MAX PLANCK INSTITUTE
FOR INTELLIGENT SYSTEMS

CombOptNet: Fit the Right NP-Hard Problem by Learning Integer Programming Constraints

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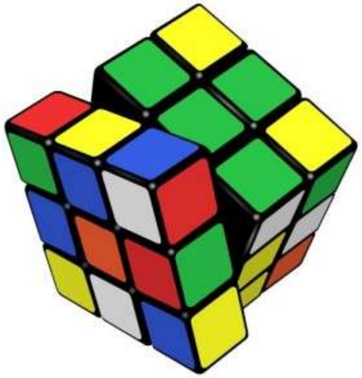
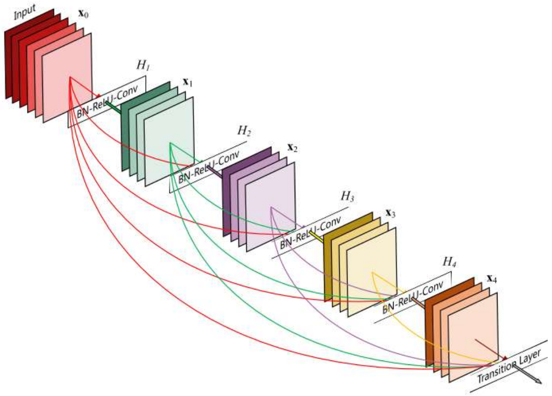
Motivation

Combinatorial optimization

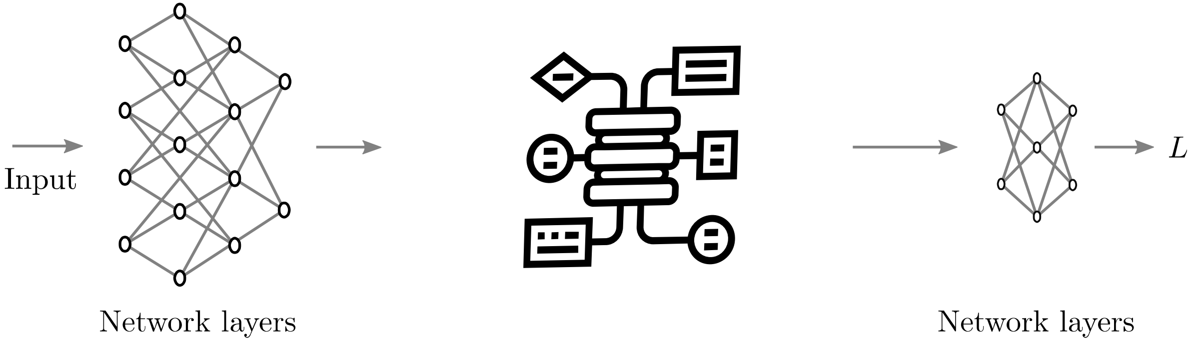


Deep Learning



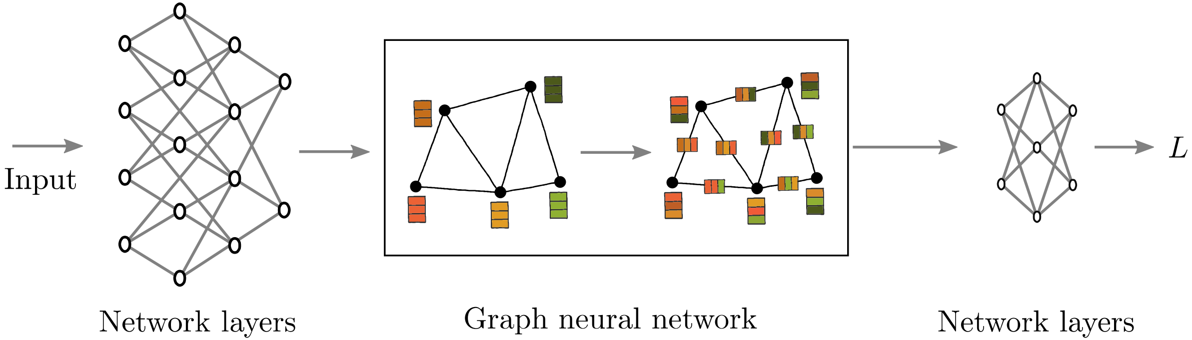
Goal

- Include combinatorial block in architecture



Approaches

- GNNs: Mimic combinatorial solver for graph problems with message passing [1]



