
A Statistical Approach to Texture Description of Medical Images: A Preliminary Study

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Abstract

The article deals with the problem of texture description. It presents a statistical approach. Specifically it introduces the use of first- and second-order statistics on texture color spaces. At the end we also give estimations of computational time complexities of calculations of parameters presented in this article and describe our experience on two application domains. This study is a preliminary preparation for application of these methods on medical images.

1. Introduction

Texture is a very commonly used term in computer vision. We all recognize texture when we see it, but it is very difficult to define it precisely. We can find many texture definitions in literature. Coggins (1982) for example collected a catalogue of texture definitions found in computer vision literature. Briefly we can regard an image texture as *a function of spatial variation in pixel values*.

The above is a computer vision viewpoint of texture perception, but let us take a quick look at psychophysics viewpoint of the matter. The question is "When is a texture pair discriminable (by human), given that they have the same brightness, contrast and color?" Julesz (1973) has studied human texture perception in the context of its discrimination. His work concentrates on the spatial statistics of texture gray levels. To summarize Julesz's work, we need to define the first- and second-order spatial statistics.

First-order statistics are calculated from the probability of observing a particular pixel value at a randomly chosen location in the image. They depend only on individual pixel values and not on

interaction of neighboring pixel values. The mean of image gray intensity is an example of the first-order statistic.

Second-order statistics are calculated from the probability of observing a pair of pixel values in the image that are some vector \vec{d} apart. Note that second-order statistics become first-order if $\vec{d} = (0, 0)$.

Julesz states that two textures are not preattentively (at first sight) discriminable by human, if their second order statistics are identical. It also means that identity of first-order statistics can not assure us, that those two textures won't be discriminable by human.

Therefore, if we are to make a system that discriminates textures at least as good as human does, we should take into account both first- and second-order statistics.

The paper is organized as follows. In the second section we define histogram and first order statistics on histogram function. In the third section we define cooccurrence function and second order statistics on cooccurrence function. The fourth section comprehends computational time complexity estimations of functions described in second and third section. In the fifth and sixth section we demonstrate the usage of solutions presented in this article on two application domains. The last section is a conclusion with some ideas for improvements and further work.

2. First-order statistics

First-order statistics are quite straightforward. They are computed from a function that measures the probability of a certain pixel occurring in an image. This function is also known as histogram.

Histogram on gray scale images is defined as follows

$$H(g) = \frac{n_g}{N}; g = 0, 1, \dots, G-1 \quad (1)$$

where N is the number of all pixels in an image, G is the number of gray levels and n_g is the number of pixels of value g in an image.

What about color images? Let's suppose we use common RGB color space representation. In that case we define histogram on color images like this

$$\chi(\vec{c}) = \frac{n_{\vec{c}}}{N}; \vec{c} \in \mathcal{L} \quad (2)$$

where N is the number of all pixels in an image, \mathcal{L} denotes a three dimensional color space $L \times L \times L$ where L is the number of intensity levels per color channel and $n_{\vec{c}}$ is the number of pixels of value \vec{c} in an image.

As we can see H and χ are probability functions of pixel values, therefore we can characterize their properties with a set of statistical parameters (also called first-order statistics). Below is a list of such parameters, where P_n represents n -th parameter of H and Π_m denotes m -th parameter of χ . Note that some Π_m are vectors from \mathcal{L} . For example mean (average color) of an image must also be a color, therefore a vector. But on the other hand some Π_m are scalars. For example standard deviation is a measure of deviation in the data, therefore not a vector. This is also obvious arithmetically, if you just calculate the parameters.

Mean (average brightness / color)

$$P_1 = \sum_{g=0}^{G-1} gH(g)$$

$$\vec{\Pi}_1 = \sum_{\vec{c} \in \mathcal{L}} \vec{c}\chi(\vec{c})$$

Standard deviation (image contrast)

$$P_2 = \sqrt{\sum_{g=0}^{G-1} (g - P_1)^2 H(g)}$$

$$\Pi_2 = \sqrt{\sum_{\vec{c} \in \mathcal{L}} (\vec{c} - \vec{\Pi}_1)^2 \chi(\vec{c})}$$

Skew

$$P_3 = \frac{\sum_{g=0}^{G-1} (g - P_1)^3 H(g)}{P_2^3}$$

$$\vec{\Pi}_3 = \frac{\sum_{\vec{c} \in \mathcal{L}} (\vec{c} - \vec{\Pi}_1)^3 \chi(\vec{c})}{\Pi_2^3}$$

Energy

$$P_4 = \sum_{g=0}^{G-1} H^2(g)$$

$$\Pi_4 = \sum_{\vec{c} \in \mathcal{L}} \chi^2(\vec{c})$$

Entropy

$$P_5 = - \sum_{g=0}^{G-1} H(g) \log_2(H(g))$$

$$\Pi_5 = - \sum_{\vec{c} \in \mathcal{L}} \chi(\vec{c}) \log_2(\chi(\vec{c}))$$

1st moment

It is the same parameter as mean $P_6 = P_1$ and $\vec{\Pi}_6 = \vec{\Pi}_1$.

