# Polyhedral Complex Extraction from ReLU Networks using Edge Subdivision

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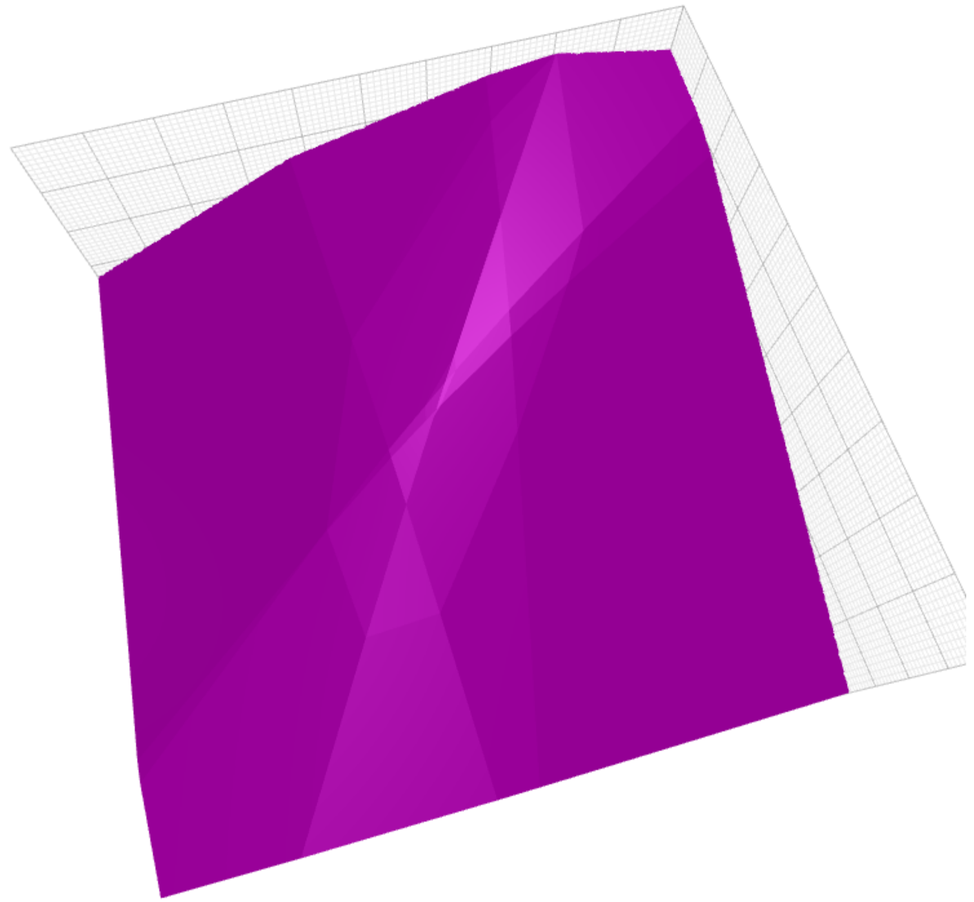
ReLU NNs are **Continuous Piece-Wise Affine**
(abs, hard hyperbolic tangent, hard sigmoid, max-pooling)

CPWA NNs induce a **polyhedral complex**

How to **extract** this complex?



