Partially Linear Additive Gaussian Graphical Models

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June 13th, 2019 @ ICML 2019
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Poster: Partially Linear Additive Gaussian Graphical Models
Thu Jun 13th 06:15 – 09:00 PM @ Pacific Ballroom
Brain Functional Connectivity Analysis

Estimation is distorted by physiological noise [Van Dijk et al., 2012, Goto et al., 2016]. The noise sources are observable e.g. motion, breathing...
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The noise sources are observable e.g. motion, breathing.
→ A general formulation of the effects caused by the noise.

→ Stronger theoretical guarantees compared to methods with hidden variables.
→ **Z** denotes the observed fMRI data, and random variable **G**, the physiological noise.

→ **Z | G = g** follows a Gaussian graphical model [Yang et al., 2015] with a parameter matrix, denoted by **Ω(g)**:

\[
P(Z = z; \Omega(g) \mid G = g) \propto \exp \left\{ \sum_{j=1}^{p} \Omega_{jj}(g)z_j \sum_{j=1}^{p} \sum_{j' > j} \Omega_{jj'}(g)z_jz_{j'} - \frac{1}{2} \sum_{j} z_j^2 \right\}.
\]

→ Parameter matrices are additive:

\[
\Omega(g) := \Omega_0 + R(g).
\]
Model Formulation: $\Omega(g)$

Goals:
- Identifiable parameters
- A general formulation

Assumptions:
- $R(g) = 0$ for any $g$ satisfying $\|g\|$.
- $R(g)$, and $\Omega(g)$ are smooth enough to be recovered by kernel methods.

Existing assumptions:
- $R(g) = 0$ [Van Dijk et al., 2012, Power et al., 2014].
- $E(R(g)) = 0$ [Lee and Liu, 2015, Geng et al., 2018].
Model Formulation: $\Omega(g)$

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Assumptions:
- $R(g) = 0$ for any $g$ satisfying $|g| \leq g^*$.
- $R(g)$, and $\Omega(g)$ are smooth enough to be recovered by kernel methods.

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Parameter Estimation

Log Pseudo Likelihood:

- We summarize the varying effects as \( M_{ij} := x_{ij}^\top \Omega_{i.j} \), where \( x_{ij}^\top \) denotes the \( i^{th} \) row vector of \( x_j \).

\[
\ell_{PL} \left( \{z_i, g_i\}_{i \in [n]} ; R(\cdot), \Omega_0 \right) = \sum_{i=1}^{n} \sum_{j=1}^{p} \left\{ z_{ij} \left( x_{ij}^\top \Omega_{0.j} + M_{ij} \right) - \frac{1}{2} z_{ij}^2 
- \frac{1}{2} \left( x_{ij}^\top \Omega_{0.j} + M_{ij} \right)^2 \right\}.
\]
Parameter Estimation

- Pseudo-Profile Likelihood [Fan et al., 2005]

Suppose that Assumptions are satisfied. Then, for any $\epsilon > 0$, with probability of at least $1 - \epsilon$, there exists $C_4 > 0$, so that $\hat{\Omega}_0$ shares the same structure with the underlying true parameter $\Omega_0^*$, if for some constant $C_5 > 0$,

$$C_5 \sqrt{\frac{\log p}{n}} \geq \lambda \geq \frac{4}{\alpha} C_4 \sqrt{\frac{\log p}{n}},$$

$$r := 4C_2 \lambda \leq \|\Omega_0^*\|_\infty,$$

and $n \geq \left(64C_5 C_2^2 C_3 / \alpha\right)^2 \log p$. 

Parameter Estimation

**Sparsity:** The underlying structure can be recovered with a high probability.

**$\sqrt{n}$ Convergence:** The smallest scale of the non-zero component that the PPL method can distinguish from zero converges to zero at a rate of $\sqrt{n}$. 
Overall Performance

- LR-GGM
- fMRI dataset with control subjects and those with Schizophrenia.
- Diagnosis using the recovered structure by two different methods.
Thank you!


