On Computing Optimal Tree Ensembles

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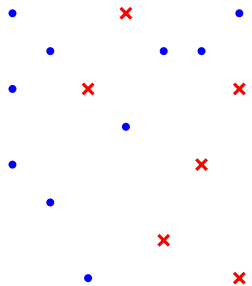

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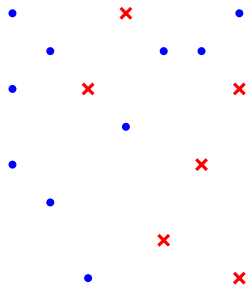
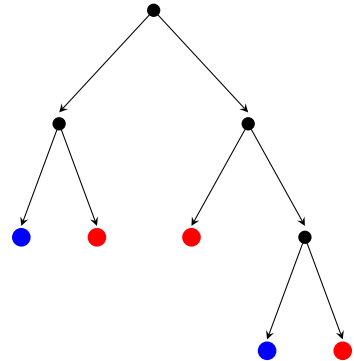
Task: Compute a decision tree of minimal size that classifies each example correctly.
(no misclassifications allowed)





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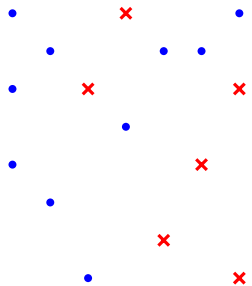

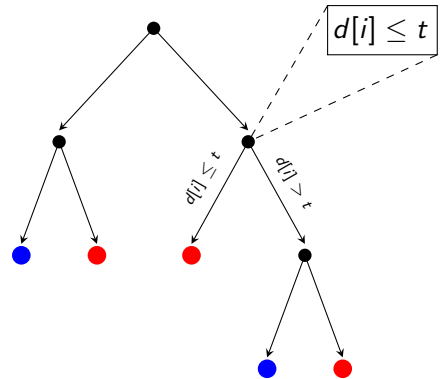

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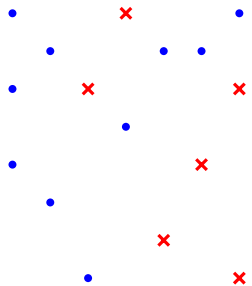

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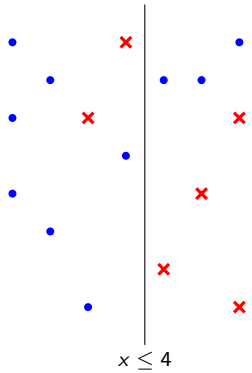

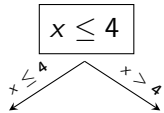

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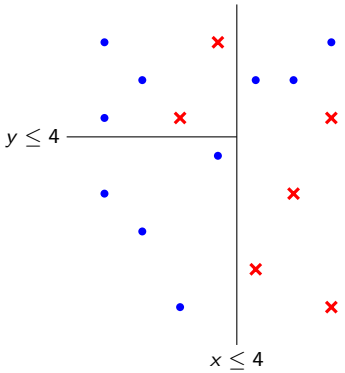

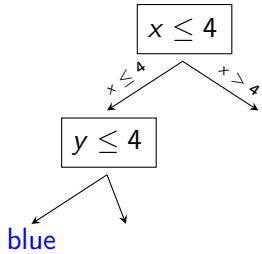

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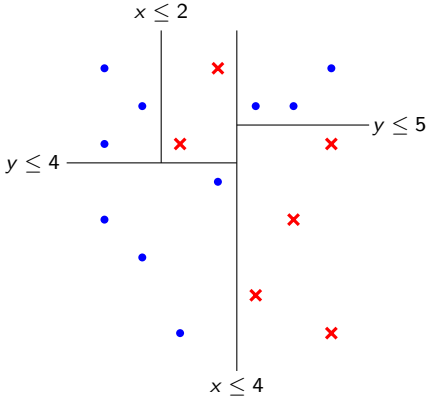