2nd moment

$$P_7 = \sum_{g=0}^{G-1} (g - P_6)^2 H(g)$$

$$\Pi_7 = \sum_{\vec{c} \in L \times L \times L} (\vec{c} - \vec{\Pi}_6)^2 \chi(\vec{c})$$

3rd moment

$$P_8 = \sum_{g=0}^{G-1} (g - P_6)^3 H(g)$$

$$\vec{\Pi}_8 = \sum_{\vec{c} \in \mathcal{L}} (\vec{c} - \vec{\Pi}_6)^3 \chi(\vec{c})$$

4th moment

$$P_9 = \sum_{g=0}^{G-1} (g - P_6)^4 H(g)$$

$$\Pi_9 = \sum_{\vec{c} \in \mathcal{L}} (\vec{c} - \vec{\Pi}_6)^4 \chi(\vec{c})$$

3. Second-order statistics

As it was already mentioned above, second-order statistics operate on probability function, that measures the probability of a pair of pixel values occurring some vector \vec{d} apart in the image. This probability function is also called *cooccurrence matrix*, since it measures the probability of cooccurrence of two pixel values. Since cooccurrence matrix becomes "weird" in color space, let us use the term *cooccurrence function* (CF) instead.

Let us now define CF on the gray scale image. Suppose we have a $M \times N$ image with color space of G gray levels. Given a certain displacement vector $+\vec{d} = (dx, dy)$

the CF looks like this

$$C_{+\vec{d}}(i, j) = \frac{\left\{ \left\{ \begin{array}{l} (r, s) = i, (t, v) = j, \\ ((r, s), (t, v)); (r, s), (t, v) \in M \times N, \\ (t, v) = (r + dx, s + dy) \end{array} \right\} \right\}}{(M - dx)(N - dy)} \quad (3)$$

Note that so defined CF is not symmetrical in terms of $C_{+\vec{d}}(i, j) \neq C_{+\vec{d}}(j, i)$. A symmetrical CF can be computed using the formula

$$C_{\vec{d}}(i, j) = C_{+\vec{d}}(i, j) + C_{-\vec{d}}(i, j) \quad (4)$$

Suppose we have a set of displacement vectors $\Delta = \{\vec{d}_1, \vec{d}_2, \dots, \vec{d}_n\}$. The final CF C used in second-order statistics is then the average at certain gray level (i, j) over all CFs based on displacement vectors from Δ .

$$C(i, j) = \text{avg} \left\{ C_{\vec{d}_k}(i, j); \vec{d}_k \in \Delta \right\} \quad (5)$$

CFs on color images can be defined in a similar way. Suppose we have a $M \times N$ image with an ordinary three dimensional RGB color space. Let's say that each color channel has L values. Therefore each image pixel is a vector from \mathcal{L} which is a $L \times L \times L$ space. Given a certain displacement vector $+\vec{d} = (dx, dy)$ we get the CF

$$\Gamma_{+\vec{d}}(\vec{c}_1, \vec{c}_2) = \frac{\left\{ \left\{ \begin{array}{l} (r, s) = \vec{c}_1, (t, v) = \vec{c}_2, \\ ((r, s), (t, v)); (r, s), (t, v) \in M \times N, \\ (t, v) = (r + dx, s + dy) \end{array} \right\} \right\}}{(M - dx)(N - dy)} \quad (6)$$

Notice the difference: $\Gamma_{+\vec{d}}$ is a function of two three dimensional vectors (two RGB colors).

