Bayesian Joint Spike-and-Slab Graphical Lasso

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Joint work with Tyler H. McCormick (UW) and Samuel J. Clark (OSU)

\[ p(x|\mu, \Omega) = \text{Normal}(\mu, \Omega^{-1}) \]

\[ \Omega = \begin{pmatrix}
\omega_{11} & 0 & \omega_{13} & 0 \\
0 & \omega_{22} & \omega_{23} & 0 \\
\omega_{33} & \omega_{34} & \omega_{44}
\end{pmatrix} \]
Contributions

- A new graphical lasso type penalty for learning multiple related graphs.
- Joint graphical lasso (Danaher et al., 2014)
- Reducing bias from over-shrinkage and automatic tuning parameter selection.
- EM algorithm for graphical model (Li and McCormick, 2019)
A new graphical lasso type penalty for learning multiple related graphs.

Joint graphical lasso (Danaher et al., 2014)
A new graphical lasso type penalty for learning **multiple related graphs**.

- Joint graphical lasso (Danaher et al., 2014)
- **Reducing bias** from over-shrinkage and automatic tuning parameter selection.
- EM algorithm for graphical model (Li and McCormick, 2019)
Doubly spike-and-slab joint graphical lasso priors

\[
pen(\{\Omega\}) = \frac{\lambda_0}{2} \sum_g \sum_j |\omega^{(g)}_{jj}| + \lambda_1 \sum_g \sum_{j<k} \frac{|\omega^{(g)}_{jk}|}{\nu_{\delta_{jk}}} + 
\]

spike-and-slab mixture penalties
Doubly spike-and-slab joint graphical lasso priors

\[
\text{pen}(\{\Omega\}) = \frac{\lambda_0}{2} \sum_g \sum_j |\omega^{(g)}_{jj}| + \lambda_1 \sum_g \sum_{j<k} \frac{|\omega^{(g)}_{jk}|}{\nu_{\delta_{jk}}} + \lambda_2 \sum_{j<k} \frac{\tilde{\text{pen}}(\omega_{jk})}{\nu_{\xi^*_{jk}}}
\]

spike-and-slab mixture penalties

Gaussian mixture

Laplace mixture

similarity
Doubly spike-and-slab joint graphical lasso priors

![Graphical Representation]

**Gaussian mixture**

**Laplace mixture**

**Spike-and-slab mixture penalties**

\[
pen(\{\Omega\}) = \frac{\lambda_0}{2} \sum_{g} \sum_{j} |\omega_{jj}^{(g)}| + \lambda_1 \sum_{g} \sum_{j<k} |\omega_{jk}^{(g)}| + \lambda_2 \sum_{j<k} \tilde{pen}(\omega_{jk}) - \log(p(\delta, \xi))
\]

**Fully Bayesian characterization via scale mixture of normal priors**
Fully Bayesian treatment can be expensive. Let’s just find the posterior mode using EM algorithm (Ročková and George, 2014).
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E-step:
- Calculate posterior inclusion probabilities: $p^*_{\delta_{jk}, \xi_{jk}}(j, k)$.
- Impute missing values.
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E-step:
- Calculate posterior inclusion probabilities: $p^*_{\delta,\xi}(j, k)$.
- Impute missing values.

M-step:
- Solve the joint graphical lasso problem with ADMM

$$\{\hat{\Omega}\} = \arg\max_{\Omega}\left\{ \ldots - \sum_{j<k} \lambda_1 \left( \frac{p^*_{0,0}(j, k)}{v_0} + \frac{1 - p^*_{0,0}(j, k)}{v_1} \right) \sum_{g} |\omega_{jk}^{(g)}| \\
- \sum_{j<k} \lambda_2 \left( \frac{1 - p^*_{1,1}(j, k)}{v_0} + \frac{p^*_{1,1}(j, k)}{v_1} \right) \text{pen}(\omega_{jk}) \right\}$$

- Maximize over $\pi_\delta$ and $\pi_\xi$ have closed form solutions
Dynamic posterior exploration

Class 1

\[ n_1 = n_2 = 150, \, p = 100 \]
Dynamic posterior exploration

Class 1

Class 2

Class 1 : F-norm=6.8

Class 2 : F-norm=23.8

Class 1

Class 2

Class 1 : F-norm=1.8

Class 2 : F-norm=6.6

\[ n_1 = n_2 = 150, \; p = 100 \]
Verbal Autopsy and more...come to poster tonight!

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Questionnaire item</th>
</tr>
</thead>
<tbody>
<tr>
<td>belly pain</td>
<td>For how long before death did [name] have belly pain? [days]</td>
</tr>
<tr>
<td>protruding belly</td>
<td>For how long before death did [name] have a protruding belly? [days]</td>
</tr>
<tr>
<td>mass belly</td>
<td>For how long before death did [name] have a mass in the belly [days]</td>
</tr>
<tr>
<td>headaches</td>
<td>For how long before death did [name] have headaches? [days]</td>
</tr>
<tr>
<td>stiff neck</td>
<td>For how long before death did [name] have stiff neck? [days]</td>
</tr>
<tr>
<td>unconsciousness</td>
<td>For how long did the period of loss of consciousness last? [days]</td>
</tr>
</tbody>
</table>

Get in touch? @zrichardli
References

