Neural Network Accelerated Implicit Filtering: Integrating Neural Network Surrogates With Provably Convergent Derivative Free Optimization Methods

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Black Box Optimization

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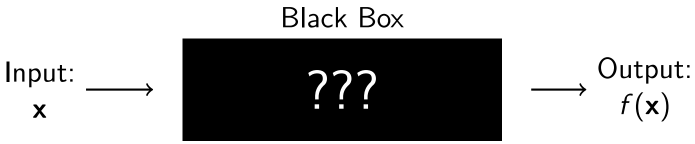

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Black Box

Input: $\mathbf{x}$ $\longrightarrow$??? $\longrightarrow$ Output: $f(\mathbf{x})$

- BBO problems naturally arise when the structure of the objective function is *unknown* (e.g. someone else's computer simulation) or *very laborious* to differentiate (e.g. your legacy FORTRAN code).

- What if the black box output $f(\mathbf{x})$ is a *noise corrupted* version of a smooth function $f_s(\mathbf{x})$ that can be decomposed as

$$f(\mathbf{x}) = f_s(\mathbf{x}) + \phi(\mathbf{x}) \tag{1}$$

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- Our goal is to solve

$$\min_{\mathbf{x}\in\mathbb{R}^{N_\mathbf{x}}} \left\{ f_s(\mathbf{x}) \right\} \tag{2}$$

using *only* samples from the noisy black box given in (1) *instead of the inaccessible* smooth function $f_s(\mathbf{x})$.

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High frequency periodic function	Draw IID uniform samples
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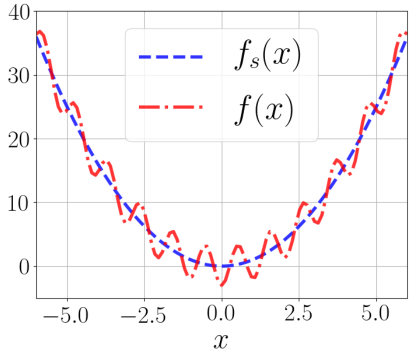	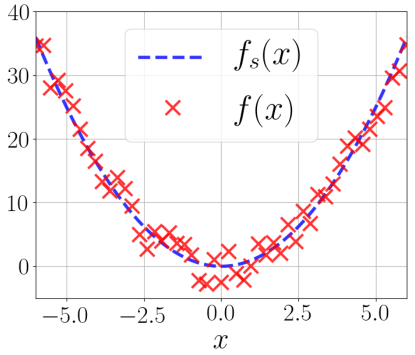

- Minimize the Rosenbrock function

$$f_s(x_1, x_2) = \big(1 - x_1\big)^2 + 100\big(x_2 - (x_1)^2\big)^2 \tag{3}$$

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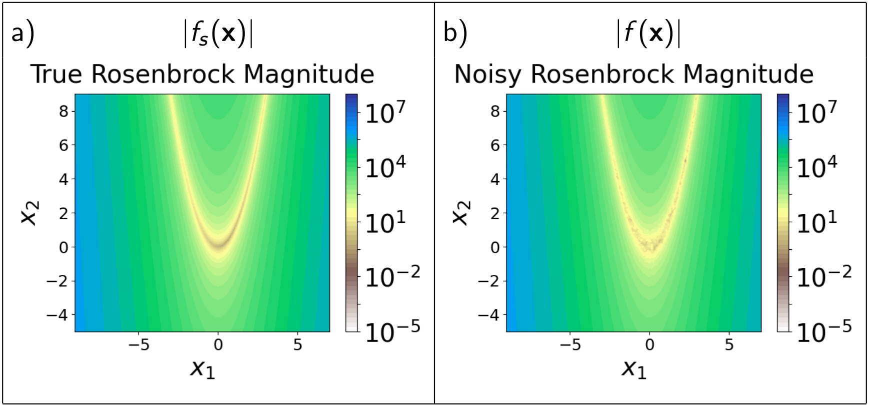