And again we can define a symmetrical $\Gamma_{\vec{d}}$

$$\Gamma_{\vec{d}}(\vec{c}_1, \vec{c}_2) = \Gamma_{+\vec{d}}(\vec{c}_1, \vec{c}_2) + \Gamma_{-\vec{d}}(\vec{c}_1, \vec{c}_2) \quad (7)$$

The final CF Γ used in second-order statistics on color images is the average at certain RGB color pair (\vec{c}_1, \vec{c}_2) over all CFs based on displacement vectors from Δ .

$$\Gamma(\vec{c}_1, \vec{c}_2) = \text{avg} \left\{ \Gamma_{\vec{d}_k}(\vec{c}_1, \vec{c}_2); \vec{d}_k \in \Delta \right\} \quad (8)$$

The next important issue, which is important on color as well as on gray scale images, is: Exactly which displacement vectors \vec{d} are in Δ ? There's no definite answer, but there are some clues that came from practice. Mostly used set of distance vectors is $\Delta = \{(1, 0), (1, 1), (0, 1), (-1, 1)\}$. This actually means, that we are considering four neighboring pixels. It is empirically proven that this is a rather successful set, but there is no proof that this is a universally good

set. This set holds pretty good information on local (short distance) pixel dependancies, but lacks of global (long distance) pixel dependancies. The more we extend Δ with vectors that represent larger distances, the more local information we loose. This loss of information happens because of the averaging in the last step of calculation of CF. There is obviously a trade off between local and global information that CF could hold.

In order to describe CF C of gray scale image Haralick (1973) proposed a set of second-order statistics. These statistics and their generalized versions (statistics for Γ) are listed below in this article. In the list below F_n represents n-th statistics of C and Φ_m denotes m-th statistic of Γ .

Energy

$$F_1 = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} C^2(i, j)$$

$$\Phi_1 = \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \Gamma^2(\vec{c}_1, \vec{c}_2)$$

Entropy

$$F_2 = - \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} C(i, j) \log_2(C(i, j))$$

$$\Phi_2 = - \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \Gamma(\vec{c}_1, \vec{c}_2) \log_2(\Gamma(\vec{c}_1, \vec{c}_2))$$

Contrast

$$F_3 = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - j)^2 C(i, j)$$

$$\Phi_3 = \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} (\vec{c}_1 - \vec{c}_2)^2 \Gamma(\vec{c}_1, \vec{c}_2)$$

Homogeneity

$$F_4 = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{C(i, j)}{1 + (i - j)^2}$$

$$\Phi_4 = \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \frac{\Gamma(\vec{c}_1, \vec{c}_2)}{1 + (\vec{c}_1 - \vec{c}_2)^2}$$

Correlation of C

Let us first define marginal probabilities of C

$$C'(i) = \sum_{j=0}^{G-1} C(i, j) \quad C''(j) = \sum_{i=0}^{G-1} C(i, j)$$

$C' = C''$ since C is symmetrical. Notice what is happening here: A probability of a certain $C'(i)$ is calculated so that probabilities over all possible pairs of i are summed. What we get is a probability of pixel value i itself. That is actually a histogram of an image. The same holds for C'' as well.

And let $\mu_{C'}$ and $\mu_{C''}$ be the means of marginal probabilities

$$\mu_{C'} = \sum_{i=0}^{G-1} iC'(i) \quad \mu_{C''} = \sum_{j=0}^{G-1} jC''(j)$$

Note that $\mu_{C'} = \mu_{C''}$. If C' and C'' are histograms of an image and $\mu_{C'}$ and $\mu_{C''}$ are their means, we can easily see what $\mu_{C'}$ and $\mu_{C''}$ represent. They simply represent the average pixel value of an image.

Further let $\sigma_{C'}$ and $\sigma_{C''}$ be the standard deviations of C' and C''

$$\sigma_{C'} = \sqrt{\sum_{i=0}^{G-1} (i - \mu_{C'})^2 C'(i)}$$

$$\sigma_{C''} = \sqrt{\sum_{j=0}^{G-1} (j - \mu_{C''})^2 C''(j)}$$

Obviously $\sigma_{C'} = \sigma_{C''}$. Now we are ready to define a measure of correlation for C , that is

$$F_5 = \frac{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu_{C'}) (j - \mu_{C''}) C(i, j)}{\sigma_{C'} \sigma_{C''}}$$

Correlation of Γ

Let us first define marginal probabilities of Γ

$$\Gamma'(\vec{c}_1) = \sum_{\vec{c}_2 \in \mathcal{L}} \Gamma(\vec{c}_1, \vec{c}_2) \quad \Gamma''(\vec{c}_2) = \sum_{\vec{c}_1 \in \mathcal{L}} \Gamma(\vec{c}_1, \vec{c}_2)$$

$\Gamma' = \Gamma''$ since Γ is symmetrical. Γ' and Γ'' are also histograms of an image, as we explained it before in gray scale case.