# Region subdivision

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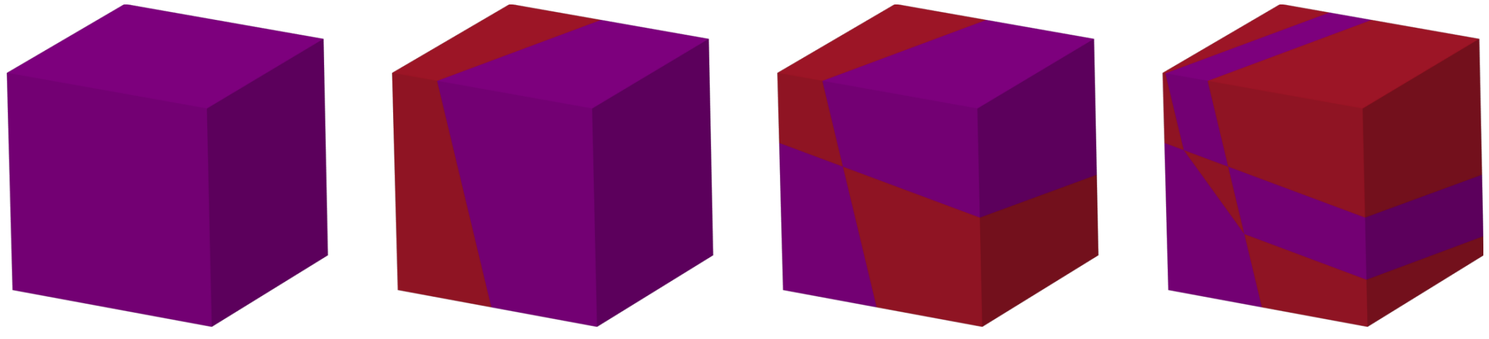
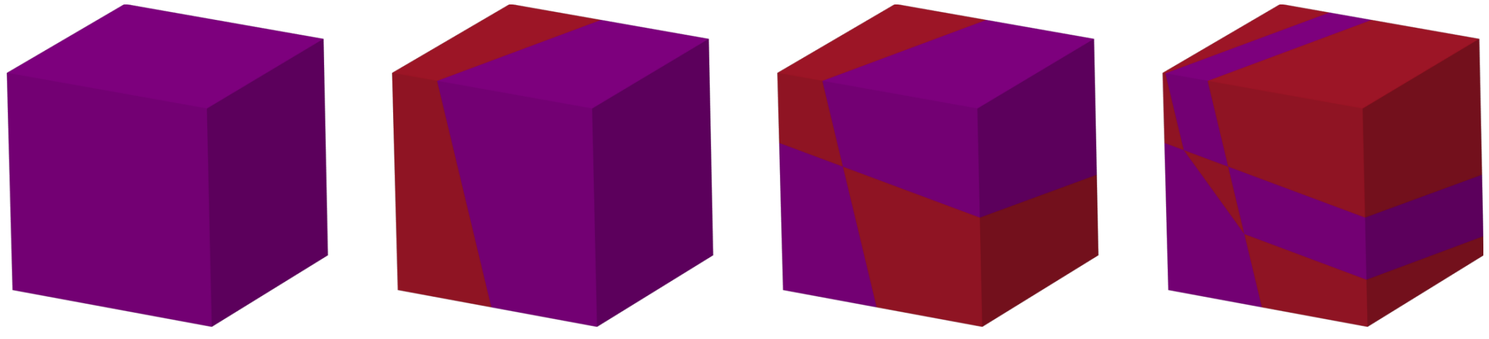


cut each **region** with its affine hyperplane

# Region subdivision



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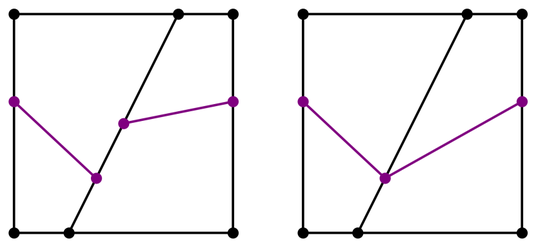
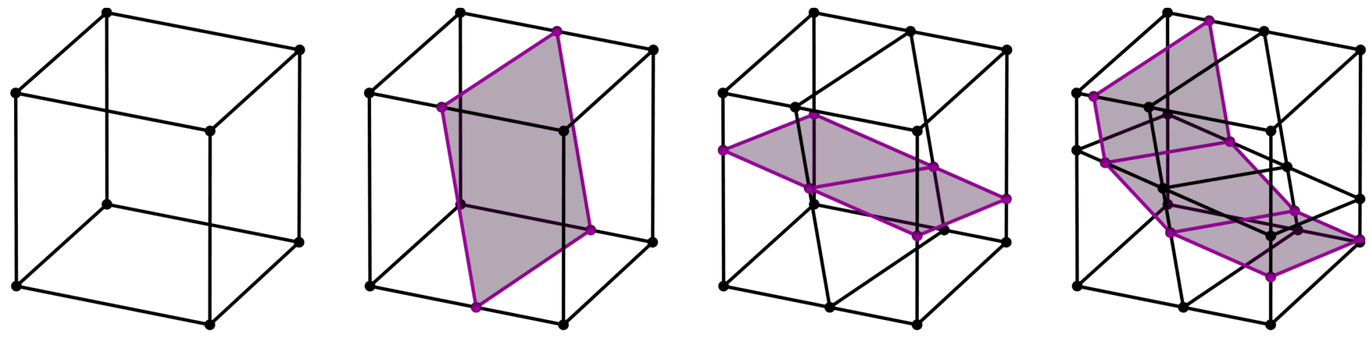


