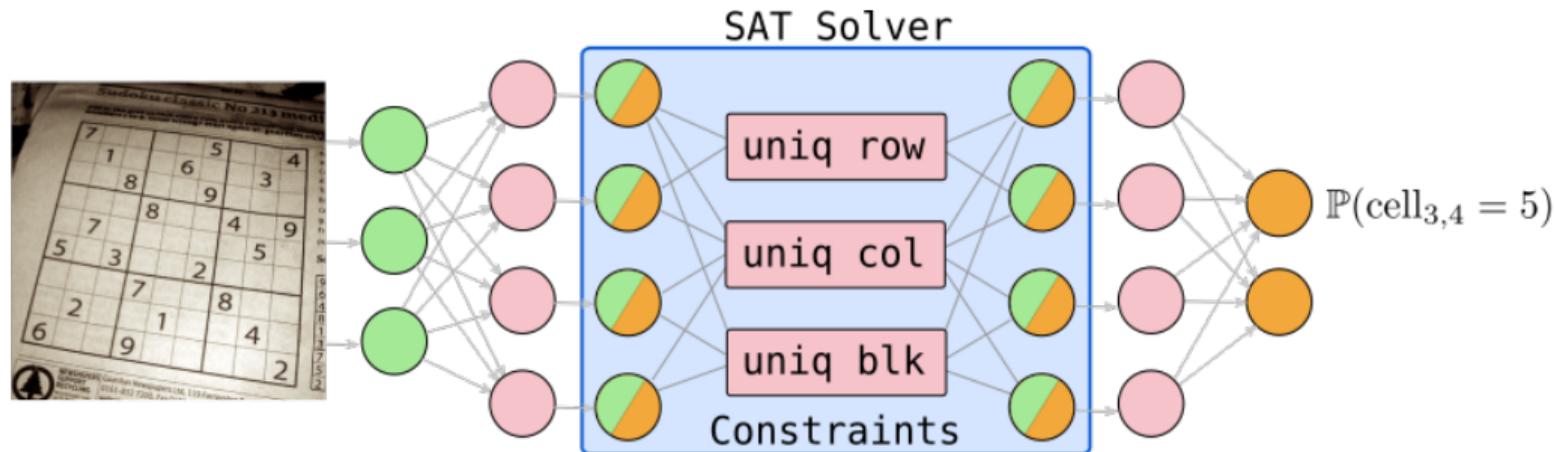
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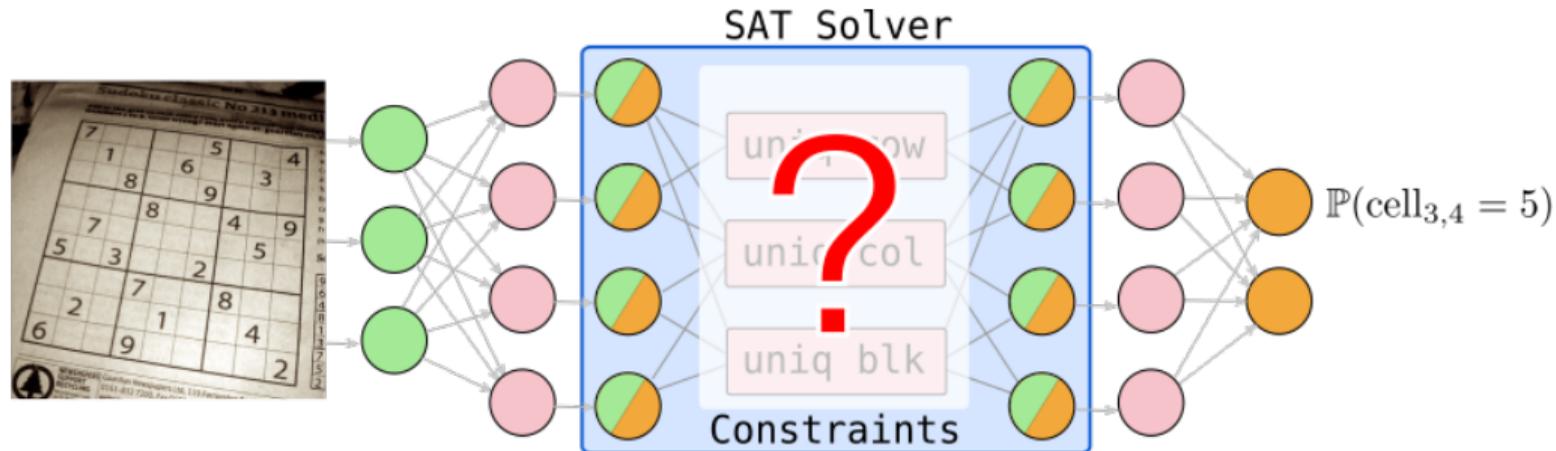
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- but about learning both **constraints** and **solution** from examples

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Not about using DL and SAT in a multi-staged manner

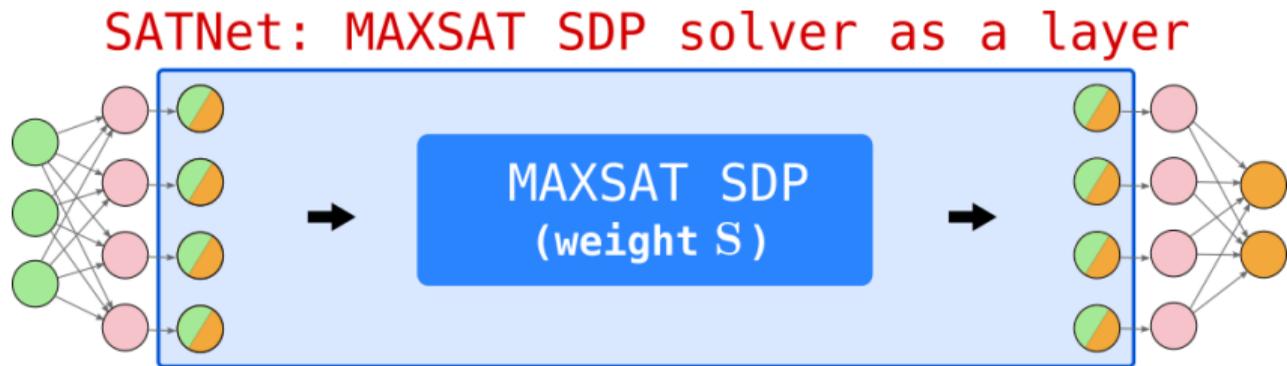
- doing so requires prior knowledge on the structure and constraints
- further, current SAT solvers cannot accept **probability inputs**

This talk is about

- A layer that enables end-to-end learning of **both** the **constraints** and **solutions** of logic problems within deep networks...

