A Random Matrix Analysis of Data Stream Clustering: Coping With Limited Memory Resources

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Random matrix framework for data stream clustering

- Observed data: $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_t, \ldots \in \mathbb{R}^p$ $\qquad \mathbf{x}_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\pm\boldsymbol{\mu}, \mathbf{I}_p)$

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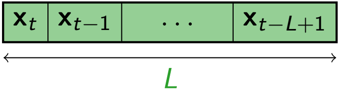
- Memory



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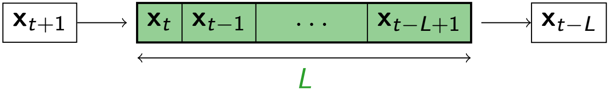

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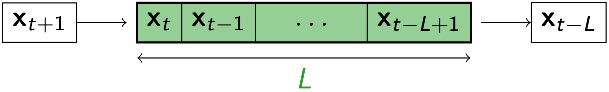

- **Clustering?**

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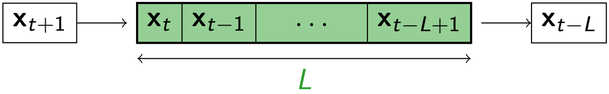

- **Clustering** on the $n \geqslant L$ previous points
  $\mathbf{X} = \begin{bmatrix} \mathbf{x}_{t-n+1} & \mathbf{x}_{t-n+2} & \ldots & \mathbf{x}_t \end{bmatrix} \in \mathbb{R}^{p \times n}$

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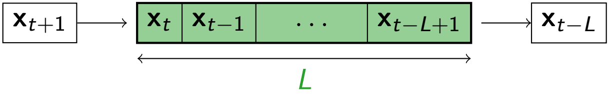

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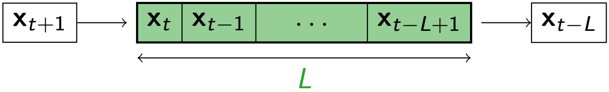

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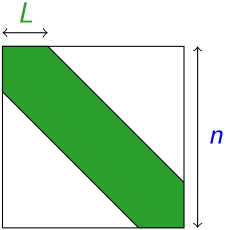

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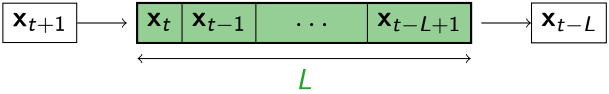

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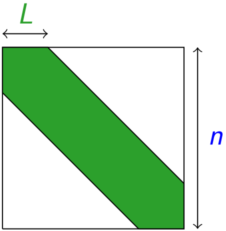

$$\mathbf{K} = \frac{\mathbf{X}^\top \mathbf{X}}{p}$$

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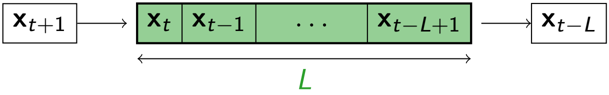

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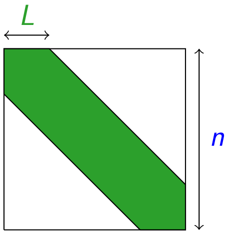

$$\mathbf{K}_L = \frac{\mathbf{X}^\top \mathbf{X}}{p} \odot \underbrace{\begin{bmatrix} 1 & \ldots & 1 & & 0 \\ \vdots & \ddots & & \ddots & \\ 1 & & \ddots & & 1 \\ & \ddots & & \ddots & \vdots \\ 0 & & 1 & \ldots & 1 \end{bmatrix}}_{\mathbf{T}}$$

Data stream clustering: improving over batch clustering

- Size-L memory $\rightsquigarrow$ *batch clustering* vs. $\mathbf{K}_L = \frac{1}{p}\mathbf{X}^\top\mathbf{X} \odot \mathbf{T}$

