

# Classification from Positive, Unlabeled and Biased Negative Data

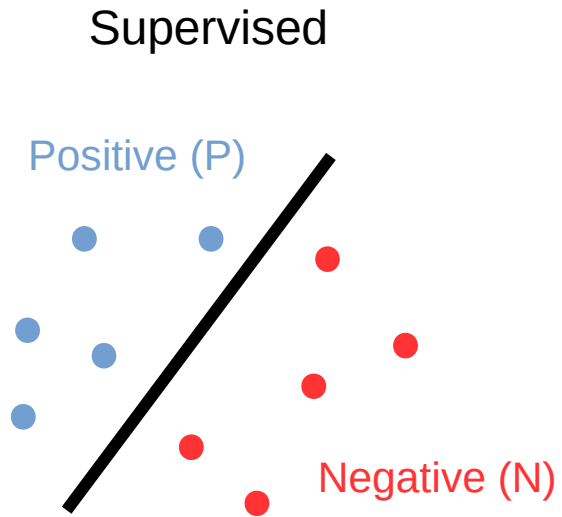
Poster #180

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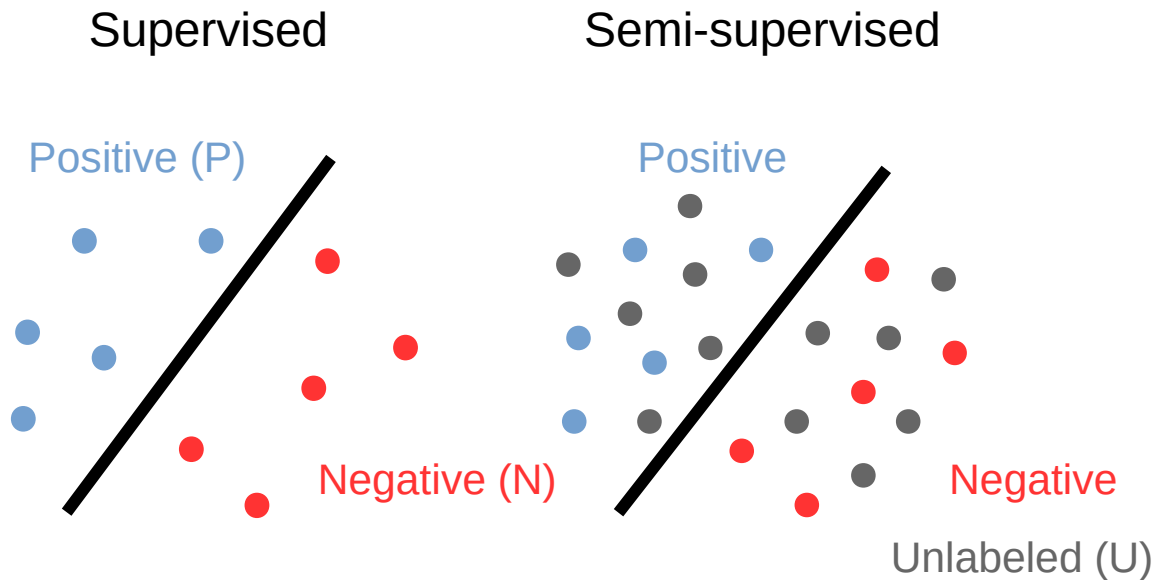
<sup>1</sup>ENS Paris, France   <sup>2</sup>RIKEN, Japan   <sup>3</sup>The University of Tokyo, Japan