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- A layer that enables end-to-end learning of **both** the **constraints** and **solutions** of logic problems within deep networks...
- A smoothed **differentiable (maximum) satisfiability solver** that can be integrated into the loop of deep learning systems.



Review of SAT problems

Example SAT problem:

$$v_2 \wedge (v_1 \vee \neg v_2) \wedge (v_2 \vee \neg v_3)$$

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\Downarrow

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} v_2 \\ v_1 \vee \neg v_2 \\ v_2 \vee \neg v_3 \end{matrix}$$

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Typical SAT: Clause matrix given, find satisfying assignment

Our setting: Clause matrix is parameters of the layer (to be learned)

MAXSAT Problem

MAXSAT is the optimization variant of SAT solving

SAT: Find feasible v_i s.t. $v_2 \wedge (v_1 \vee \neg v_2) \wedge (v_2 \vee \neg v_3)$

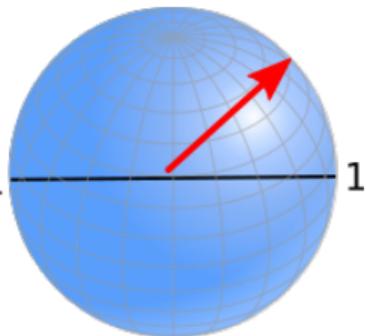
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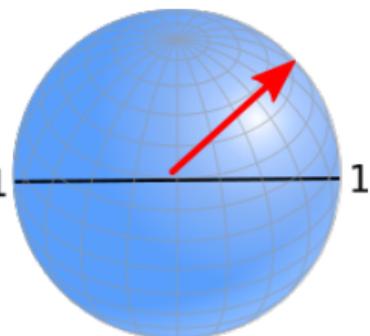
Relax the binary variables to smooth & continuous spheres

$$v_i \in \{+1, -1\} \xrightarrow{\text{equiv}} |v_i| = 1, \quad v_i \in \mathbb{R}^1 \xrightarrow{\text{relax}} \|v_i\| = 1, \quad v_i \in \mathbb{R}^k$$

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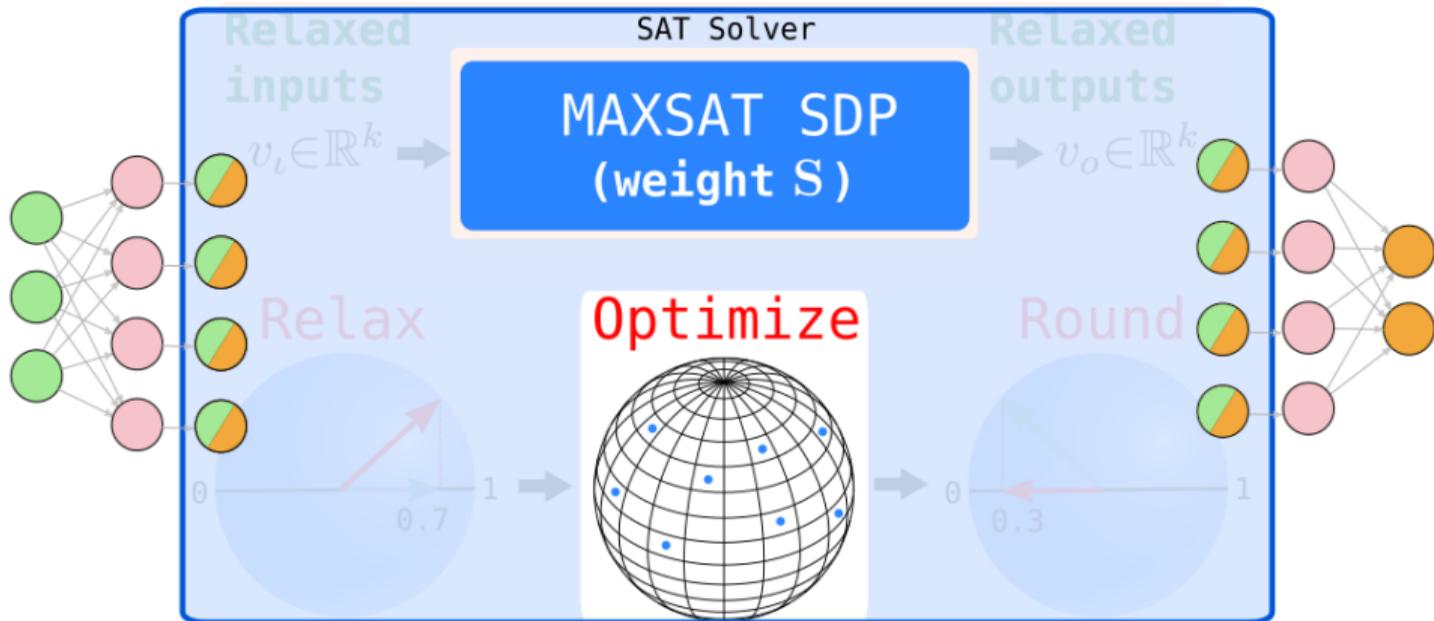
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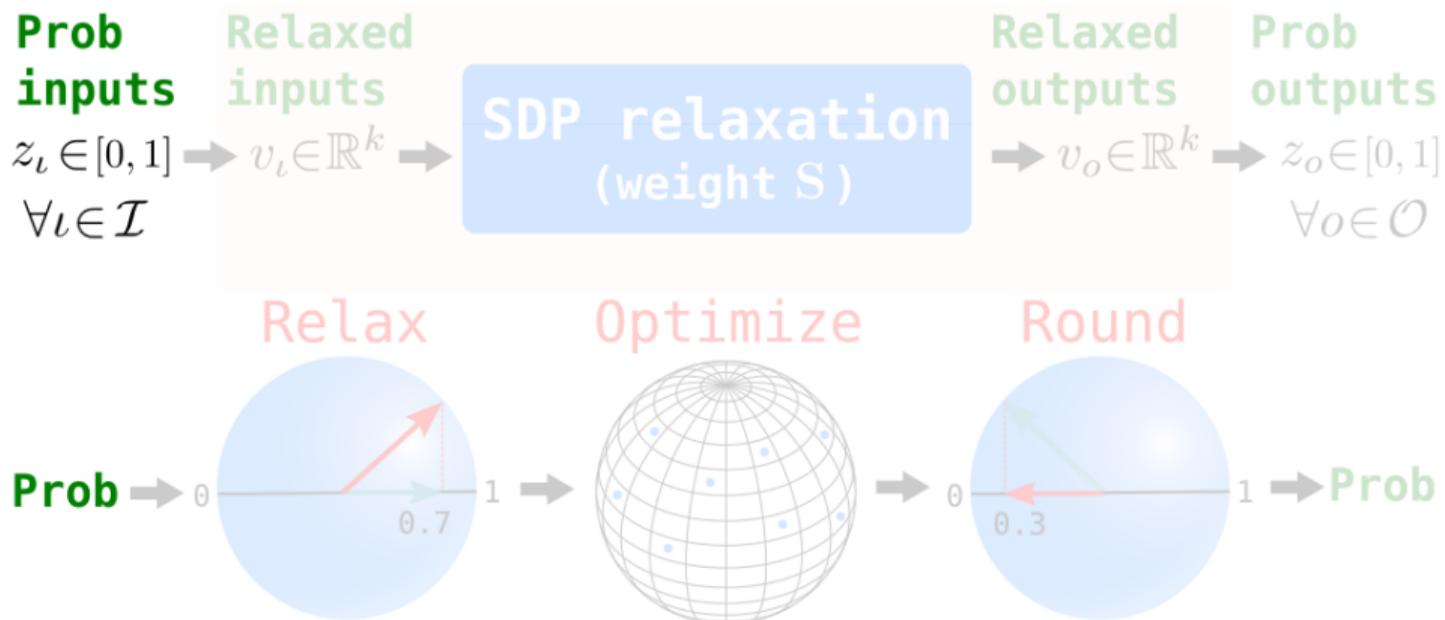
Semidefinite relaxation (Goemans-Williamson, 1995), $X = V^T V$