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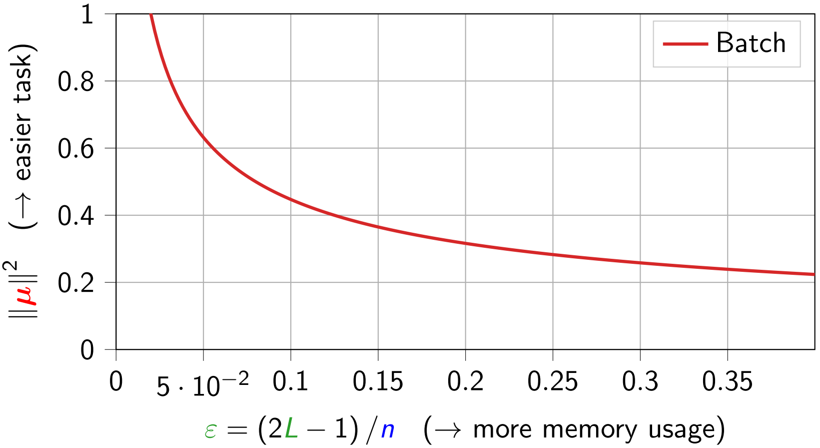

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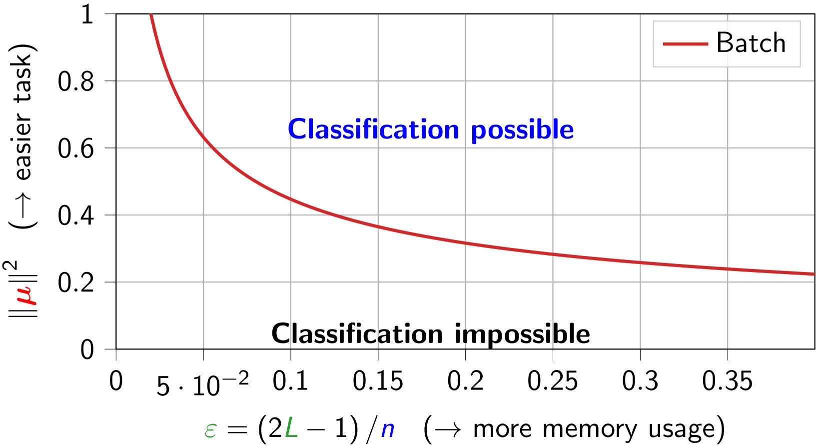

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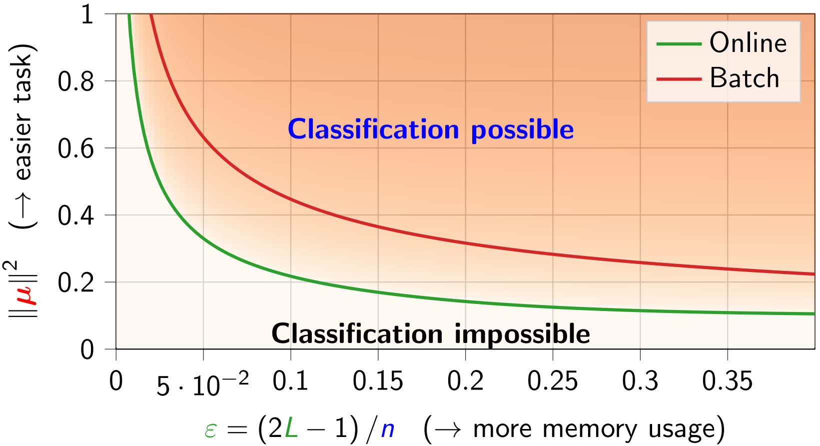