# Background and problem setup

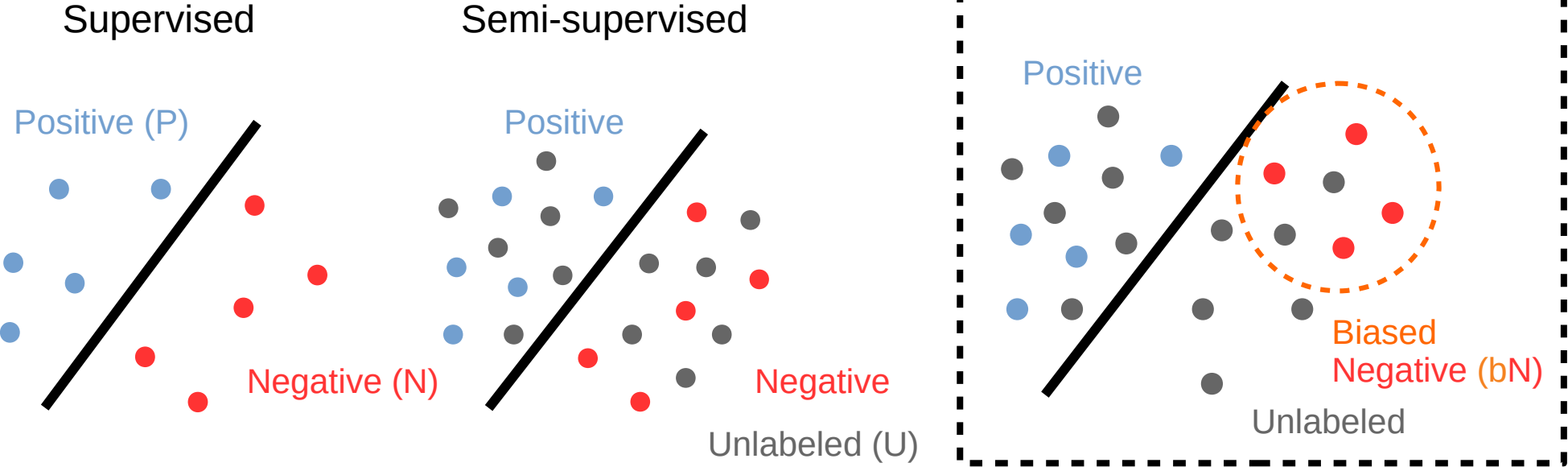
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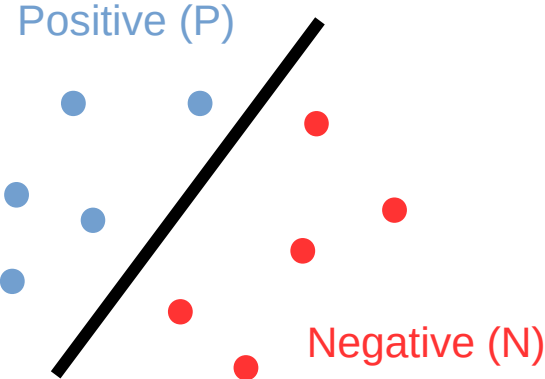


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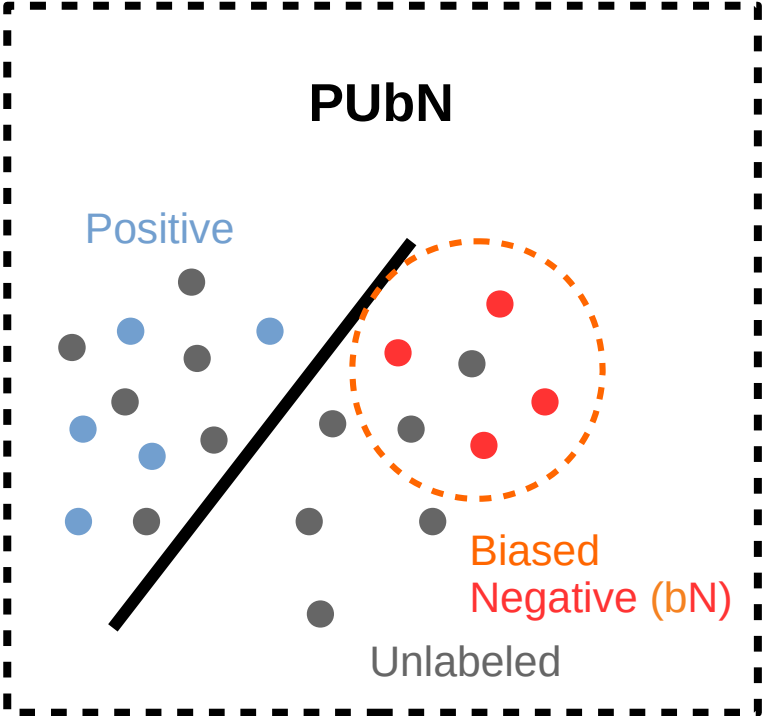
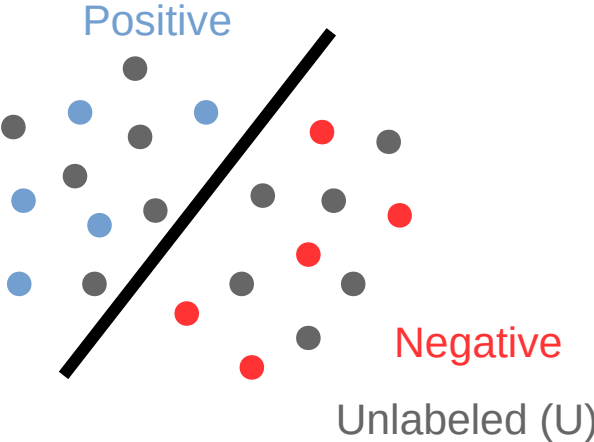


# Background and problem setup

Supervised



Semi-supervised



<span style="color: blue;">●</span> $\sim p_P(x) := p(x   y = +1)$	<span style="color: grey;">●</span> $\sim p(x)$	$y = +1 \Rightarrow s = +1$
<span style="color: red;">●</span> $\sim p_N(x) := p(x   y = -1)$	<span style="border: 1px dashed orange; border-radius: 50%; padding: 2px;">●</span> $\sim p_{bN}(x) := p(x   y = -1, s = +1)$	

# Motivating examples

Poster #180

The diagram is divided into three vertical panels, each with a dashed border. The left panel, labeled 'Positive Samples' in blue text, contains five images of cats of various breeds and colors. The middle panel, labeled 'Labeled Negative Samples' in red text, contains six images of animals that are not cats: a lion, a dog, a lioness, a wolf, an arctic fox, and a leopard. The right panel, labeled 'Other Negative Samples' in purple text, contains four images of marine mammals: two dolphins and two seals.