And let $\mu_{\Gamma'}$ and $\mu_{\Gamma''}$ be the means of marginal probabilities

$$\vec{\mu}_{\Gamma'} = \sum_{\vec{c}_1 \in \mathcal{L}} \vec{c}_1 \Gamma'(\vec{c}_1) \quad \vec{\mu}_{\Gamma''} = \sum_{\vec{c}_2 \in \mathcal{L}} \vec{c}_2 \Gamma''(\vec{c}_2)$$

Note that now $\vec{\mu}_{\Gamma'}$ and $\vec{\mu}_{\Gamma''}$ are vectors (remember they represent average colors) and it still holds that $\vec{\mu}_{\Gamma'} = \vec{\mu}_{\Gamma''}$.

Further let $\sigma_{\Gamma'}$ and $\sigma_{\Gamma''}$ be the standard deviations of Γ' and Γ''

$$\sigma_{\Gamma'} = \sqrt{\sum_{\vec{c}_1 \in \mathcal{L}} (\vec{c}_1 - \vec{\mu}_{\Gamma'})^2 \Gamma'(\vec{c}_1)}$$

$$\sigma_{\Gamma''} = \sqrt{\sum_{\vec{c}_2 \in \mathcal{L}} (\vec{c}_2 - \vec{\mu}_{\Gamma''})^2 \Gamma''(\vec{c}_2)}$$

Obviously $\sigma_{\Gamma'} = \sigma_{\Gamma''}$. And finally we are able to define a measure of correlation for Γ , that is

$$\Phi_5 = \frac{\sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} (\vec{c}_1 - \vec{\mu}_{\Gamma'}) (\vec{c}_2 - \vec{\mu}_{\Gamma''}) \Gamma(\vec{c}_1, \vec{c}_2)}{\sigma_{\Gamma'} \sigma_{\Gamma''}}$$

Variance

$$F_6 = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu_{C'})^2 C(i, j)$$

$$\Phi_6 = \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} (\vec{c}_1 - \vec{\mu}_{\Gamma'})^2 \Gamma(\vec{c}_1, \vec{c}_2)$$

Mean of pair-sums

First define a new probability function. A function that represents a probability of a particular pair-sum of pixel values. Let's denote it with S in gray scale and Θ in color space.

$$S(x) = \sum_{i+j=x} C(i, j) \quad \Theta(\vec{y}) = \sum_{\vec{c}_1 + \vec{c}_2 = \vec{y}} \Gamma(\vec{c}_1, \vec{c}_2)$$

Now we can calculate their means

$$F_7 = \sum_{x=0}^{2(G-1)} xS(x) \quad \vec{\Phi}_7 = \sum_{\vec{y} \in \hat{\mathcal{L}}} \vec{y} \Theta(\vec{y})$$

where $\hat{\mathcal{L}}$ is a $2L \times 2L \times 2L$ space.

Variance of pair-sums

$$F_8 = \sum_{x=0}^{2(G-1)} (x - F_7)^2 S(x)$$

$$\Phi_8 = \sum_{\vec{y} \in \hat{\mathcal{L}}} (\vec{y} - \vec{\Phi}_7)^2 \Theta(\vec{y})$$

Entropy of pair-sums

$$F_9 = - \sum_{x=0}^{2(G-1)} S(x) \log(S(x))$$

$$\Phi_9 = - \sum_{\vec{y} \in \hat{\mathcal{L}}} \Theta(\vec{y}) \log_2 \Theta(\vec{y})$$

Variance of pair-differences

Like we defined a probability function for pair-sums we can also define a probability function for all possible pair-differences. We will denote it with D in gray scale and Λ in color space.

$$D(x) = \sum_{|i-j|=x} C(i, j)$$

$$\Lambda(\vec{y}) = \sum_{\substack{|r_1 - r_2| = r \\ |g_1 - g_2| = g \\ |b_1 - b_2| = b}} \Gamma(\vec{c}_1, \vec{c}_2)$$

where $\vec{c}_1 = (r_1, g_1, b_1)$, $\vec{c}_2 = (r_2, g_2, b_2)$ and $\vec{y} = (r, g, b)$. Now it is possible to define both means of pair-differences

$$\mu_D = \sum_{x=0}^{G-1} xD(x) \quad \bar{\mu}_\Lambda = \sum_{\vec{y} \in \mathcal{L}} \vec{y}\Lambda(\vec{y})$$