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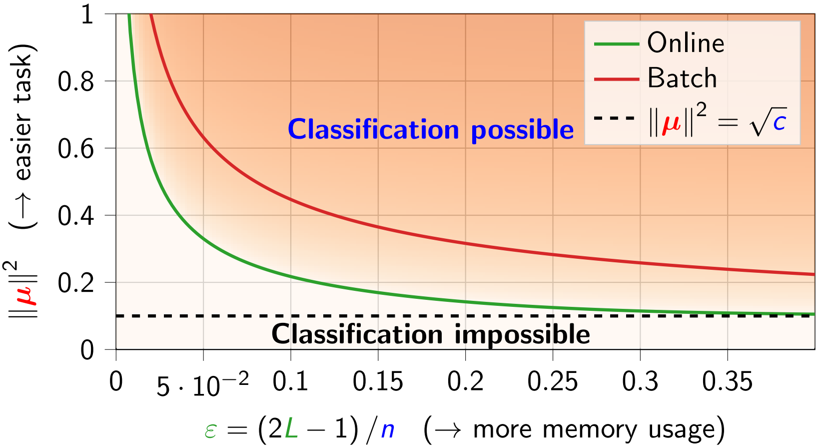

From theory to practice: online kernel spectral clustering

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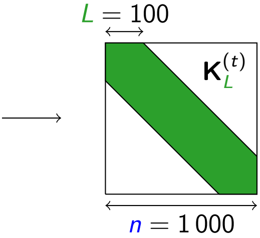

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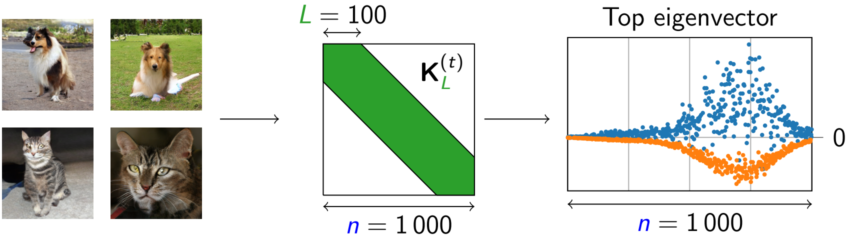

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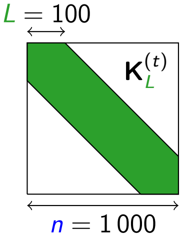

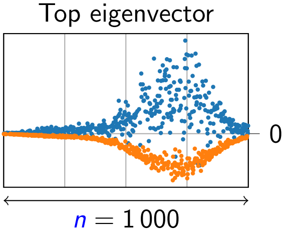

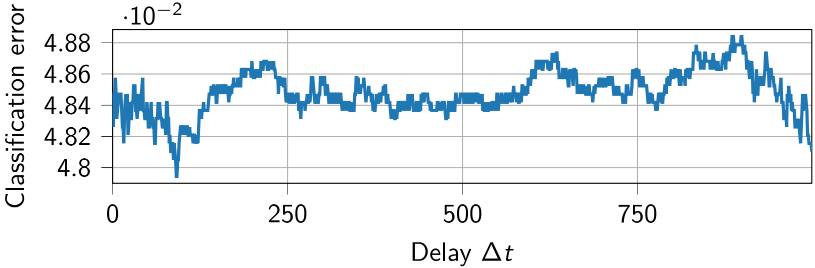

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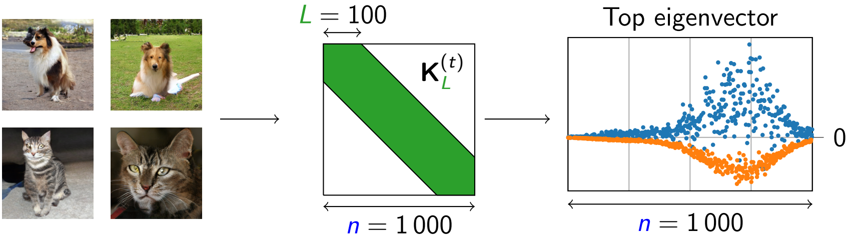

