Lipschitz Generative Adversarial Nets

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Generalized Formulation for GANs

\[
\begin{align*}
\min_{f \in F} & \mathbb{E}_{z \sim P_z}[\phi(f(g(z)))] + \mathbb{E}_{x \sim P_r}[\psi(f(x))], \\
\min_{g \in G} & \mathbb{E}_{z \sim P_z}[\varphi(f(g(z)))] ,
\end{align*}
\tag{1}
\]

where

- \(P_z\): the source distribution in \(\mathbb{R}^{nz}\);  
- \(P_r\): the target (real) distribution in \(\mathbb{R}^{nr}\);  
- \(g\): the generative function \(\mathbb{R}^{nz} \rightarrow \mathbb{R}^{nr}\);  
- \(f\): the discriminative function \(\mathbb{R}^{nr} \rightarrow \mathbb{R}\);  
- \(G\): the generative function space;  
- \(F\): the discriminative function space;  
- \(\phi, \psi, \varphi: \mathbb{R} \rightarrow \mathbb{R}\) are loss metrics.

We denote the generation distribution by \(P_g\).
The Gradient Uninformativeness

The problem that the gradient from the discriminator does not contain any informative about the real distribution.

A new perspective for the training instability / convergence issue of GANs.

For GANs with unrestricted $\mathcal{F}$:

$$f^*(x) = \arg\min_{f(x) \in \mathbb{R}} \mathbb{P}_g(x)\phi(f(x)) + \mathbb{P}_r(x)\psi(f(x)), \forall x. \quad (2)$$

- $f^*(x)$ is independently defined and only reflects the local densities $\mathbb{P}_r(x)$ and $\mathbb{P}_g(x)$;
- $\nabla_x f^*(x)$ does not reflect any information about the other distribution, if the supports of two distributions are disjoint.
<table>
<thead>
<tr>
<th>Type</th>
<th>Impact on Problem</th>
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<tbody>
<tr>
<td>Unrestricted GANs</td>
<td>MUST suffer from</td>
</tr>
<tr>
<td>Restricted GANs</td>
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</tr>
<tr>
<td>GANs with W-Distance</td>
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<tr>
<td>Lipschitz GANs</td>
<td>DO NOT suffer from</td>
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</table>

The Gradient Un informativeness

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Lipschitz Generative Adversarial Nets

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Lipschitz Generative Adversarial Nets (LGANs)

\[
\begin{align*}
\min_{f \in F} & \quad \mathbb{E}_{z \sim P_z}[\phi(f(g(z)))] + \mathbb{E}_{x \sim P_r}[\psi(f(x))] + \lambda \cdot k(f)^2, \\
\min_{g \in G} & \quad \mathbb{E}_{z \sim P_z}[\varphi(f(g(z)))] .
\end{align*}
\]

(3)

We require \( \phi \) and \( \psi \) to satisfy:

\[
\begin{align*}
\phi'(x) & > 0, \\
\phi''(x) & \geq 0, \\
\text{and} \quad \phi(x) & = \psi(-x).
\end{align*}
\]

(4)

Any increasing function with non-decreasing derivative.
Lipschitz Generative Adversarial Nets (LGANs)

Theoretically guaranteed properties:

- The optimal discriminative function $f^*$ exists;
- If $\phi$ is strictly convex, then $f^*$ is unique;
- There exists a unique Nash equilibrium where $P_r = P_g$ and $k(f^*) = 0$;
- Do not suffer from gradient uninformativeness;
- For each generated sample, the gradient directly points towards a real sample.
Gradient uninformativeness practically behaves as noisy gradient.

(a) Disjoint Case
(b) Overlapping Case
Experiments: $\nabla_x f^*(x)$ in LGANs

$\nabla_x f^*(x)$ directly point towards real samples.
Experiments: $f^*$ is Unique

Uniqueness of $f^*$ leads to stabilized discriminative functions.

(a) $f(x)$ in WGAN.

(b) $f(x)$ in LGANs.
Experiments: Unsupervised Image Generation

LGANs, with different choices of $\phi(x)$, consistently outperform WGAN.

(a) Training curves on CIFAR.

(b) Training curves on Tiny.
Summary

- Gradient uninformativeness
  - Unrestricted GANs: MUST suffer from this problem
  - Restricted GANs: May suffer from this problem
  - GANs with W-Distance: May suffer from this problem
  - Lipschitz GANs: DO NOT suffer from this problem

- Lipschitz GANs:
  - Penalize the Lipschitz constant of $f$;
  - Set $\phi(x)$ to be any increasing function with non-decreasing derivative;
  - If $\phi$ is strictly convex, then $f^*$ is unique;
  - The gradients directly point towards real samples.

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Poster: this evening, #19