$2^{D-1}$ fold **redundancy** due to continuity

# Edge subdivision



cut **edges** with folded hyperplanes

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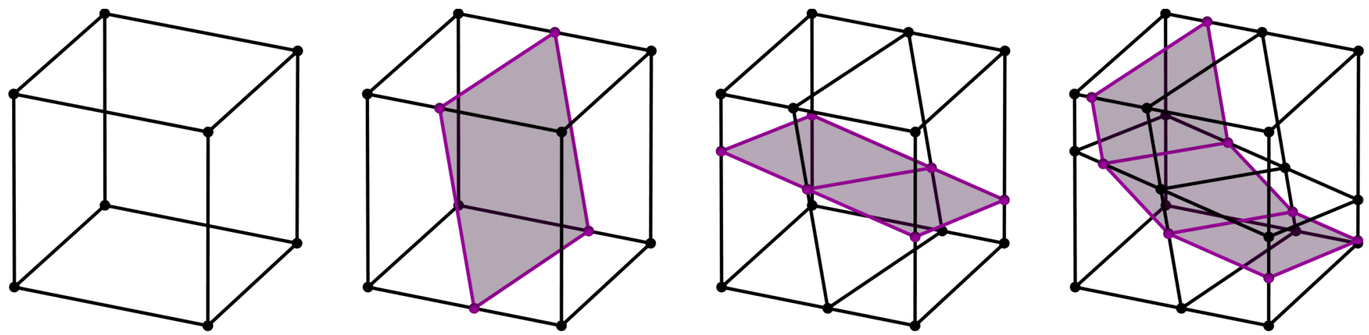