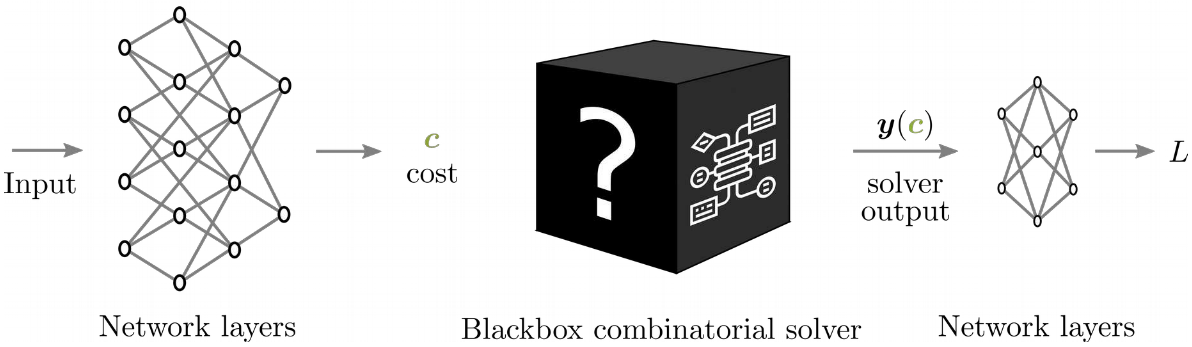
- Drawback: Suboptimal from combinatorial perspective

Approaches

- **Softened solver**: Design differentiable softened versions of solvers [2, 3]

Approaches

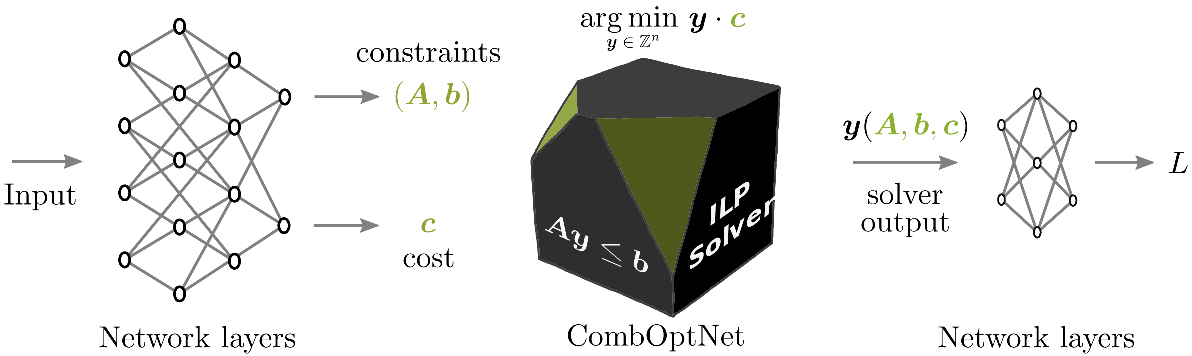
- Blackbox Differentiation: Blackbox backprop [4, 5, 6], differentiable perturbed optimizers [7]



- Drawback: Commit to specific combinatorial problem

Our Approach

- CombOptNet: ILP solver as differentiable layer

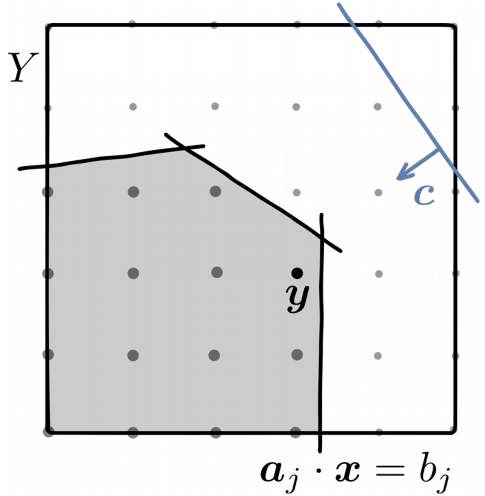


Integer Linear Programs

ILP

$$y(A, b, c) = \arg\min_{y \in Y} \; y \cdot c$$
$$\text{subject to } Ay \le b$$

Y .... bounded subset of $\mathbb{Z}^n$
$c \in \mathbb{R}^n$ .... cost
$A = [a_1, \ldots, a_k] \in \mathbb{R}^{k \times n}$... constraints
$b \in \mathbb{R}^k$ .... bias term