$$\text{minimize } \langle S^T S, X \rangle, \quad \text{s.t. } X \succeq 0, \quad \text{diag}(X) = 1.$$

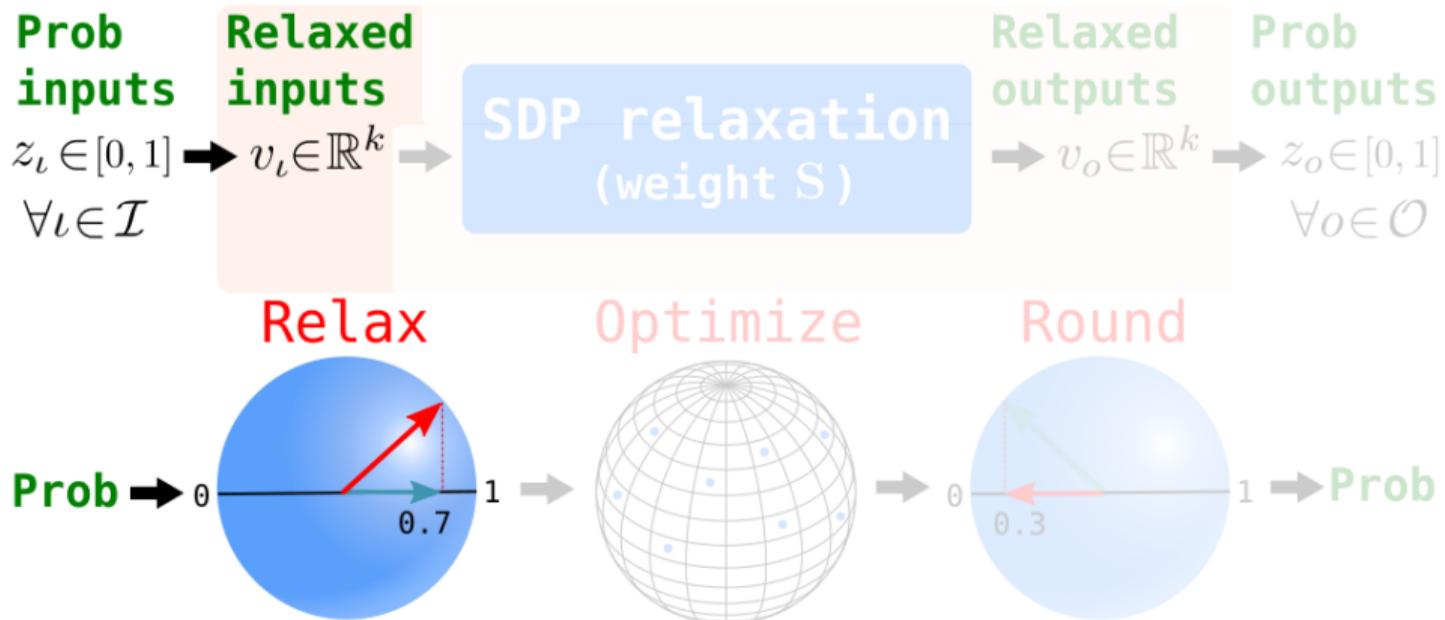
SATNet: MAXSAT SDP as a layer



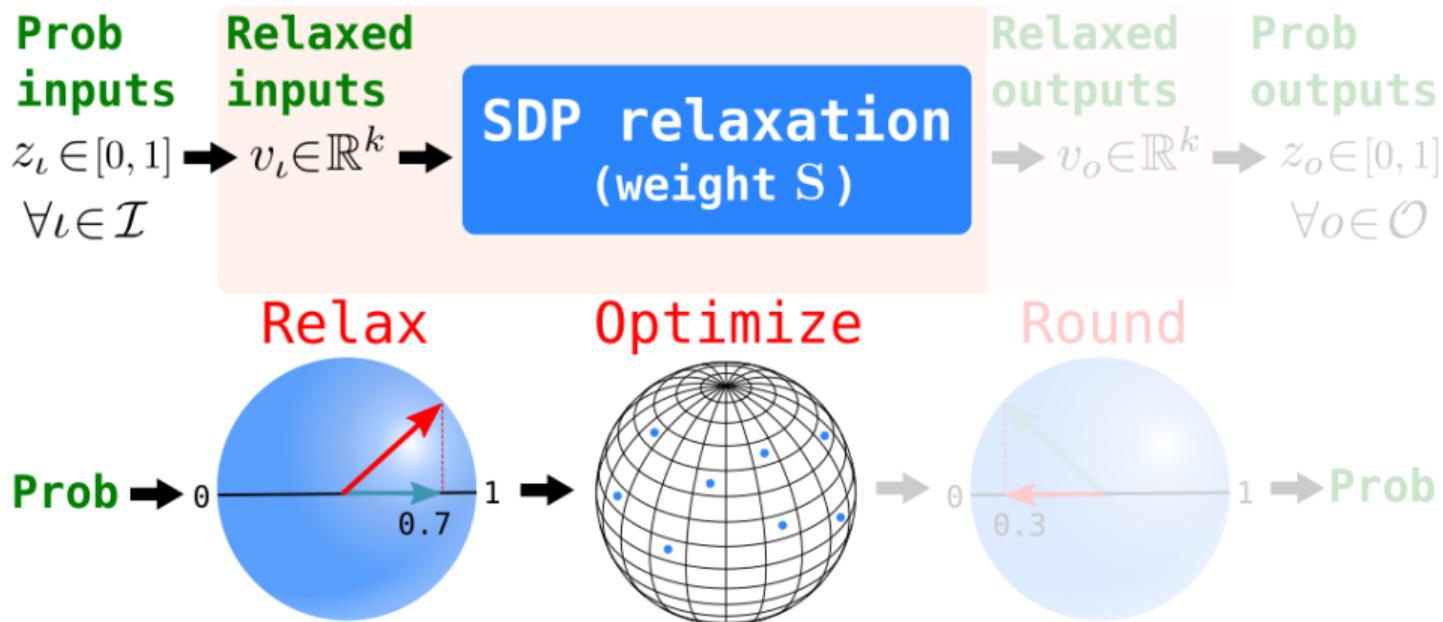