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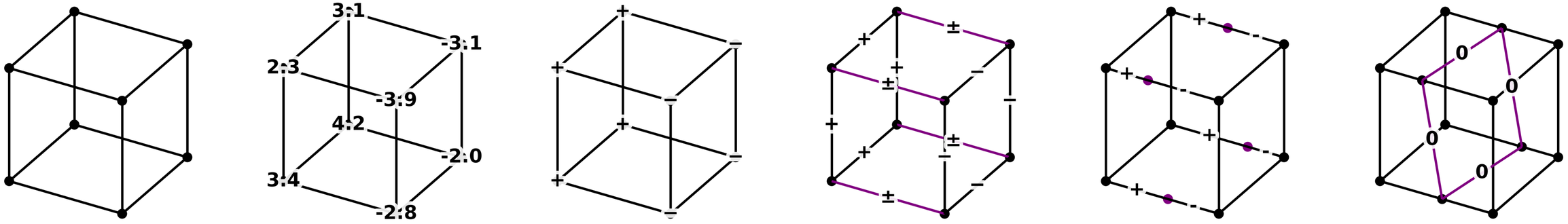
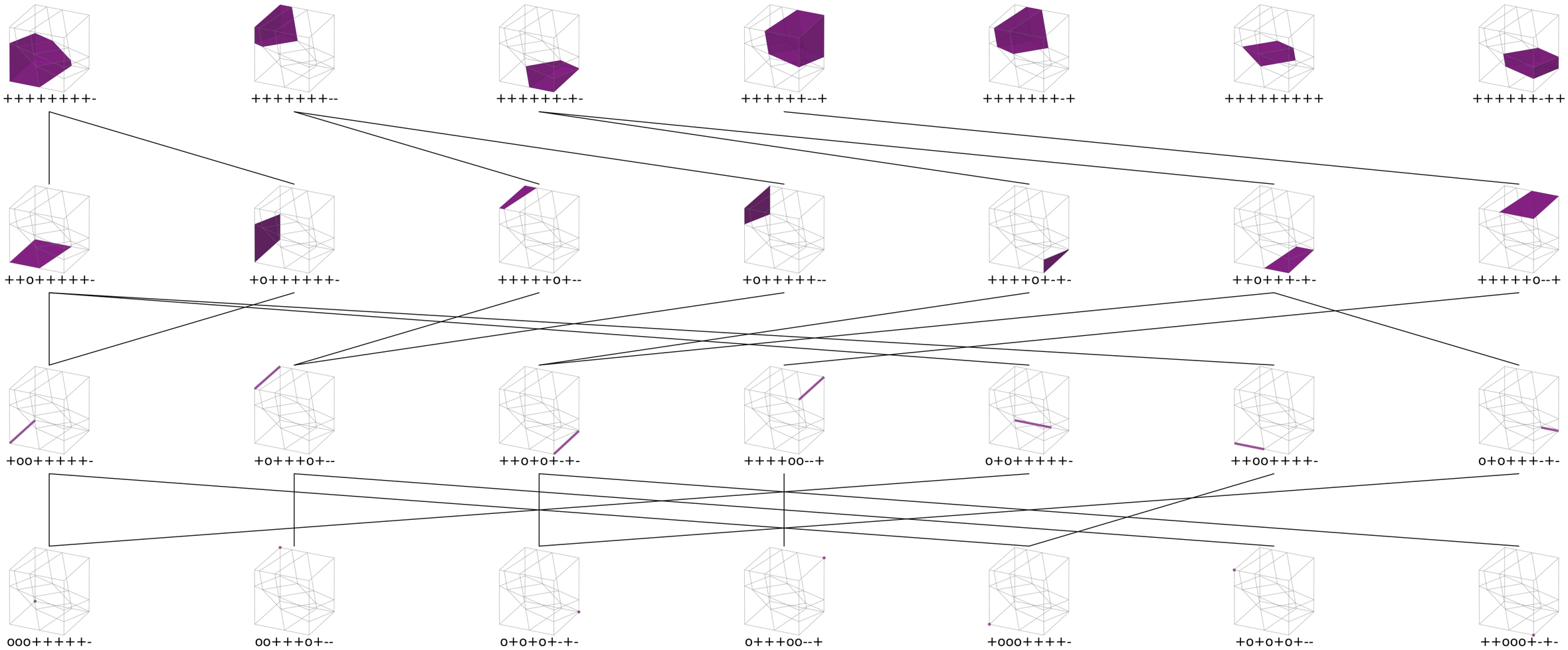



(0) (1) (2) (3) (4) (5)

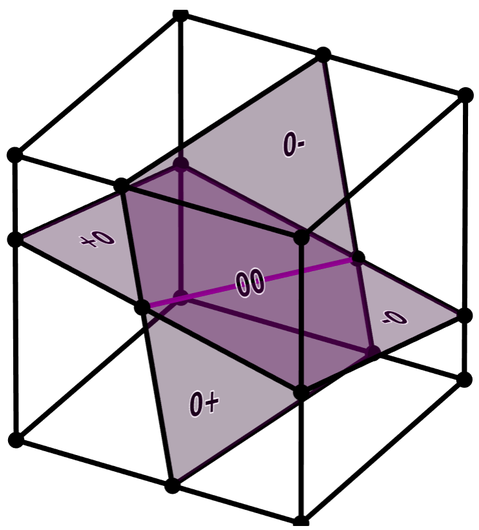
.. in five steps

# Sign-vectors



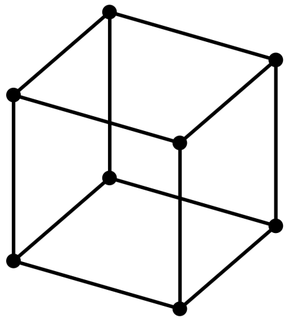

sign-vectors indicate the pre-activation signs of the neurons
they encode the combinatorial structure of the complex