- Information retrieval, text classification, sentiment analysis
- Medical diagnosis: healthy population requesting physical exams is biased

## Method: Empirical risk estimator

$$\min_{g \in \mathcal{G}} \underbrace{\mathbb{E}_{(x,y) \sim p(x,y)} [\ell(yg(x))]}_{R(g)}$$

Risk Minimization

Unbiased  
Estimator

$$\min_{g \in \mathcal{G}} \frac{1}{n} \sum_{i=1}^n \underbrace{\ell(y_i g(x_i))}_{\hat{R}(g)}$$

Unbiased labeled data  
Empirical Risk Minimization



## Method: Empirical risk estimator

$$\begin{aligned}
 R(g) = & \pi \mathbb{E}_{x \sim p_P(x)} [\ell(g(x))] + \pi \mathbb{E}_{x \sim p_P(x)} \left[ \mathbf{1}_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right] \\
 & + \rho \mathbb{E}_{x \sim p_{bN}(x)} [\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{bN}} \left[ \mathbf{1}_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right] \\
 & + \mathbb{E}_{x \sim p(x)} [\mathbf{1}_{\sigma(x) \leq \eta} \ell(-g(x))(1 - \sigma(x))]
 \end{aligned}$$

$\sigma(x) = p(s=+1|x)$  probability of  $x$  being labeled

$\eta > 0$  determining how much we rely on the  $U$  data to approximate the risk

## Method: Empirical risk estimator

#P data

$$R(g) = \pi \mathbb{E}_{x \sim p_P(x)} [\ell(g(x))] + \pi \mathbb{E}_{x \sim p_P(x)} \left[ \mathbf{1}_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]$$

$$+ \rho \mathbb{E}_{x \sim p_{bN}(x)} [\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{bN}} \left[ \mathbf{1}_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]$$

$$+ \mathbb{E}_{x \sim p(x)} [\mathbf{1}_{\sigma(x) \leq \eta} \ell(-g(x)) (1 - \sigma(x))] \quad \#U \text{ data}$$