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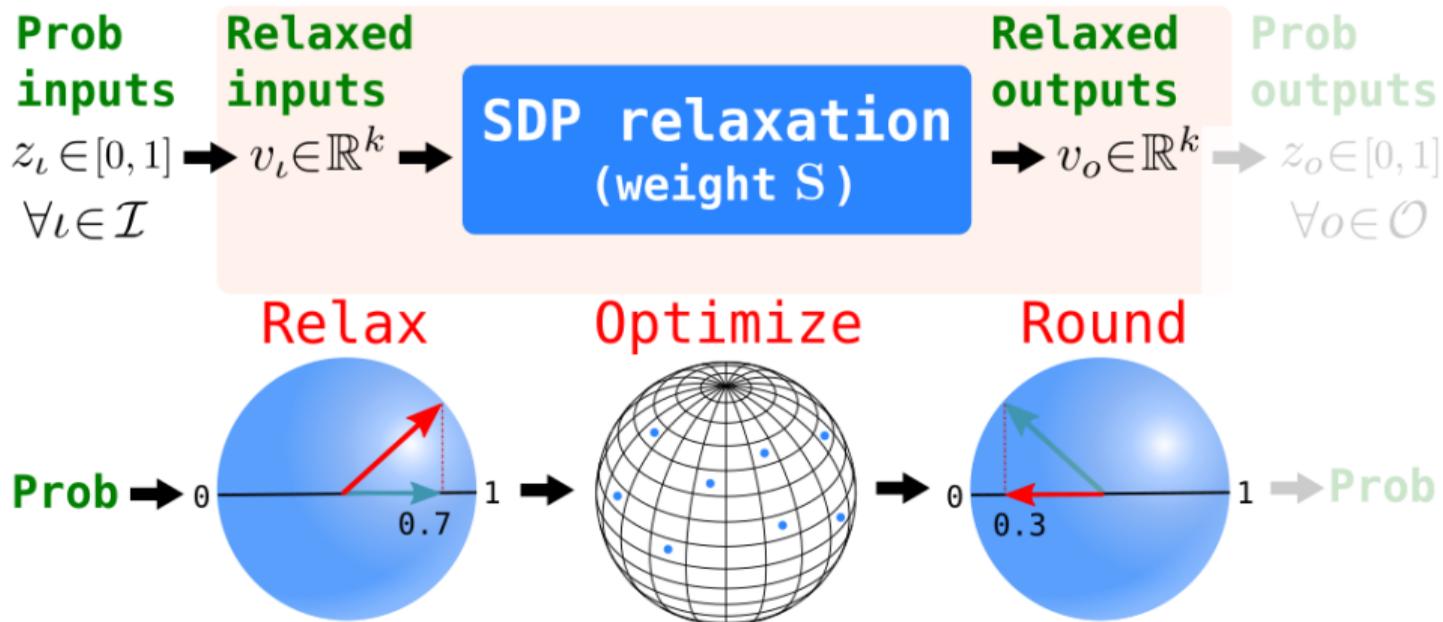
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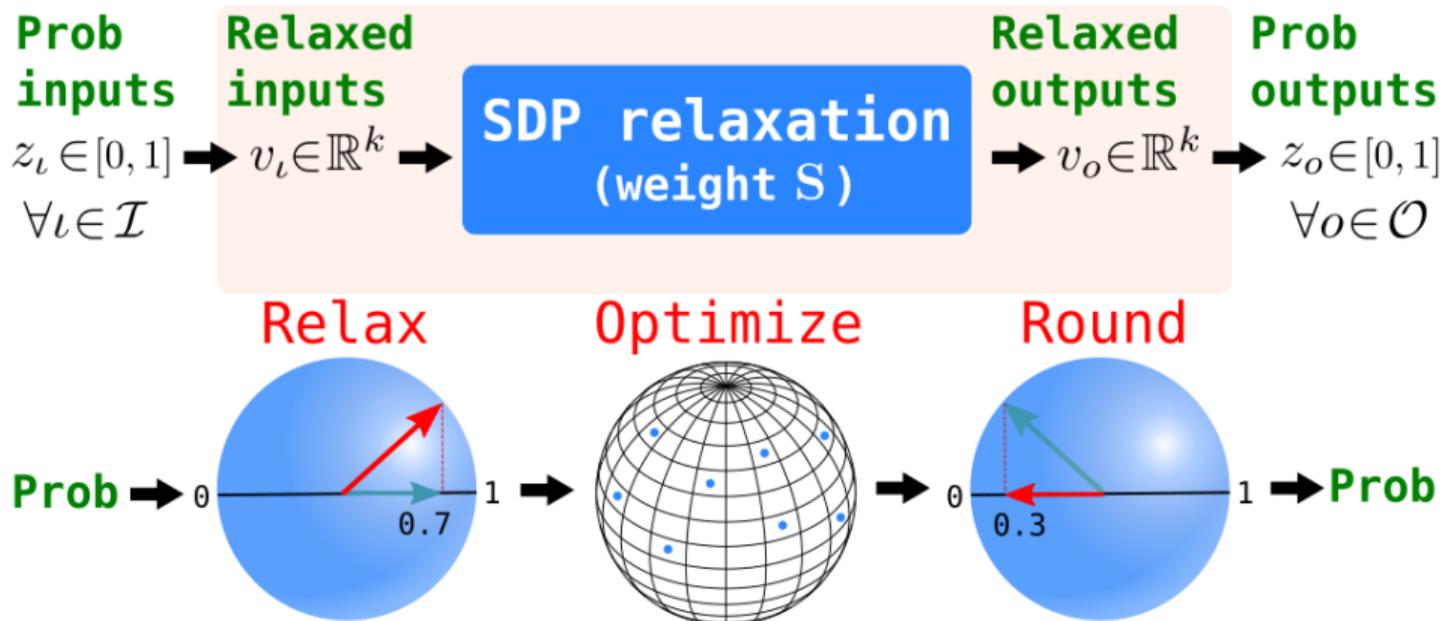
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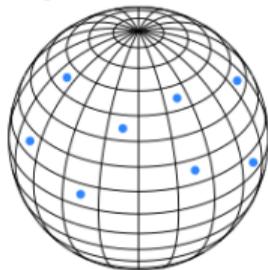