# Perturbation

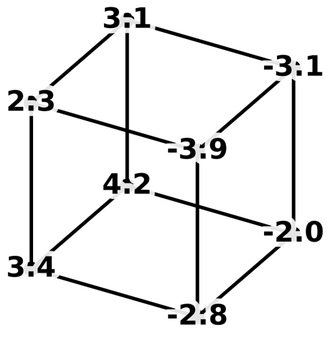



parents of a cell can be built by perturbing the zeros

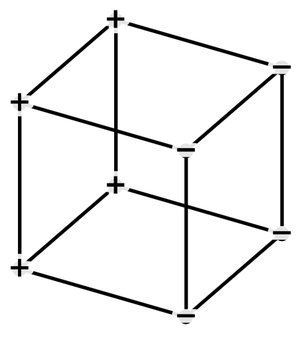
# Edge subdivision



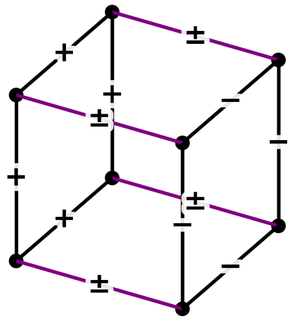
(0)



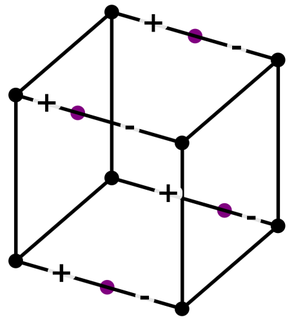

(1)



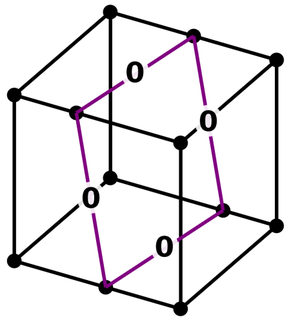
(2)



(3)



(4)




(5)

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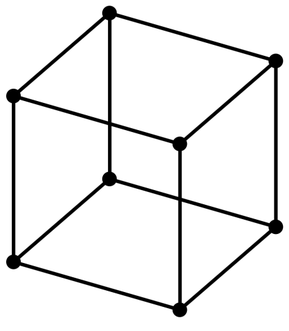

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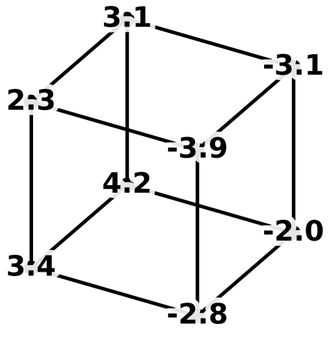


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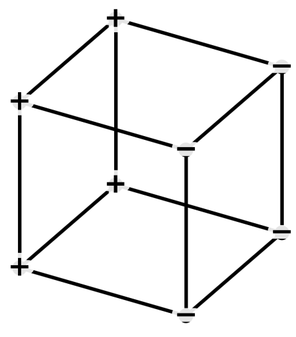
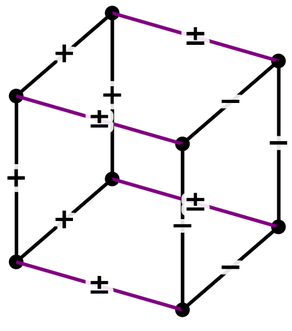


(2)



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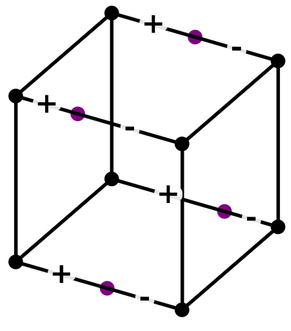

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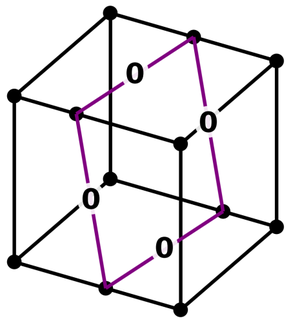