And finally we are able to calculate variances

$$F_{10} = \sum_{x=0}^{G-1} (x - \mu_D)^2 D(x)$$

$$\Phi_{10} = \sum_{\vec{y} \in \mathcal{L}} (\vec{y} - \bar{\mu}_\Lambda)^2 \Lambda(\vec{y})$$

Entropy of pair-differences

$$F_{11} = - \sum_{x=0}^{G-1} D(x) \log(D(x))$$

$$\Phi_{11} = - \sum_{\vec{y} \in \mathcal{L}} \Lambda(\vec{y}) \log_2 \Lambda(\vec{y})$$

Information measure of correlation 1

$$F_{12} = \frac{F_2 + \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} C(i, j) \log_2(C'(i)C''(j))}{-\sum_{i=0}^{G-1} C'(i) \log_2(C'(i))}$$

$$\Phi_{12} = \frac{\Phi_2 + \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \Gamma(\vec{c}_1, \vec{c}_2) \log_2(\Gamma'(\vec{c}_1)\Gamma''(\vec{c}_2))}{-\sum_{\vec{c}_1 \in \mathcal{L}} \Gamma'(\vec{c}_1) \log_2(\Gamma'(\vec{c}_1))}$$

Information measure of correlation 2

$$X = -2 \left(- \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} C'(i)C''(j) \log_2(C'(i)C''(j)) + F_2 \right)$$

$$F_{13} = \sqrt{|1 - e^X|}$$

$$Y = -2 \left(- \sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \Gamma'(\vec{c}_1)\Gamma''(\vec{c}_2) \log_2(\Gamma'(\vec{c}_1)\Gamma''(\vec{c}_2)) + \Phi_2 \right)$$

$$\Phi_{13} = \sqrt{|1 - e^Y|}$$

4. Time Complexity

It is important to analyse computational time complexity of these functions. As they may be simply written in math formulae they tend to be quite algorithmically time and space consuming.

In the following subsections we will use: M and N to denote a $M \times N$ image, G for gray scale channel and L for the size of color channel.

4.1. Time Complexity of Histogram function

What we do here is run over all image pixels and count them. The time complexity of this operation is $O(mn)$. That is usually feasible for all normal image sizes in gray scale and color space.

4.2. Time Complexity of First-order Statistics

Note that all first-order statistics require the sum over all points of gray/color space. That is the most time consuming part of them.

In gray scale space

$$\sum_{g=0}^{G-1} \text{Function_of}(H(g)) \quad (9)$$

which gives time complexity $O(n)$. Considering usual values for G the computational complexity is acceptable with current hardware speed.

In color space we have

$$\sum_{\vec{c} \in \mathcal{L}} \text{Function_of}(\chi(\vec{c})) \quad (10)$$

which requires L^3 loops. Time complexity is $O(n^3)$. Now imagine, we use usual quantization of a signal per color channel ($L = 256$), then L^3 is approximately 16.7 million! Therefore, it is recommended to use a rougher quantization, say $L = 16$.

4.3. Time Complexity of CF

Remember how we calculate CF. First we count all pair pixels that are some vector $+\vec{d}$ apart. We can do that in approximately $M \cdot N$ steps. Note the size of the CF space. It is $G \times G$ for gray scale and $L \times L \times L \times L \times L \times L$ for color images! Then we calculate a symmetrical CF, which requires summing over all gray/color space. That means G^2 summations in gray scale and L^6 summations in color space. And lastly we average CFs for all displacement vectors. Which again requires G^2 and L^6 operations respectively. Time complexity in gray scale space is $O(n^2)$ and $O(n^6)$ in color space. Needless to say, it is recommended to use a rougher quantizations than 256 values per channel.

4.4. Time Complexity of Second-order Statistics

The most time consuming part of second-order statistics is obviously summing over all points from its space.

In gray scale space this part looks like this

$$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \text{Function_of}(C(i, j)) \quad (11)$$

which requires G^2 operations. Time complexity is $O(n^2)$.

And in color space we have

$$\sum_{\vec{c}_1 \in \mathcal{L}} \sum_{\vec{c}_2 \in \mathcal{L}} \text{Function_of}(\Gamma(\vec{c}_1, \vec{c}_2)) \quad (12)$$

which requires L^6 operations. Time complexity is $O(n^6)$.