Relax

Optimize

Complexity:

Primal-dual interior point method for SDP



$O(n^6)$

Too expensive

Fast solution to MAXSAT SDP approximation

Efficiently solve via low-rank factorization $X = V^T V$, $V \in \mathbb{R}^{k \times n}$, $\|v_i\| = 1$ (a.k.a. Burer-Monteiro method), and block coordinate descent iters

$$v_i = -\text{normalize}(VS^T s_i - \|s_i\|^2 v_i).$$

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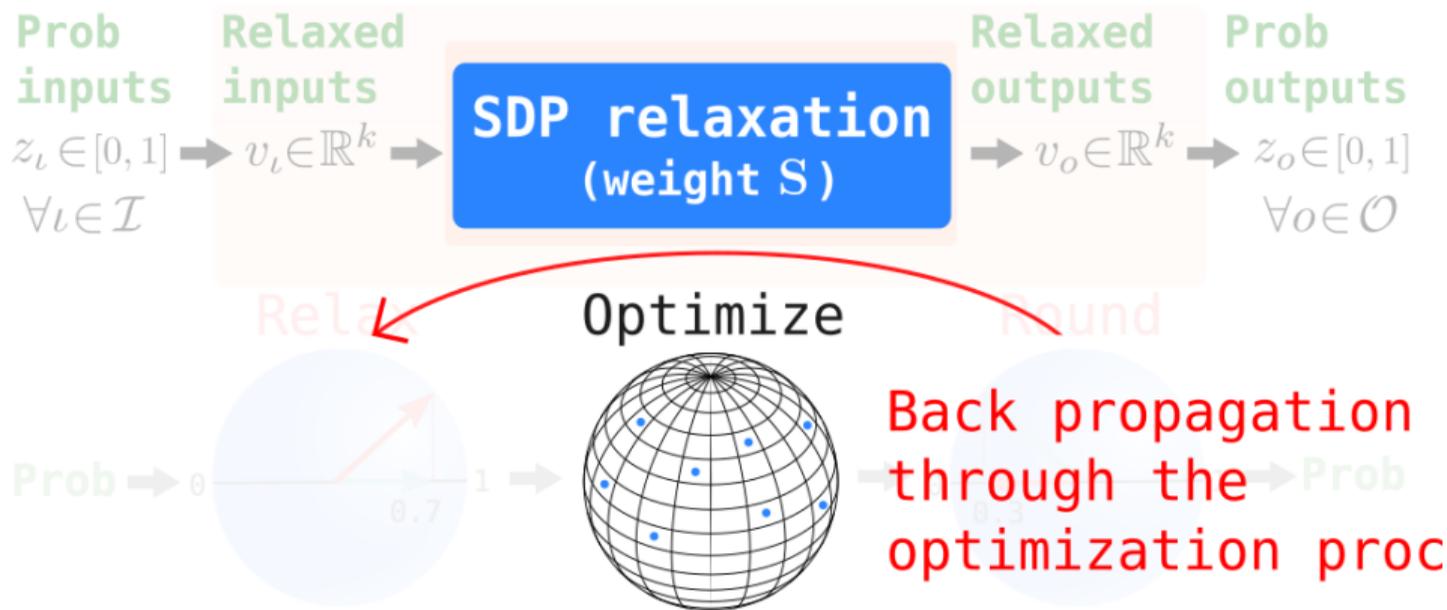
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Complexity reduced from $O(n^6 \log \log \frac{1}{\epsilon})$ of interior point methods to $O(n^{1.5} m \log \frac{1}{\epsilon})$ of our method, where m is #clauses.