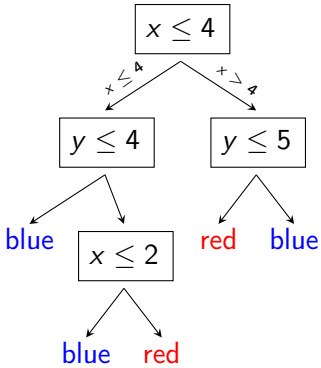

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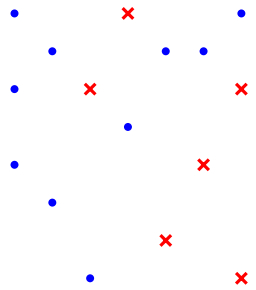

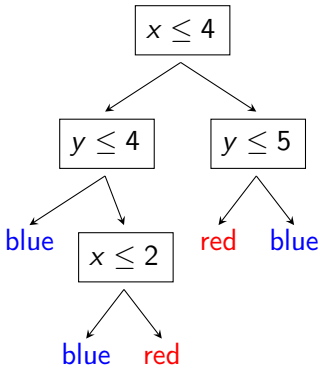

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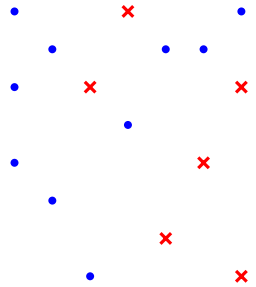

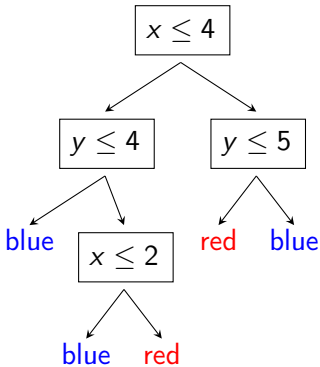

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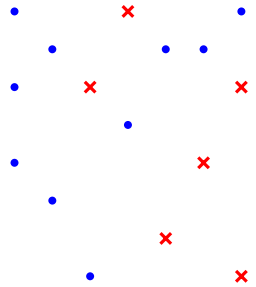

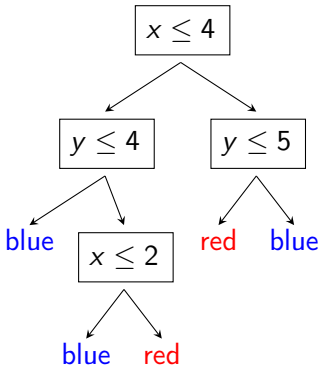

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Theorem (*Our Work*)

An optimal decision tree can be computed in $\mathcal{O}((6 \cdot \delta D s)^s sn)$ time.

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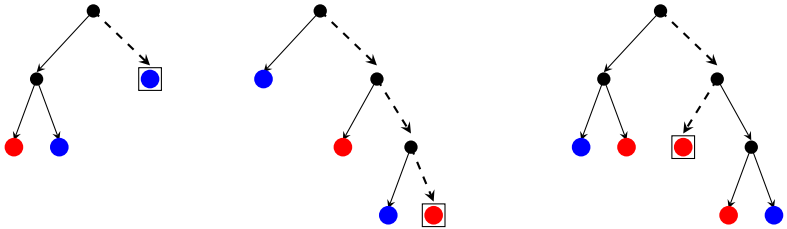
Tree Ensembles: Majority vote of ℓ decision trees



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example classified as red

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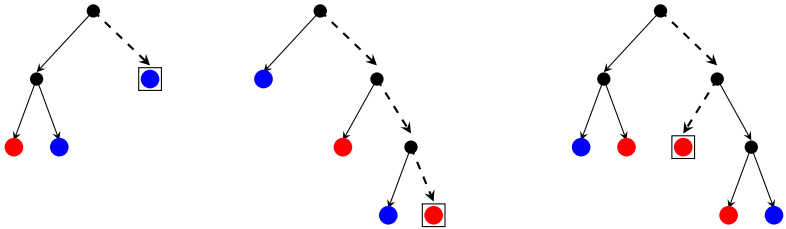

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Future Work

- Algorithm engineering for our algorithm for decision trees
- Study variant which allows misclassifications