5. Application: Droplets

In this section we describe one application of the approach presented in previous sections. The task is to discriminate two chemical suspensions on the basis of their images of dried drops.

For our experiment, the first suspension was 0.1% of Al_2O_3 in H_2O with acidity of Ph 6, the second suspension was also 0.1% of Al_2O_3 in H_2O but with acidity of Ph 10.

5.1. Procedure

The procedure was as follows. First the suspensions were prepared. The second task was to make drop samples of the suspensions. Drop samples were made with a sterile syringe on a clean microscopic slide. 8 drops per slide were made (4 from Ph 6 suspension and 4 from Ph 10 suspension). The average diameter of a drop was about 5 mm. At this point the drops were dried before further processing. To take pictures of dried drops, we used a dark field microscope with a mounted digital camera. A relatively small magnification factor of 20 was used. Each drop separately was observed under microscope and photographed (Figure 1, Figure 2). 12 slides of drops were sampled and we got 45 images of Ph 6 and 37 images of Ph 10 suspension. Some images had to be eliminated due to inappropriate size of a drop (some drops were too big to fit on the image).

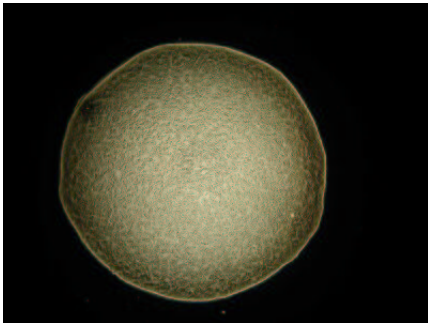


Figure 1. An image of Ph 6 droplet



Figure 2. An image of Ph 10 droplet

The size of images was 2048×1536 . The Color depth was 24 bit.

In the next step we extracted an inner rectangular area of each drop in order to obtain a discriminating pattern (Figure 3, Figure 4).

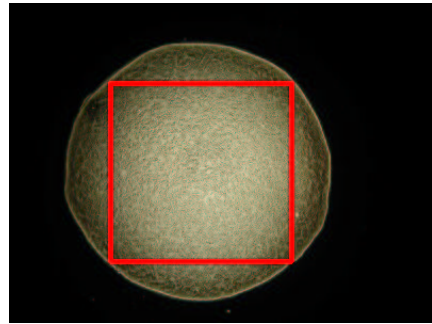


Figure 3. An area marked for extraction

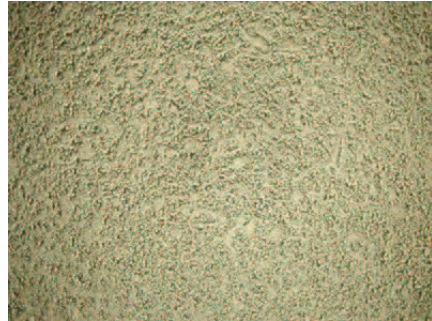


Figure 4. An extracted area

So preprocessed images were used in the next phase of the experiment, where we calculated the histogram and CF parameters. The calculation of parameters was performed by our program *ImageProcessor* (IP), that implements all the parameters described in Sec. 2 and 3. The output of IP was a list of samples (images) described by histogram and CF parameters. Since we know each sample's origin (Ph 6 or Ph 10), i.e. its class, we tried to solve this classification problem in order to verify the quality of parameters.

The task is to classify samples into one of two classes Ph 6 and Ph 10 using histogram and CF parameters. We had 45+37 samples, which means the default classification error of 45%. The aim is to minimize the classification error. We used See5 (C5.0 algorithm) (Quinlan, 1993), that builds decision trees. Each experiment was 10 fold cross validated.

5.2. Results

In the first experiment we tried the parameters on gray scale images. Each image was color-quantized to 256 levels for histogram and 16 levels for CF parameters. The classification error on gray scale images was $5.0\% \pm 2.8\%$ (Table 1a), written in the form of mean \pm standard error.

In the second experiment we tried parameters on color images. Each image was color-quantized to 16 levels per channel for histogram and 8 levels per channel for CF parameters. The classification error on color images was $1.3\% \pm 1.3\%$ (Table 1b).