(5)

all steps scale linearly with complex size

# Implementation

PyTorch

github.com/arturs-berzins/relu_edge_subdivision

steps 1-4 are linear
step 5 is log-linear due to sorting

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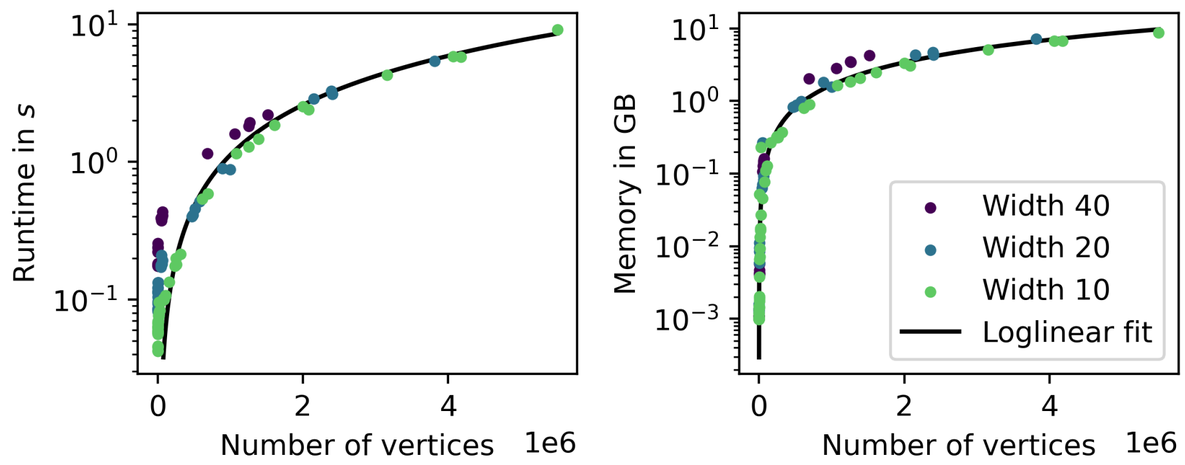
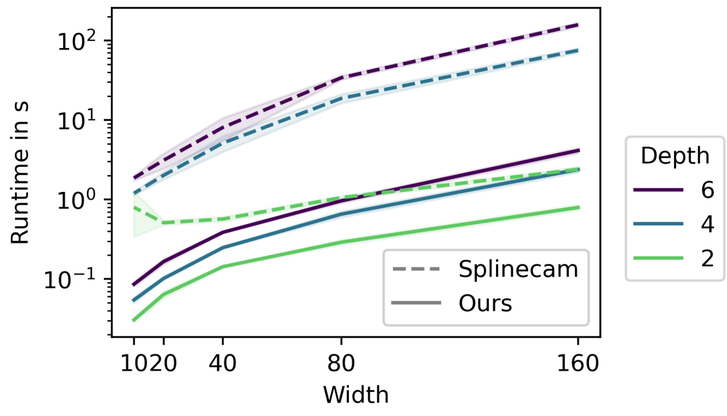


# Benchmark




Humayun, A. I., Balestriero, R., Balakrishnan, G., and Baraniuk, R.
Splinecam: Exact visualization and characterization of deep network geometry and decision boundaries.
CVPR2023.

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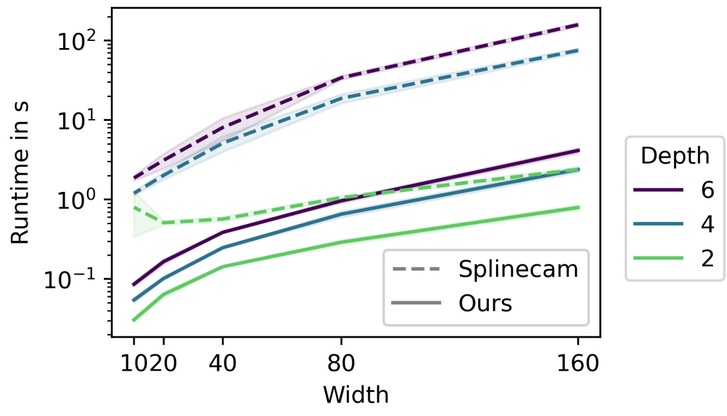