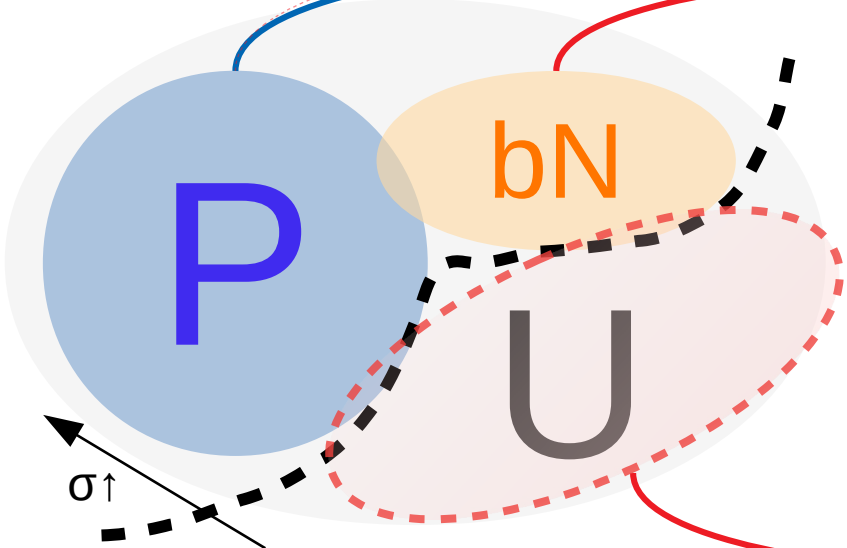
#bN data

$\sigma(x) = p(s=+1|x)$  probability of  $x$  being labeled

$\eta > 0$  determining how much we rely on the U data to approximate the risk

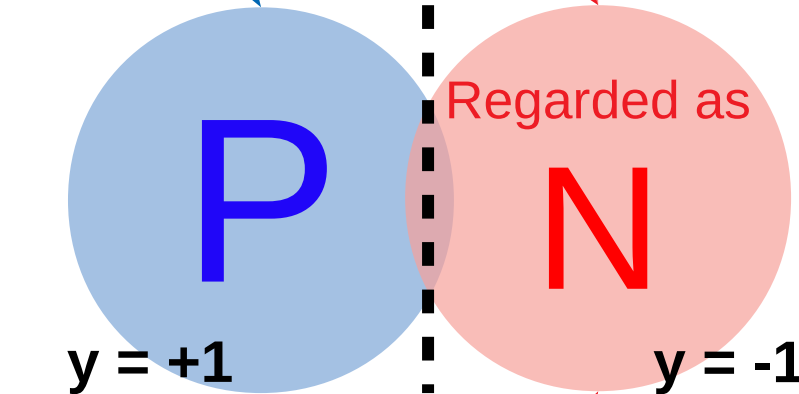
# Method: Illustration

Step 1



estimate  $\sigma = p(s=+1|.)$ : s as label  
 nnPU classifier (Kiryo+ NeurIPS 2017)

Step 2



final classifier: y as label  
 ERM:  
**pseudo labeling**  
**+ weight adjustment**

# Estimation error bound

With probability at least  $1-\delta$

$$\begin{aligned}
 R(\hat{g}) - R(g^*) \leq & \underbrace{\frac{4\pi L_\ell}{\eta} \mathfrak{R}_{n_P, p_P}(\mathcal{G}) + \frac{2\pi C_\ell}{\eta} \sqrt{\frac{\ln(6/\delta)}{2n_P}}}_{\text{\#P data}} + \underbrace{\frac{4\rho L_\ell}{\eta} \mathfrak{R}_{n_{bN}, p_{bN}}(\mathcal{G}) + \frac{2\rho C_\ell}{\eta} \sqrt{\frac{\ln(6/\delta)}{2n_{bN}}}}_{\text{\#bN data}} \\
 & + \underbrace{4L_\ell \mathfrak{R}_{n_U, p}(\mathcal{G}) + 2C_\ell \sqrt{\frac{\ln(6/\delta)}{2n_U}}}_{\text{\#U data}} + \underbrace{2C_\ell \sqrt{\zeta \epsilon} + \frac{2C_\ell}{\eta} \sqrt{(1-\zeta)\epsilon}}_{\text{Bias due to inexact approximation of } \sigma}
 \end{aligned}$$

# Experiments

Models: ConvNet / ResNet / FCN + Training: Amsgrad

Dataset	P	$\pi$	bN	$\rho$	nnPU/nnPNU	PUBN(\N)	PU $\rightarrow$ PN
MNIST	2, 4, 6, 8, 10	0.49	Not given	NA	5.76 $\pm$ 1.04	<b>4.64 <math>\pm</math> 0.62</b>	NA
			1, 3, 5	0.3	5.33 $\pm$ 0.97	<u>4.05 <math>\pm</math> 0.27</u>	<b>4.00 <math>\pm</math> 0.30</b>
			9 > 5 > others	0.2	4.60 $\pm$ 0.65	<u>3.91 <math>\pm</math> 0.66</u>	<b>3.77 <math>\pm</math> 0.31</b>
CIFAR-10	Airplane, automobile, ship, truck	0.4	Not given	NA	12.02 $\pm$ 0.65	<b>10.70 <math>\pm</math> 0.57</b>	NA
			Cat, dog, horse	0.3	10.25 $\pm$ 0.38	<u>9.71 <math>\pm</math> 0.51</u>	10.37 $\pm$ 0.65
			Horse > deer = frog > others	0.25	<u>9.98 <math>\pm</math> 0.53</u>	<b>9.92 <math>\pm</math> 0.42</b>	<u>10.17 <math>\pm</math> 0.35</u>
CIFAR-10	Cat, deer, dog, horse	0.4	Not given	NA	23.78 $\pm$ 1.04	<b>21.13 <math>\pm</math> 0.90</b>	NA
			Bird, frog	0.2	22.00 $\pm$ 0.53	<u>18.83 <math>\pm</math> 0.71</u>	<u>19.88 <math>\pm</math> 0.62</u>
			Car, truck	0.2	22.00 $\pm$ 0.74	<b>20.19 <math>\pm</math> 1.06</b>	21.83 $\pm$ 1.36
20 Newsgroups	alt., comp., misc., rec.	0.56	Not given	NA	14.67 $\pm$ 0.87	<b>13.30 <math>\pm</math> 0.53</b>	NA
			sci.	0.21	14.69 $\pm$ 0.46	<b>13.10 <math>\pm</math> 0.90</b>	13.58 $\pm$ 0.97
			talk.	0.17	14.38 $\pm$ 0.74	<u>12.61 <math>\pm</math> 0.75</u>	13.76 $\pm$ 0.66
			soc. > talk. > sci.	0.1	14.41 $\pm$ 0.76	<b>12.18 <math>\pm</math> 0.59</b>	12.92 $\pm$ 0.51