Table 1. The results of see5 on (a) gray scale images and (b) color images

Fold	Decision Tree		Fold	Decision Tree	
	Size	Errors		Size	Errors
0	3.0	12.5%	0	2.0	0.0%
1	3.0	0.0%	1	2.0	0.0%
2	3.0	0.0%	2	2.0	0.0%
3	3.0	12.5%	3	2.0	0.0%
4	3.0	0.0%	4	2.0	12.5%
5	3.0	0.0%	5	2.0	0.0%
6	3.0	25.0%	6	2.0	0.0%
7	3.0	0.0%	7	2.0	0.0%
8	3.0	0.0%	8	2.0	0.0%
9	3.0	0.0%	9	2.0	0.0%
Mean	3.0	5.0%	Mean	2.0	1.3%
SE	0.0	2.8%	SE	0.0	1.3%

The results prove the information quality of parameters. The result on color images is better than on gray scale ones, which shows the importance of color in patterns. Notice that, though we used rougher quantization on color images, the result is better than on gray scale images.

6. Application: GDV

In the second application we tested parameters on GDV (Gas Discharge Visualisation) images. GDV images are obtained from "BEO GDV Camera by Dr. Korotkov" (Korotkov, 1998). This digital camera records images of gas discharge effect. An object of consideration is grounded and exposed to high-voltage and high-frequency field. These fields cause ionisation of gas around the object. At certain voltage threshold gas discharges which is seen as a flash of light. This light is captured with camera. Camera

captures 320×240 gray scale images. Camera comes with software GDV Analysis (Korotkov, 1998), that also generates parametric image description (Table 2). GDV Analysis uses different approach to parametrization of images and computes the following parameters: area and relative area of corona, amount of deleted noise, fractality (shape) of corona and fractal dimension, brightness and brightness deviation, number of fragments, average area of fragments and its deviation. The results from IP parameters were compared with results from GDV Analysis.

Table 2. A list of GDV parameters

Area of GDV-gram
Noise deleted from image
Form coefficient I
Fractal dimension
Brightness coefficient
Brightness deviation
of separated fragments
Average area per fragment
Deviation of fragments' areas
Relative area
Rel. coef. of glow inside inner oval
Rel. coef. of glow for 25% area
Rel. coef. of glow for 50% area
Rel. coef. of glow for 75% area
Rel. coef. of glow for 100% area
of sectors
Areas in sectors
Quantile coefficient for the sectors

6.1. Metallic Cylinder (temperature)

We captured two sets of GDV images of metallic cylinder.



Figure 5. Metallic cylinder at $21^{\circ}C$

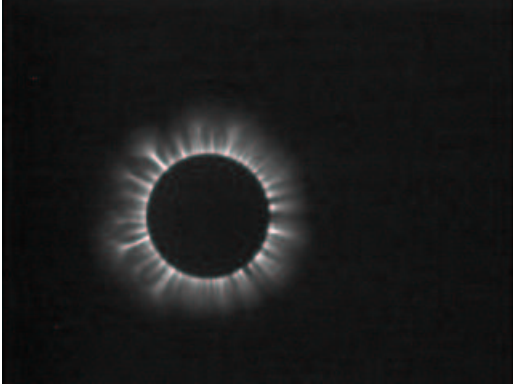


Figure 6. Metallic cylinder at $-20^{\circ}C$



Figure 8. Cylinder at 15kV



Figure 9. Cylinder at 17kV

The first class (S) of 30 images was captured at temperature of $21^{\circ}C$. The second class (H) of 20 images was captured at $-20^{\circ}C$. A two-class problem with 40% default classification error. With IP we calculated parametric description of each image. Then we used see5 with 10 fold cross validation to classify parametric samples. The classification error was $2.0\% \pm 2.0\%$. The classification error with GDV Analysis was $6.0\% \pm 3.1\%$.

6.2. Metallic Cylinder (voltage)

Later we performed another experiment with metallic cylinder. We captured three sets of GDV images at three different voltages.



Figure 7. Cylinder at 13.4kV

The first class (S) of 30 images was captured at 13.4kV. The second class (J) of 20 images was captured at 15kV. The third class ($J3$) of 20 images was captured at 17kV. A three-class problem with 57% default classification error. With IP we calculated parametric description of each image. Then we used see5 with 10 fold cross validation to classify parametric samples. The classification error was $4.3\% \pm 2.3\%$. The classification error with GDV Analysis was $5.7\% \pm 3.2\%$.

6.3. Metallic Cone

Here we captured two sets of GDV images of metallic cone at different voltages.