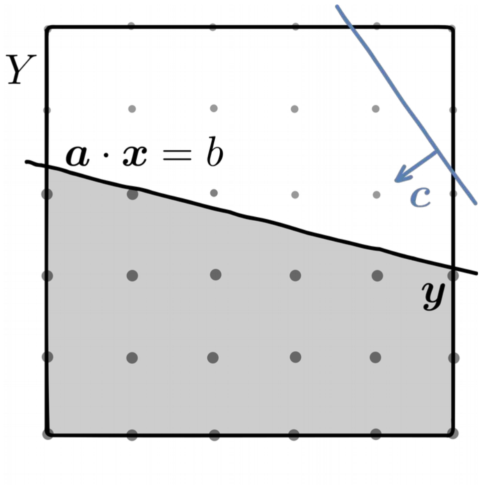
Integer Linear Programs

Knapsack as ILP

$$\boldsymbol{y}(\boldsymbol{a}, b, \boldsymbol{c}) = \arg\min_{\boldsymbol{y} \in Y} \boldsymbol{y} \cdot \boldsymbol{c}$$

$$\text{subject to } \boldsymbol{a} \cdot \boldsymbol{y} \leq b$$

$Y = \{0,1\}^n$ .... binary hypercube
$\boldsymbol{c} \in \mathbb{R}^n$................. neg. item prices
$\boldsymbol{a} = [a_1, \ldots, a_n] \in \mathbb{R}^n$... item weights
$b \in \mathbb{R}$.................. knapsack capacity



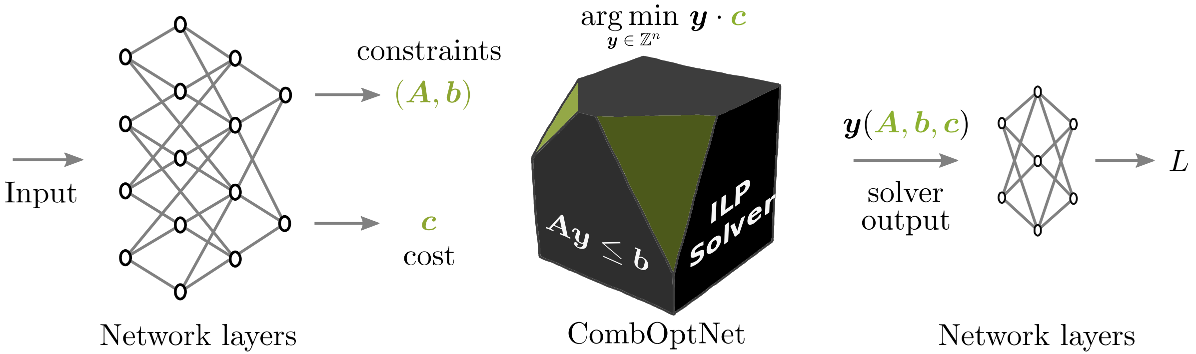
- Express NP-hard problems as ILPs
- Learning constraints = learning combinatorial nature

ILP

$$y(A, b, c) = \arg\min_{y \in Y} \; y \cdot c$$

$$\text{subject to } Ay \leq b$$

Architecture



- Generality: Aspires universal combinatorial expressivity
- Competitive combinatorics: SOTA ILP solver (Gurobi [8])