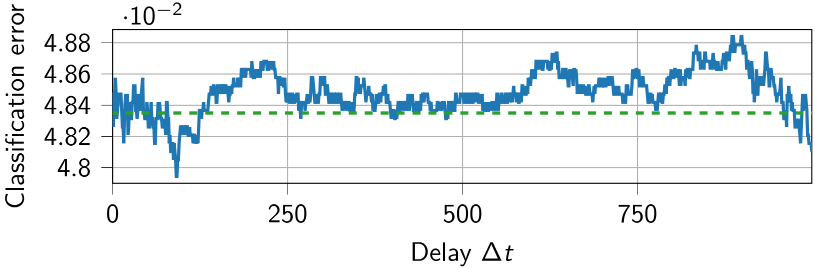

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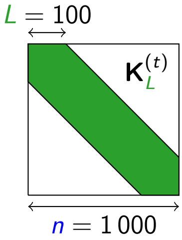

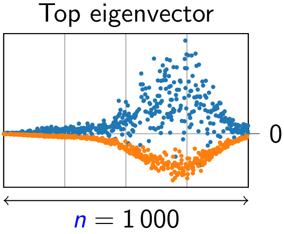

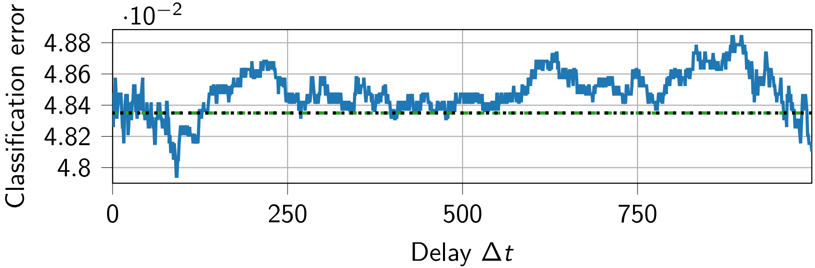

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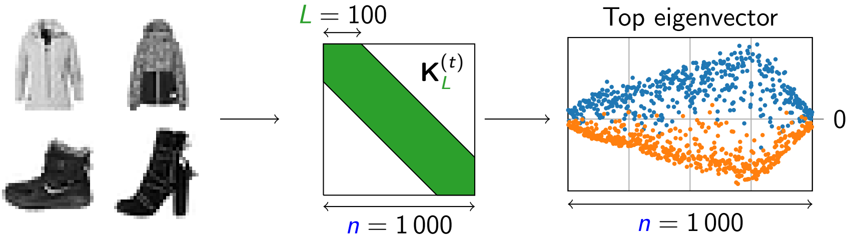
- **Fashion-MNIST images** (raw, $p = 784$) $T = 14\,000$



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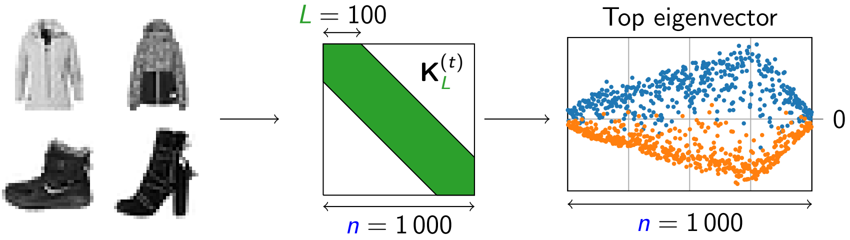

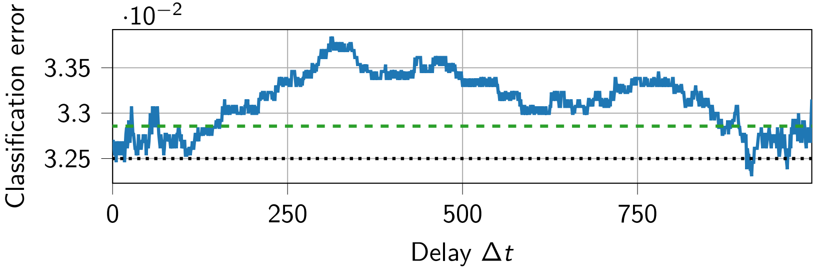

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