Figure 10. Metallic cone at 13.4kV



Figure 11. Metallic cone at 15kV

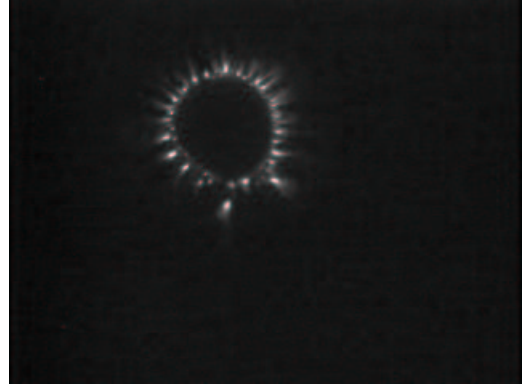


Figure 12. Forefinger at 33.2°C

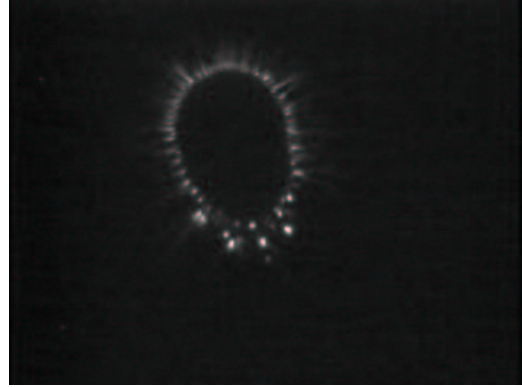


Figure 13. Forefinger at 40.0°C

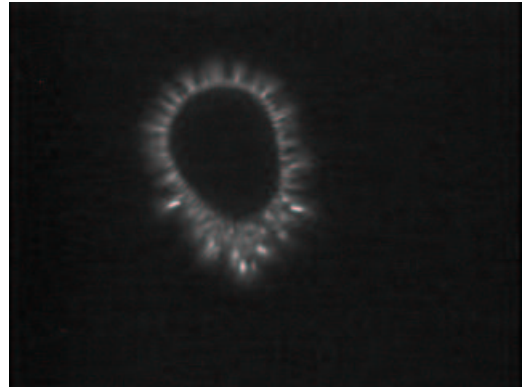


Figure 14. Forefinger at 20.0°C

The first class (S) of 10 images was captured at 13.4kV. The second class (J) of 20 images was captured at 15kV. A two-class problem with 33% default classification error. With IP we calculated parametric description of each image. Then we used see5 with 10 fold cross validation to classify parametric samples. The classification error was $6.7\% \pm 4.4\%$. The classification error with GDV Analysis was $3.3\% \pm 3.3\%$.

6.4. Forefinger

We also tested parameters on GDV images of forefinger. We captured three sets of GDV images at three different forefinger temperatures.

The first class (S) of 30 images was captured at 33.2°C. The second class (G) of 10 images was captured at 40.0°C. The third class (H) of 10 images was captured at 20.0°C. A three-class problem with 40% default classification error. With IP we calculated parametric description of each image. Then we used see5 with 10 fold cross validation to classify parametric samples. The classification error was $14.0\% \pm 4.3\%$. The classification error with GDV Analysis was $18.0\% \pm 5.5\%$.

7. Conclusion

It is possible to describe a certain texture statistically. The quality of description depends on complexity of statistics, in other words, on computational time and space. Histogram alone may be too simple to give a good description, whereas CFs could do the job. The use of CFs makes sense also because they can compete with humans (Julesz & Gilbert & Shepp & Firsch, 1973). Anyway, further work could continue in two directions:

- To find new better second-order statistics.
- To try statistics of higher orders.

It would also be helpful, if we found a computationally less expensive representation of CFs. Here is a potential solution. Generally we have a relatively huge color space with relatively small number of points in it. For example: Say we have a 1024×768 ($M = 1024$, $N = 768$) color image, with 256 values per color channel ($L = 256$). Calculating its CF would require more than $256^6 \doteq 2.8 \cdot 10^{14}$ steps. But we know that we have at most $1024 \cdot 768 \doteq 7.8 \cdot 10^5$ points in that space. It would be much faster, if we store the relevant points and their values (probabilities) and then scan through points rather than through the whole space.

In section 6 we compared histogram and CF parameters with GDV parameters by measuring classification accuracy of C5.0. This comparison shows better accuracy using histogram and CF parameters in 3 of 4 cases, which is surprising considering that GDV parameters are designed specially for the problem of GDV image description.

This is a preliminary study of image analysis. The aim is to use these methods in medical image analysis. We plan to apply our algorithms on the analysis of pulmonary scintigraphic images (diagnosis of lungs) and myocardical scintigraphic images (diagnosis of ischaemic heart disease).

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