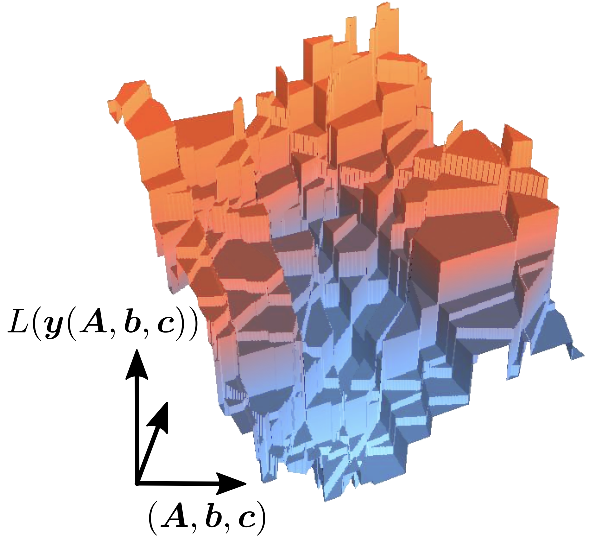
How? — The Difficulty

ILP

$$\boldsymbol{y}(\boldsymbol{A},\boldsymbol{b},\boldsymbol{c}) = \arg\min_{\boldsymbol{y}\in Y} \boldsymbol{y}\cdot\boldsymbol{c}$$

$$\text{subject to } \boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{b}$$

- Small perturbation typically does not change solution
- True gradient exists but is uninformative
- Cost:
  - Well-studied [4, 7, 9, 10, 11]
- Constraints:
  - Mostly unexplored
  - Difficult (no active constraints)



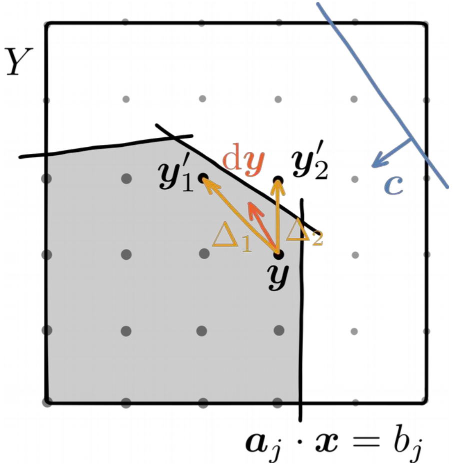
How? — Components

ILP

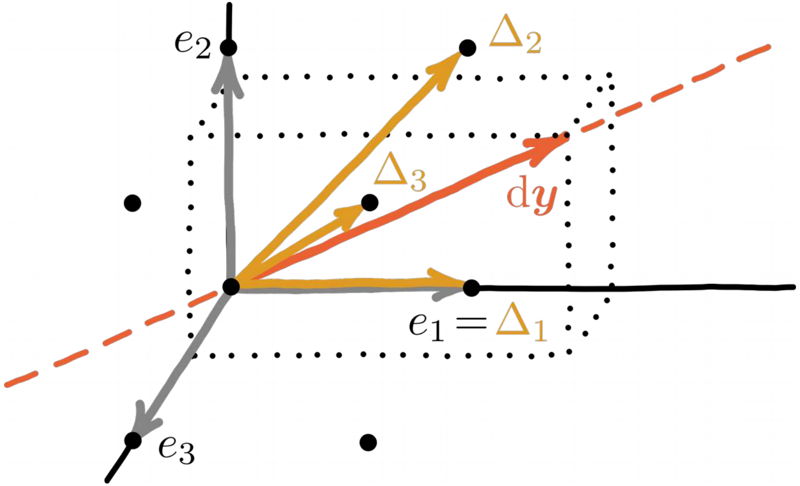
$$\boldsymbol{y}(\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}) = \arg\min_{\boldsymbol{y} \in Y} \boldsymbol{y} \cdot \boldsymbol{c}$$