Differentiate through the optimization problem



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When converged, the procedure satisfies the fixed-point equation

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$$F_i(S, V(S)) = v_i + \text{normalize}(VS^T s_i - \|s_i\|^2 v_i) = 0, \forall i$$

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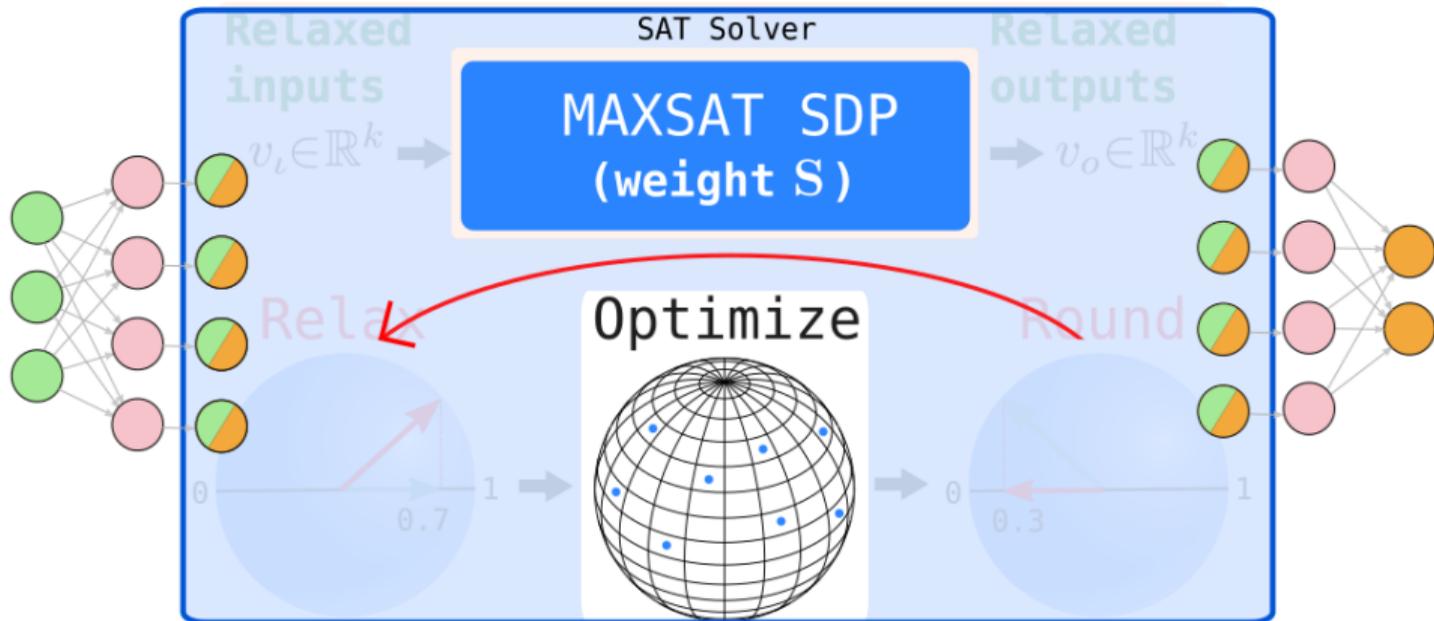
$$F_i(S, V(S)) = v_i + \text{normalize}(VS^T s_i - \|s_i\|^2 v_i) = 0, \forall i$$

Thus, can apply implicit function theorem on the total derivatives

$$\frac{\partial \vec{F}(\vec{S}, \vec{V}(S))}{\partial \vec{S}} = 0 \implies \frac{\partial \vec{F}(\vec{S}, \vec{V})}{\partial \vec{S}} + \frac{\partial \vec{F}(\vec{S}, \vec{V})}{\partial \vec{V}} \cdot \frac{\partial \vec{V}}{\partial \vec{S}} = 0$$

Solve the above **linear system** of $\partial \vec{V} / \partial \vec{S}$ to backprop

SATNet: MAXSAT SDP as a layer



Other ingredients in SATNet

Low-rank regularization on S

- Doubly-exponentially many possible Boolean functions!

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- Adding auxiliary variable (gadget) increases representation power

Illustration: Learning Parity from single bit supervision

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- Parity problem is surprisingly hard for most deep networks to learn [Shalev-Swartz et al., 2017]
- Chained (recurrent) SATNet-based network learns parity function for up to length 40 strings from 10K examples

