Implicit Rate-Constrained Optimization of Non-decomposable Objectives

Abhishek Kumar, Harikrishna Narasimhan, Andrew Cotter

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Problem setting

Optimize

- FNR at a fixed FPR
- Precision at a fixed Recall
- Precision at k
- Area under the ROC curve or area under the PR curve
- Fairness constraints

All these problems can be cast as:

$$\min_{\theta \in \mathbb{R}^p} f(\theta, \lambda) \quad \text{s.t.} \quad g(\theta, \lambda) = \mathbf{0}$$

where θ are the model parameters and λ are the thresholds that act on the model predictions.

f and g are replaced by smooth surrogates $\tilde{f}$ and $\tilde{g}$ for first-order optimization.

Thresholds as a function of model parameters

Most existing methods formulate Lagrangian based primal dual problem to solve these problems.

We propose ICO (implicit constrained optimization) where we directly express thresholds λ as function of model parameters θ using the Implicit Function Theorem.

$$\min_{\theta} \tilde{f}\,(\theta, \tilde{h}(\theta))$$

We can compute the derivative using the Implicit Function Theorem:

$$\nabla_\theta \tilde{f}\,(\theta, \tilde{h}(\theta)) = \nabla_\theta \tilde{f}\,(\theta, \lambda) - \frac{\frac{\partial \tilde{f}\,(\theta,\lambda)}{\partial \lambda}}{\frac{\partial \tilde{g}(\theta,\lambda)}{\partial \lambda}} \nabla_\theta \tilde{g}(\theta, \lambda)$$

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Updating model and thresholds

Model parameters are updated using

$$\theta^{t+1} = OPT(\theta^t, \nabla_\theta \tilde{f}\,(\theta^t, \tilde{h}(\theta^t)))$$

Using IFT, we approximate the new threshold as

$$\begin{aligned} \lambda^{t+1} &= \tilde{h}(\theta^{t+1}) = \tilde{h}(\theta^t + \Delta\theta) \\ &\approx \tilde{h}(\theta^t) + \langle \nabla_\theta \tilde{h}(\theta^t), \Delta\theta \rangle \\ &= \lambda^t + \langle \nabla_\theta \tilde{h}(\theta^t), \Delta\theta \rangle \end{aligned}$$

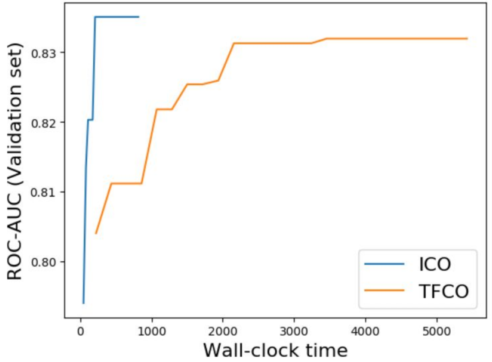
We use a correction step every τ steps that sets the thresholds to satisfy the constraint exactly on k minibatches.

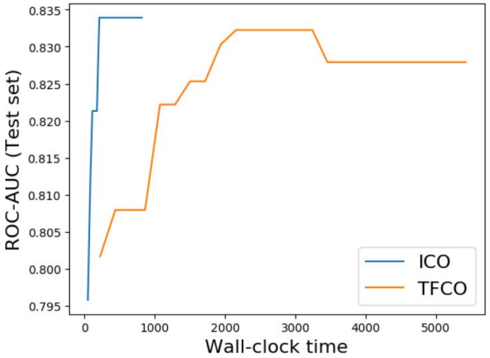
Results

Minimizing FNR at a fixed FPR for CelebA: CE / TFCO[1] / ICO (proposed)

FPR	High-cheekbones	Heavy-makeup	Wearing-lipstick	Smiling	Black-hair	Blond-hair
1%	53.5/ 49.0/ **46.9**	57.0/ 57.0/ **49.6**	44.0/ 42.6/ **37.5**	37.4/ 35.9/ **33.7**	69.3/ 64.4/ **63.2**	40.4/ 38.6/ **36.8**
2%	44.8/ 40.9/ **39.8**	45.6/ 41.2/ **38.9**	32.7/ 30.4/ **26.7**	29.4/ 27.8/ **26.1**	56.4/ 52.0/ **50.5**	28.9/ 25.6/ **24.2**
5%	32.9/ 30.1/ **28.5**	28.2/ 25.4/ **23.1**	16.3/ 14.9/ **13.1**	18.7/ 17.0/ **16.9**	36.7/ 32.4/ **32.5**	13.4/ 11.6/ **10.8**
10%	22.9/ 20.4/ **19.7**	15.1/ 13.6/ **12.4**	6.6/ 5.9/ **4.7**	11.7/ 10.7/ **10.2**	23.0/ 19.2/ **18.6**	6.5/ 4.9/ **4.7**

Timing comparison:





[1] https://github.com/google-research/tensorflow_constrained_optimization

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