Riemannian adaptive stochastic gradient algorithms on matrix manifolds

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Optimization on manifolds

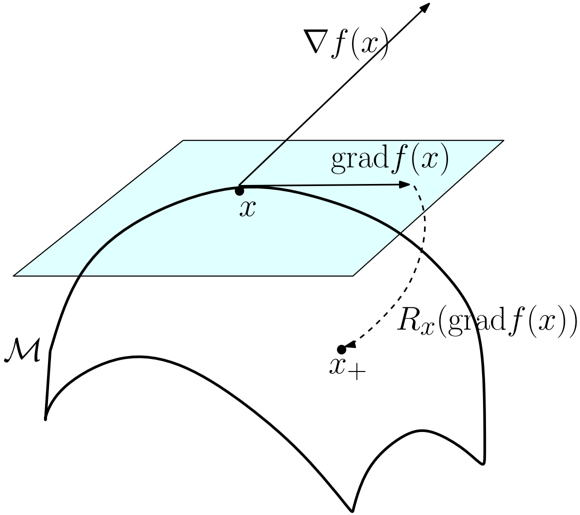
$$\min_{x \in \mathcal{M}} \qquad f(x)$$

$\mathcal{M}$ is smooth manifold.
f is smooth function.

Manifolds of interest:
Grassmann / subspaces
Low-rank matrices / tensors
Hyperbolic spaces
Positive definite matrices

Stochastic update on manifold:

$$x_{t+1} = R_{x_t}(-\alpha_t \mathrm{grad} f_t(x_t))$$



Adaptive algorithms on manifolds

Euclidean adaptive update:

$$x_{t+1} = x_t - \alpha_t \underbrace{\mathbf{V}_t^{-1/2}}_{\text{adaptive scaling}} \nabla f_t(x_t),$$

Riemannian adaptive update:

$$x_{t+1} = R_{x_t}(-\alpha_t \underbrace{\mathbf{V}_t^{-1/2}}_{\text{adaptive scaling}} \mathrm{grad} f_t(x_t)),$$

The challenge is how to compute $\mathbf{V}$ in the Riemannian setting.

Our contributions:

- Propose a principled approach for modeling adaptive weights for Riemannian stochastic gradient, i.e., model adaptive weight matrices for row and column subspaces exploiting the geometry of the manifold.

- Provide convergence analysis of our algorithm, under a set of mild conditions. Our algorithms achieve a rate of convergence order $O(\log(T)/\sqrt{T})$, where T is the number of iterations, for non-convex stochastic optimization.

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