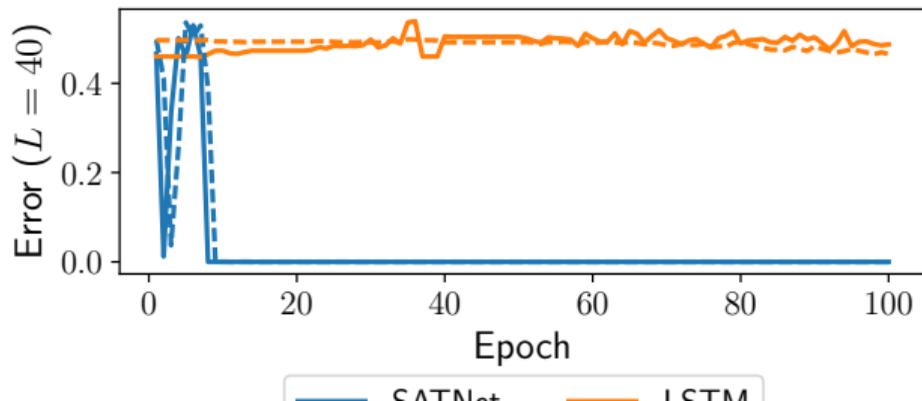


Illustration: Learning Sudoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
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- Learning 9x9 Sudoku from 9K examples
- Single SATNet layer on one-hot-encoded input puzzles

Model	Train	Test
ConvNet	72.6%	0.04%
SATNet (ours)	99.8%	98.3%

Original Sudoku.

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Original Sudoku.

Model	Train	Test
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Permuted Sudoku.

Illustration: MNIST Sudoku

0 6 2	1 0 7	0 8 0
0 3 0	0 0 8	2 5 0
8 0 0	0 0 4	0 0 0
0 0 0	0 8 0	7 0 0
4 9 1	0 6 0	0 2 8
5 0 0	3 4 0	1 0 0
0 0 3	0 7 9	0 1 0
1 7 0	0 0 0	5 0 0
0 5 0	0 0 0	9 6 0

Illustration: MNIST Sudoku

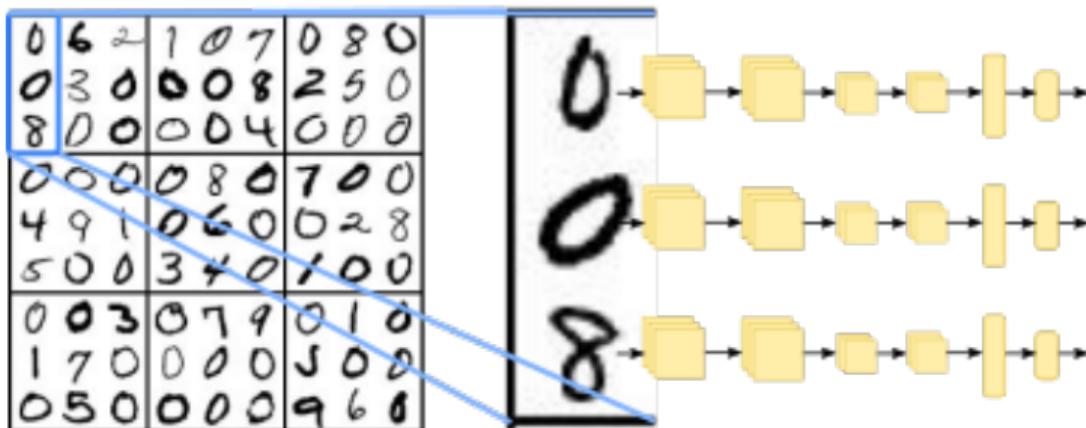


Illustration: MNIST Sudoku

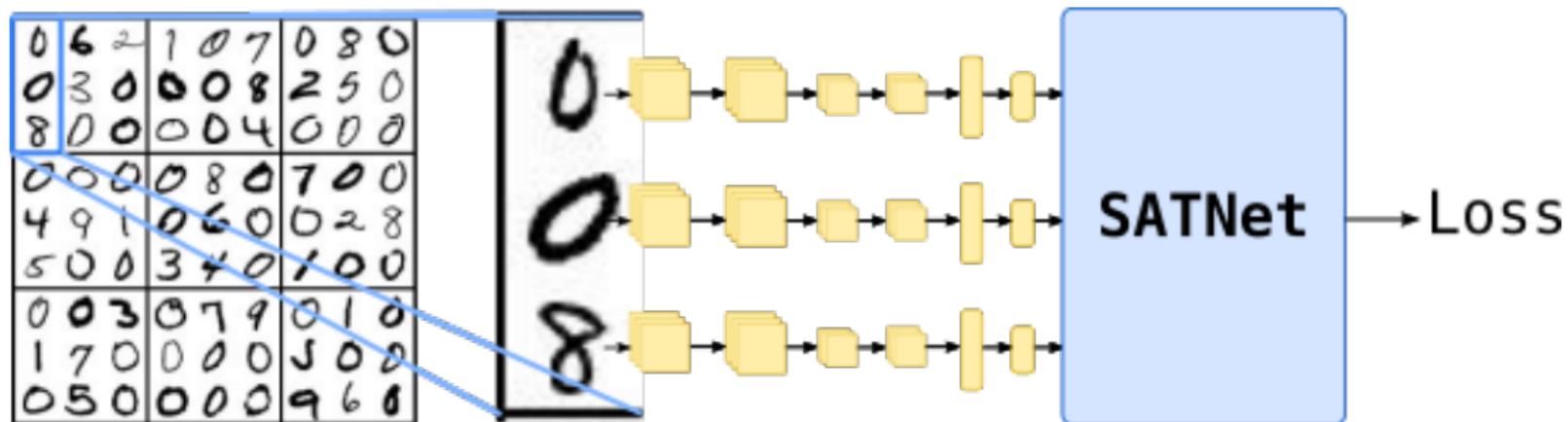
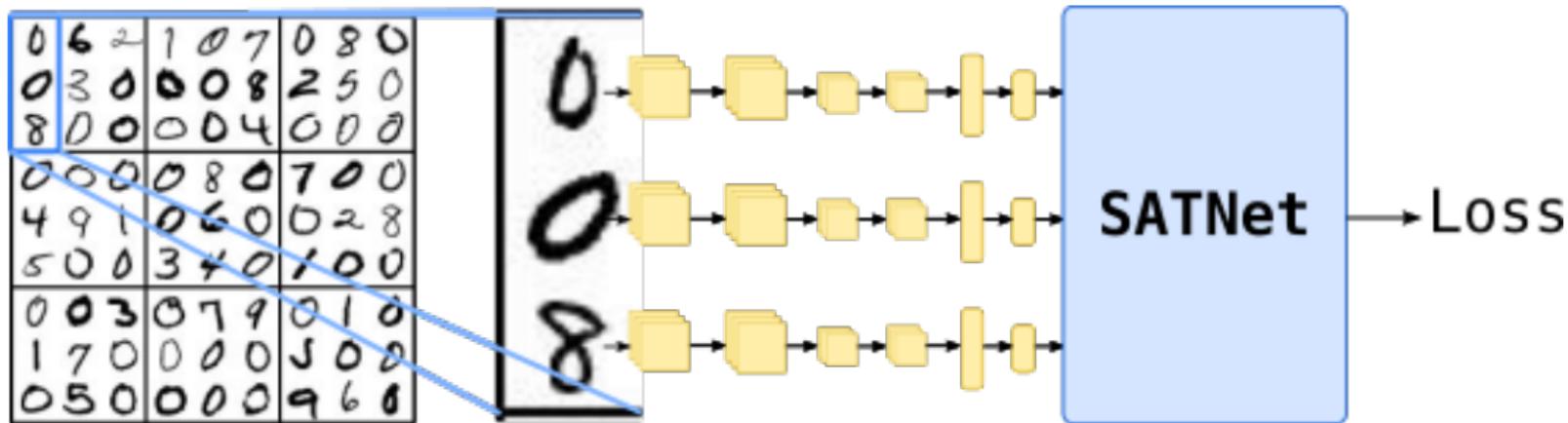