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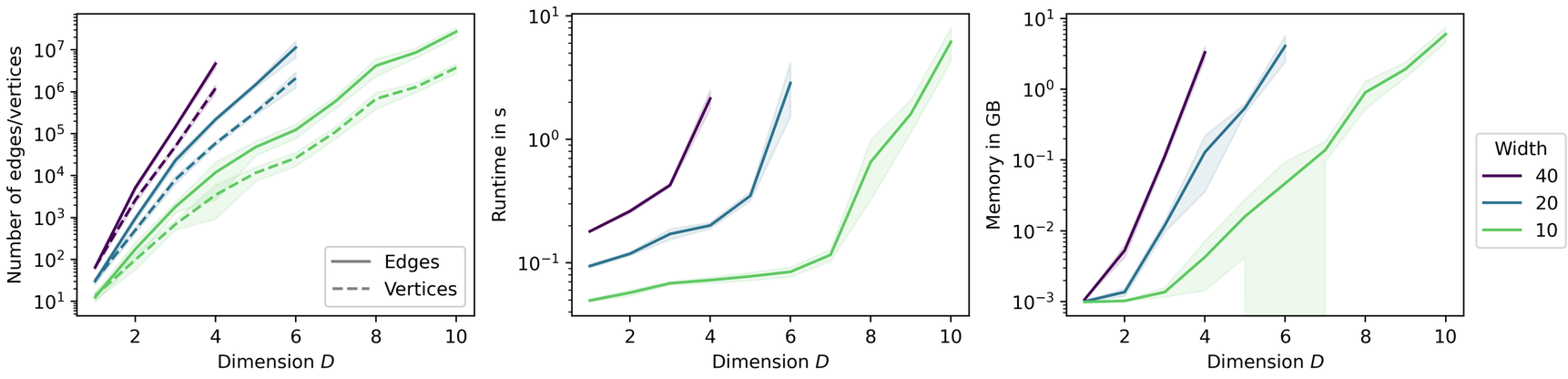
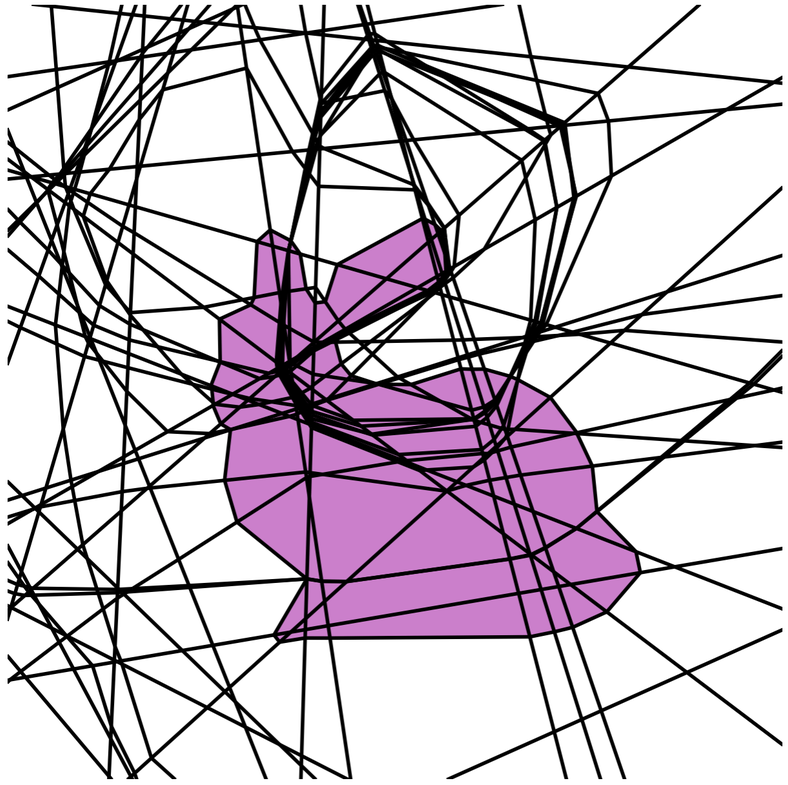


