On the Computation and Communication Complexity of Parallel SGD with Dynamic Batch Sizes for Stochastic Non-Convex Optimization

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Stochastic Non-Convex Optimization

- Stochastic non-convex optimization

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Stochastic Non-Convex Optimization

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stochastic gradient averaged from a mini-batch of size B
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- Single node training:
  - Larger \( B \) can improve the utilization of computing hardware

- Data-parallel training:
  - Multiple nodes form a bigger “mini-batch” by aggregating individual mini-batch gradients at each step.
  - Given a budget of gradient access, larger batch size yields fewer update/comm steps
Batch size for (parallel) SGD

- Question: Should always use a BS as large as possible in (parallel) SGD?
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- Recall B=1 means poor hardware utilization and huge communication cost
Dynamic BS: reduce communication without sacrificing SFO convergence

- Motivating result:
  For strongly convex stochastic opt, [Friedlander&Schmidt’12] and [Bottou et.al.’18] show that SGD with exponentially increasing BS can achieve the same $O(1/T)$ SFO convergence as SGD with fixed small BS.
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• This paper explores how to use dynamic BS for non-convex opt such that:
  • do not sacrifice SFO convergence in parallel SGD. Recall (N node parallel) SGD with (B=1) has $O(1/\sqrt{NT})$ SFO convergence for T: SFO access budge at each node.

  Linear speedup w.r.t. # of nodes; computation power perfectly scaled out
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    Linear speedup w.r.t. # of nodes; computation power perfectly scaled out.
  • reduce communication complexity (# of used batches) in parallel SGD.
Non-Convex under PL condition

- PL condition: \( \frac{1}{2} \| \nabla f(x) \|^2 \geq \mu (f(x) - f^*), \forall x \)
  
- Milder than strong convexity: strong convexity implies PL condition.
- Non-convex fun under PL is typically as nice as strong convex fun.

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Algorithm 1 CR-PSGD\((f, N, T, x_1, B_1, \rho, \gamma)\)

1: **Input:** \( N, T, x_1 \in \mathbb{R}^m, \gamma, B_1 \) and \( \rho > 1 \).
2: Initialize \( t = 1 \)
3: while \( \sum_{\tau=1}^{t} B_\tau \leq T \) do
4:  Each worker calculates batch gradient average \( \tilde{g}_{t,i} = \frac{1}{B_t} \sum_{j=1}^{B_t} F(x_t; \zeta_{i,j}) \).
5:  Each worker aggregates gradient average \( \tilde{g}_t = \frac{1}{N} \sum_{i=1}^{N} \tilde{g}_{t,i} \).
6:  Each worker updates in parallel via: \( x_{t+1} = x_t - \gamma \tilde{g}_t \).
7:  Set batch size \( B_{t+1} = [\rho^t B_1] \).
8:  Update \( t \leftarrow t + 1 \).
9: end while
10: Return: \( x_t \)
Non-Convex under PL condition

• Under PL, we show using exponentially increasing batch sizes in PSGD with $N$ workers has $O\left(\frac{1}{NT}\right)$ SFO convergence with $O(\log T)$ comm rounds

• SoA $O\left(\frac{1}{NT}\right)$ SFO convergence with $O(\sqrt{NT})$ inter-worker comm rounds attained by local SGD in [Stich’18] for strongly convex opt only
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- How about general non-convex without PL?
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- How about general non-convex without PL?

- Inspiration from “catalyst acceleration” developed in [Lin et al.’15][Paquette et al.’18]
  
  - Instead of solving original problem directly, it repeatedly solves “strongly convex” proximal minimization
General Non-Convex Opt

- A new catalyst-like parallel SGD method

Algorithm 2 CR-PSGD-Catalyst($f, N, T, y_0, B_1, \rho, \gamma$)

1: **Input:** $N, T, \theta, y_0 \in \mathbb{R}^m$, $\gamma$, $B_1$ and $\rho > 1$.
2: Initialize $y^{(0)} = y_0$ and $k = 1$.
3: while $k \leq \lceil \sqrt{NT} \rceil$ do
4: Define $h_{\theta}(x; y^{(k-1)}) \triangleq f(x) + \frac{\theta}{2} \| x - y^{(k-1)} \|^2$
5: Update $y^{(k)}$ via

$$
y^{(k)} = \text{CR-PSGD}(h_{\theta}(\cdot; y^{(k-1)}), N, \lceil \sqrt{T/N} \rceil, y^{(k-1)}, B_1, \rho, \gamma)
$$

6: Update $k \leftarrow k + 1$.
7: end while

- We show this catalyst-like parallel SGD (with dynamic BS) has $O(1/\sqrt{NT})$ SFO convergence with $O(\sqrt{NT \log(T/N)})$ comm rounds
  - SoA is $O(1/\sqrt{NT})$ SFO convergence with $O(N^{3/4}T^{3/4})$ inter-worker comm rounds
Experiments

Distributed Logistic Regression: N=10

[Graph 1: Loss vs. Number of SFO access]

[Graph 2: Loss vs. Number of communication rounds]
Experiments

Training ResNet20 over Cifar10: N=8
Thanks!

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