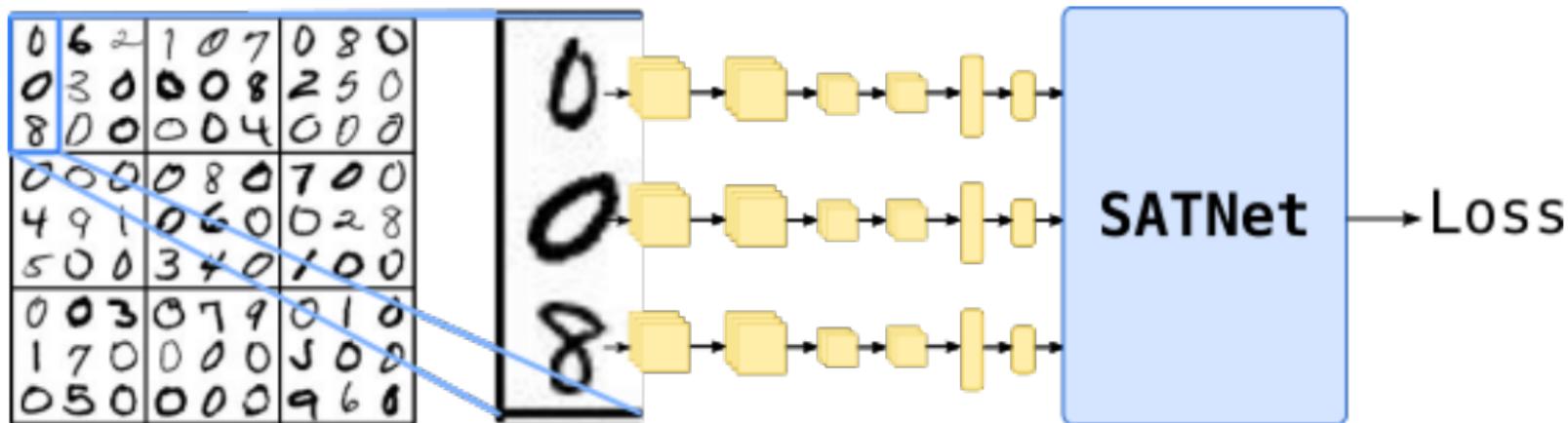
Illustration: MNIST Sudoku



Model	Train	Test
ConvNet	0.31%	0%
SATNet (ours)	93.6%	63.2%

- Getting example “correct” requires correct Sudoku solution *and* predicting all MNIST test digits correctly

Illustration: MNIST Sudoku



Model	Train	Test
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- Getting example “correct” requires correct Sudoku solution *and* predicting all MNIST test digits correctly
- 85% accuracy on correct ConvNet input

Code and Colab



Code available at <https://github.com/locuslab/SATNet>

The screenshot shows a Google Colab notebook titled "Learning and Solving Sudoku via SATNet.ipynb". The interface includes a top menu bar with "File", "Edit", "View", "Insert", "Runtime", "Tools", and "Help". Below the menu is a toolbar with buttons for "+ CODE", "+ TEXT", "↑ CELL", and "↓ CELL". On the left side, there is a "Table of contents" sidebar with the following items: "Introduction to SATNet", "Building SATNet-based Models", "The Sudoku Datasets" (with sub-items "Sudoku", "One-hot encoded Boolean Sudoku", and "MNIST Sudoku"), and "The 9x9 Sudoku Experiment". The main area displays a code cell with the following content:

```
!git clone https://github.com/locuslab/SATNet
%cd SATNet
!python setup.py develop > install.log 2>&1
```

The terminal output for the first cell is:

```
Cloning into 'SATNet'...
remote: Enumerating objects: 47, done.
remote: Counting objects: 100% (47/47), done.
remote: Compressing objects: 100% (36/36), done.
remote: Total 47 (delta 12), reused 43 (delta 8), pack
Unpacking objects: 100% (47/47), done.
/content/SATNet
```

Below this, there are two more code cells:

```
[ ] !wget -cq powei.tw/sudoku.zip && unzip -qq sudoku.zip
!wget -cq powei.tw/parity.zip && unzip -qq parity.zip
```

```
[ ] import os
import shutil
import argparse
```

Conclusion

We presented

- **SATNet**, the first **differentiable MAXSAT solver** as a layer

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Possible extensions:

- Incorporating known rules into the system
- Exploiting structures of the clause matrix

Poster at Pacific Ballroom #26