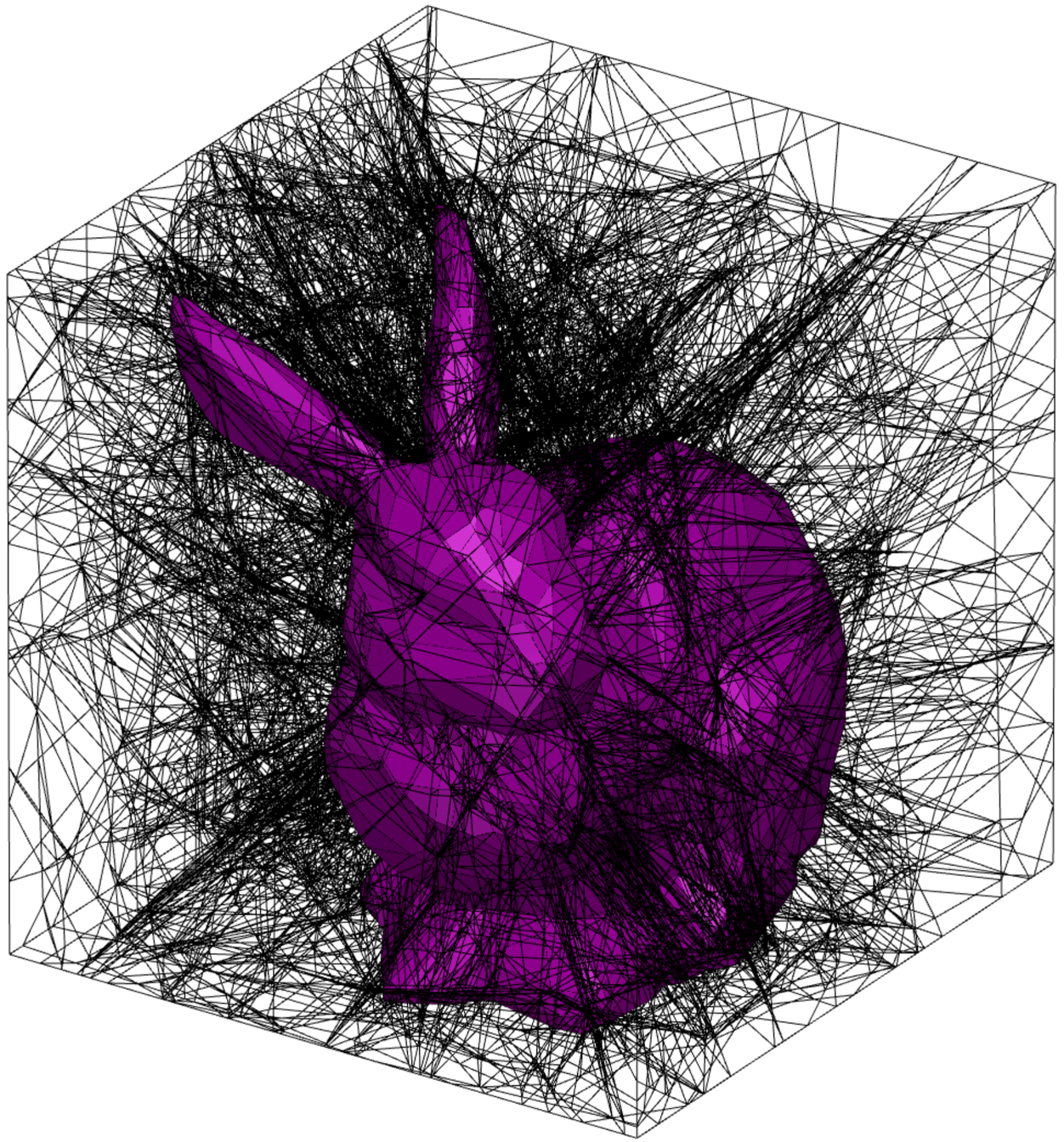
# Optimizing the complex



shape compactness

in general, any vertex position based objective

(edge lengths, angles, areas, volumes, curvatures)



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