Learning to Convolve: 
A Generalized Weight-Tying Approach 

Nichita Diaconu* & Daniel Worrall*
Philips Lab c AMLAB, University of Amsterdam
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Symmetry
Symmetry
Symmetry

Dog ≠ Dog
Symmetry
Symmetry
Symmetry

In

Out
Equivariance & Convolution

Standard convolution

\[ [f \ast \psi](g) = \sum_{x \in X} f(x) \psi(x - g) \]

e.g. LeCun et al. (1998)
Equivariance & Convolution

Standard convolution

\[(f \ast \psi)(g) = \sum_{x \in \mathcal{X}} f(x) \psi(x - g)\]

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Equivariance & Convolution

**Standard convolution**

\[
[f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \psi(x - g)
\]

e.g. LeCun et al. (1998)

**Group convolution**

\[
[f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x)
\]

e.g. Cohen & Welling (2015)
Equivariance & Convolution

Standard convolution

\[ [f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) \psi(x - g) \]

*Input*  
*Filter*

e.g. LeCun et al. (1998)

Group convolution

\[ [f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

g-transformed filter

e.g. Cohen & Welling (2015)
Group Convolutions

Group convolution

\[ [f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

g-transformed filter
Group Convolutions

Group convolution

\[ [f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

\( \mathcal{L}_0[\psi] \quad \mathcal{L}_{45}^\circ[\psi] \quad \cdots \quad \cdots \quad \mathcal{L}_{315}^\circ[\psi] \)

Nearest-neighbor
Group Convolutions

Group convolution

\[ [f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

\[ \mathcal{L}_{0^\circ}[\psi] \quad \mathcal{L}_{45^\circ}[\psi] \quad \cdots \quad \cdots \quad \mathcal{L}_{315^\circ}[\psi] \]

Nearest-neighbor

Bilinear
Unitary Group Convolutions

Group convolution

\[(f \ast \psi)(g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x)\]
Unitary Group Convolutions

Group convolution

\[ (f \star \psi)(g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

Unitarity

\[ \sum_{x \in \mathcal{X}} \mathcal{L}_g[f](x) \mathcal{L}_g[\psi](x) = \sum_{x \in \mathcal{X}} f(x) \psi(x) \]
Unitary Group Convolutions

**Group convolution**

\[
[f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) L_g[\psi](x)
\]

**Unitarity**

\[
\sum_{x \in \mathcal{X}} L_g[f](x) L_g[\psi](x) = \sum_{x \in \mathcal{X}} f(x) \psi(x)
\]
Learning Convolutions

Group convolution

\[
(f \star \psi)(g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x)
\]
Learning Convolutions

Group convolution

\[ [f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) L_g [\psi](x) \]

\[ \psi(x) = \sum_{i} \hat{\psi}_i e^i(x) \]
Learning Convolutions

Group convolution

\[ [f \ast \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x) \]

\[ \psi(x) = \sum_i \hat{\psi}_i e^{i}(x) \]

Coefficients
Learning Convolutions

**Group convolution**

\[
[f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x)
\]

\[
\psi(x) = \sum_{i} \hat{\psi}_i e^i(x)
\]

**Coefficients** | **Basis**
Learning Convolutions

Group convolution

\[
[f \star \psi](g) = \sum_{x \in \mathcal{X}} f(x) \mathcal{L}_g[\psi](x)
\]

\[
\mathcal{L}_g[\psi](x) = \sum_i \hat{\psi}_i \mathcal{L}_g[e^i](x)
\]

Coefficients  Basis
Experiments: MLP $\rightarrow$ CNN

With thanks to Nichita Diaconu, Andrei Pauliuc, Daniel Maaskant, and Jens Dudink
Experiments: MLP → CNN

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Experiments: MLP $\rightarrow$ CNN

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MNIST Test Error

- MLP: 1.4%
- Us: 0.7%
- CNN: 0.5%
Experiments: Filters

$\mathcal{L}_{0^\circ} [\psi]$  $\mathcal{L}_{45^\circ} [\psi]$  $\cdots$  $\cdots$  $\mathcal{L}_{315^\circ} [\psi]$

Learned (ours)
Experiments: Filters

$\mathcal{L}_{0^\circ}[\psi]$ $\mathcal{L}_{45^\circ}[\psi]$ $\cdots$ $\cdots$ $\mathcal{L}_{315^\circ}[\psi]$

Learned (ours)
Experiments: Filters

\[ \mathcal{L}_{0^\circ}[\psi] \quad \mathcal{L}_{45^\circ}[\psi] \quad \cdots \quad \cdots \quad \mathcal{L}_{315^\circ}[\psi] \]

- Learned (ours)
- Nearest-neighbor
- Bilinear
Transformation robustness

Weiler from Weiler et al. (2018)
Transformation robustness

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Abstract
Recent work (Cohen & Welling, 2016a) has shown that generalizations of convolutions, based on group theory, provide powerful inductive biases for learning. In these generalizations, filters are not only translated but can also be rotated, flipped, etc. However, coming up with exact models of how to rotate a $3 \times 3$ filter on a square pixel-grid is difficult. In this paper, we learn how to transform filters for use in the group convolution, focusing on roto-translation. For this, we learn a filter basis and all rotated versions of that filter basis. Filters are then encoded by a set of rotation invariant coefficients. To rotate a filter, we switch the basis. We demonstrate we can produce feature maps with low sensitivity to input rotations, while achieving high performance on MNIST and CIFAR-10.

group convolutions extend standard translational convolution to the setting where the symmetry is a discrete algebraic group (explained in Section 2.2). In other words, these are convolutions over invertible transformations, so kernels are not only translated but also rotated, flipped, etc. One of the key assumptions with Cohen & Welling (2016a) and associated approaches is that the set of transformations forms a group. We cannot pick an arbitrary set of transformations. For instance, in Cohen & Welling (2016a) the authors choose the group of pixelwise translations, 90° rotations, and flips, that is the set of all transformations that map the regular square-lattice into itself; and in Hoogeboom et al. (2018) the authors consider the set of all transformations that map the hexagonal lattice into itself. However, in general the set of $\frac{\pi}{N}$ rotations for integer $N$ and pixelwise translations does not form a group because of pixelwise discretization, yet in Bekkers et al. (2018) and Wellershof et al. (2018a), they use these sets of transformations. Their

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https://deworrall92.github.io/