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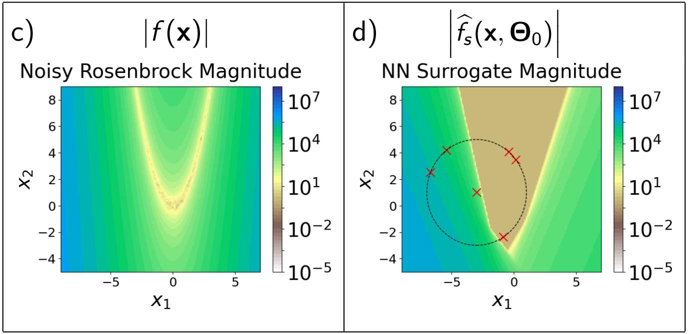
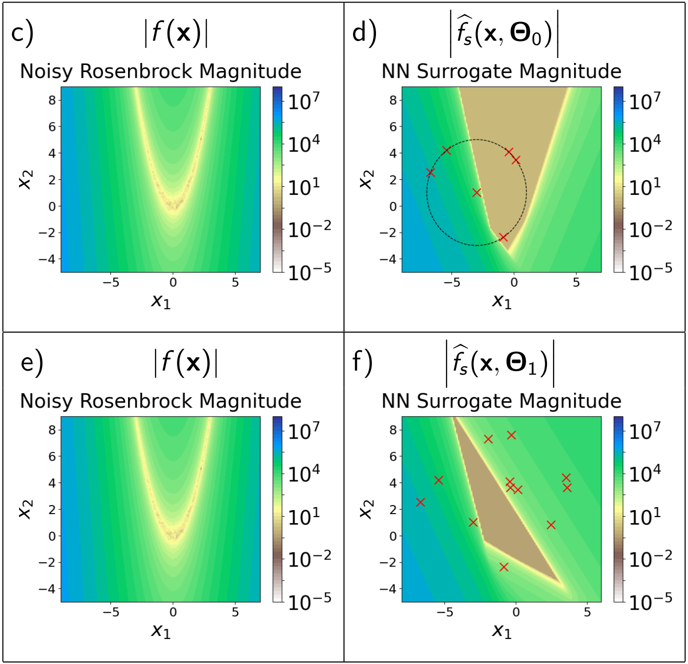
- If surrogate model $\widehat{f_s}$ is bad, fall back to convergence of IF.



c)
$|f(\mathbf{x})|$
Noisy Rosenbrock Magnitude
d)
$\left|\widehat{f_s}(\mathbf{x}, \mathbf{\Theta}_0)\right|$
NN Surrogate Magnitude
x_1
x_2
10^7
10^4
10^1
10^{-2}
10^{-5}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f)
$\left|\widehat{f_s}(\mathbf{x}, \mathbf{\Theta}_1)\right|$
NN Surrogate Magnitude
x_2
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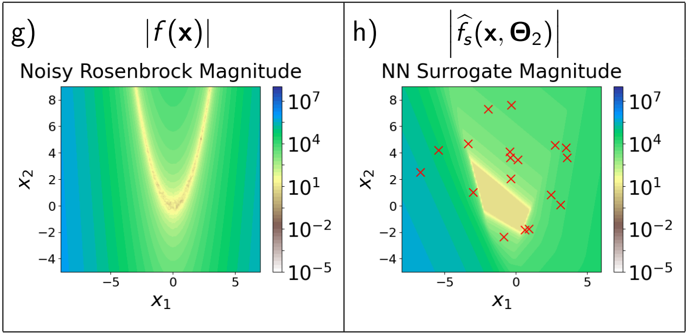

g)
$|f(\mathbf{x})|$
Noisy Rosenbrock Magnitude
h)
$\left|\widehat{f_s}(\mathbf{x}, \mathbf{\Theta}_2)\right|$
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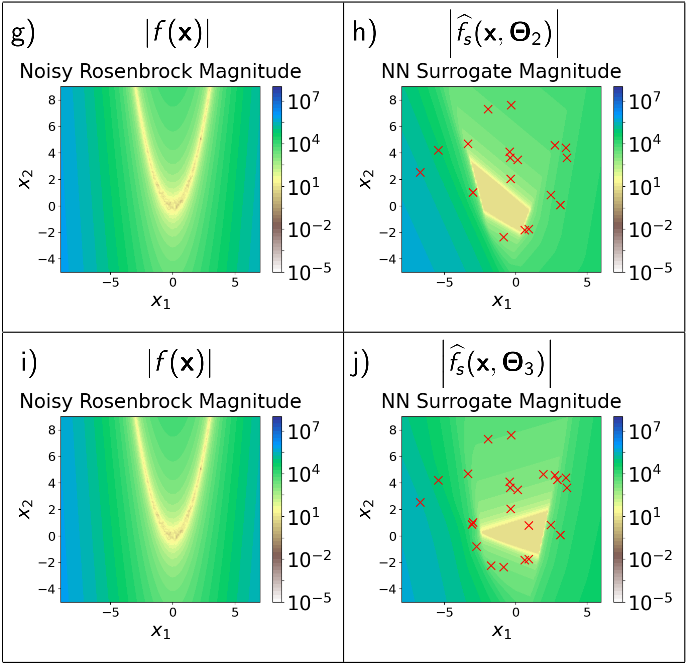
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- Code and documentation available at: `https://github.com/0x4249/NNAIF`