Calibrated Approximate Bayesian Inference

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Approximation schemes are key to Bayesian inference.

Does a $\alpha$ level approximate credible set have the right coverage?

Let $C_y$ and $\tilde{C}_y$ be the exact and approximate $\alpha$ level credible set,

$$
\alpha = E_\pi(1_{\phi \in C_y}) = \int_{\Omega} 1_{\phi \in C_y} \pi(\phi | y) d\phi.
$$

$$
\alpha = E_{\tilde{\pi}}(1_{\theta \in \tilde{C}_y}) = \int_{\Omega} 1_{\theta \in \tilde{C}_y} \tilde{\pi}(\theta | y) d\theta.
$$
We also define

$$b(y) = \Pr(\phi \in \tilde{C}_Y | Y = y) = \int_{\Omega} \mathbf{1}_{\phi \in \tilde{C}_y} \pi(\phi | y) d\phi$$

be the operational coverage $\tilde{C}_Y$ achieves.

We want to estimate $b(y_{obs})$, the true Bayesian coverage of the approximate credible set, as it measures the reliability of approximation at the observed data.
**Method**

- **Regression approach** Let \( \{\phi_i, y_i\}^M_{i=1} \) be samples from the generative model \( \pi(\phi)p(y|\phi) \), let \( \tilde{C}_{y_i} \) be an approximate credible set for \( y_i \), and \( c_i = 1_{\phi_i \in \tilde{C}_{y_i}} \). Conditional on \( y_i \),

\[
c_i \sim \text{Bernoulli}(b(y_i)), \quad b(y_i) = \Pr(\phi_i \in \tilde{C}_{Y_i}|Y_i = y_i)
\]

- **Weighted-sample approach** Estimate \( b(y) \) by first approximately sampling from the exact posterior using Annealed Importance Sampling algorithm. This leverages our ability to draw samples from the approximate posterior \( \tilde{\pi}(\phi|y_{obs}) \) as a good starting point for the AIS iteration.
We approximate the free boundary condition likelihood by a toroidal boundary condition likelihood.
Figure: Left: AIS Right: Regression