subject to $\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{b}$

- Decomposition of incoming gradient into integer basis
    - 2D



    - 3D



- Attainable update targets

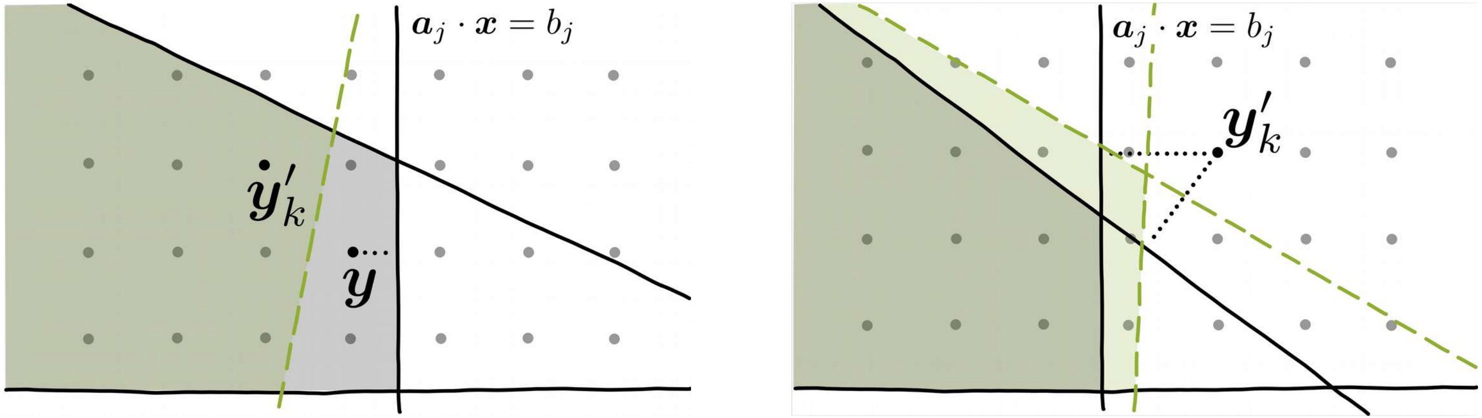
How? — Components

ILP

$$y(A, b, c) = \arg\min_{y \in Y} y \cdot c$$

subject to $Ay \leq b$

- Gradient of geometry-aware mismatch function



- Generalizes concept of active constraints
- Precise definitions & algorithm on poster

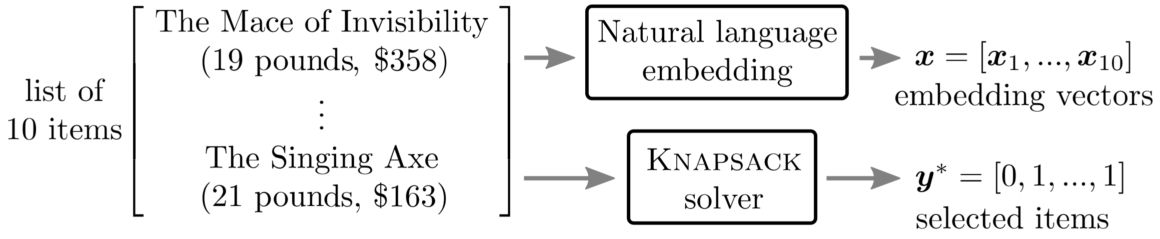
Experiments — Knapsack

ILP

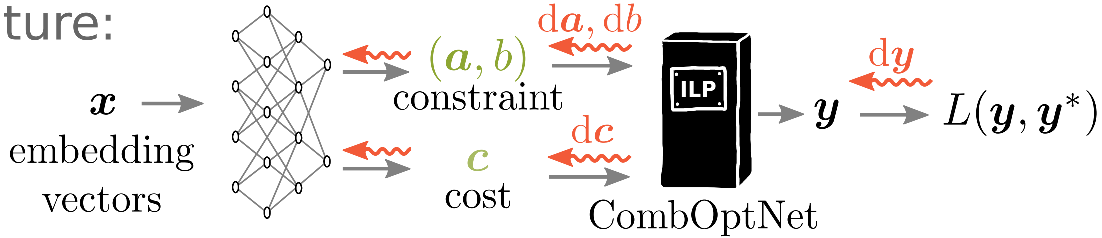
$$y(A, b, c) = \arg\min_{y \in Y} y \cdot c$$

subject to $Ay \leq b$

- Dataset:

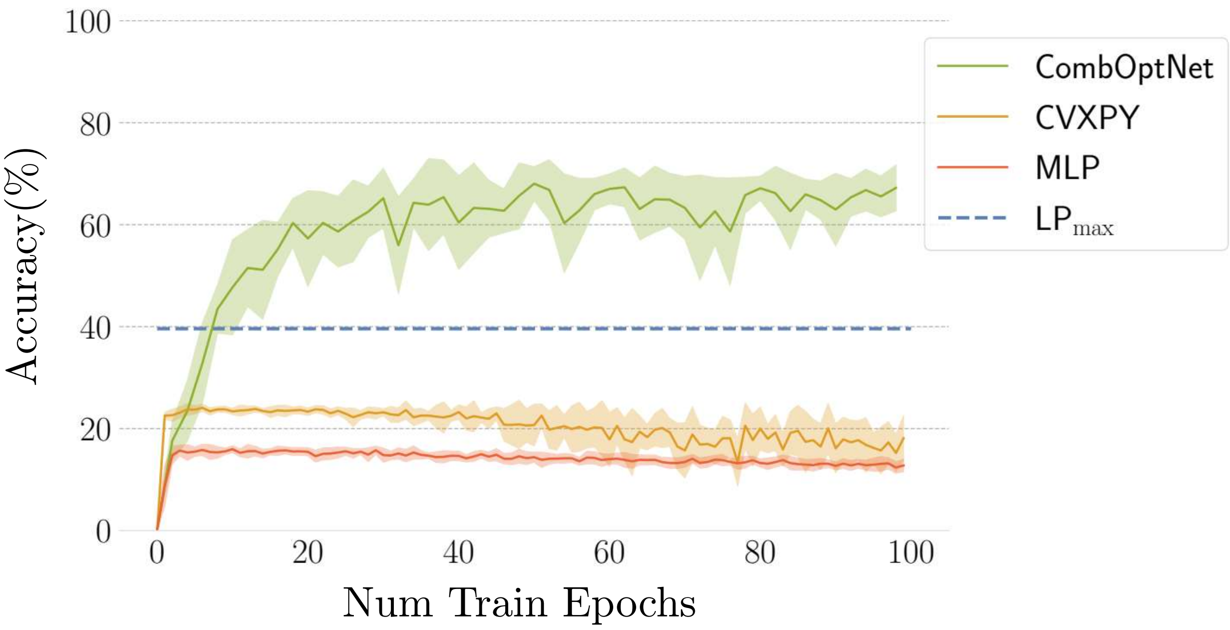


- Architecture:



Experiments — Knapsack

- Results:



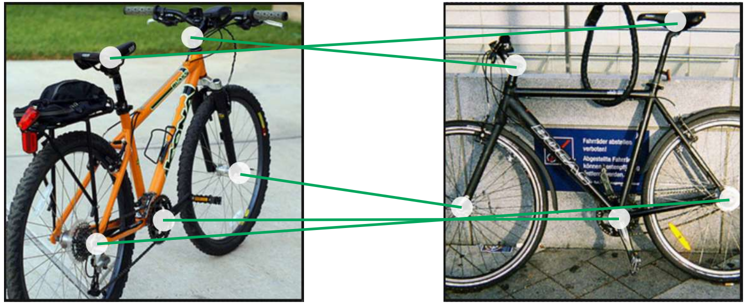
Experiments — Keypoint Matching

ILP

$$y(A, b, c) = \arg\min_{y \in Y} y \cdot c$$

subject to $Ay \leq b$

- Dataset:



- Architecture: Blackbox Graph Matching [6]

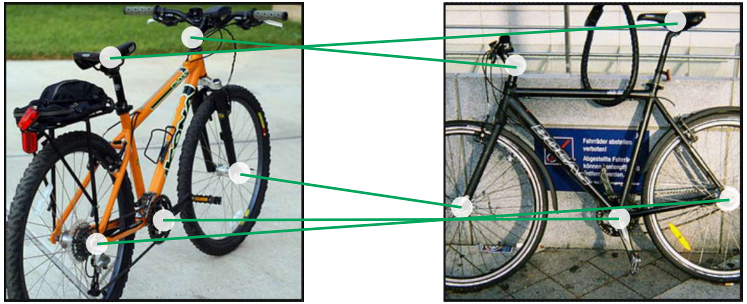
Experiments — Keypoint Matching

ILP

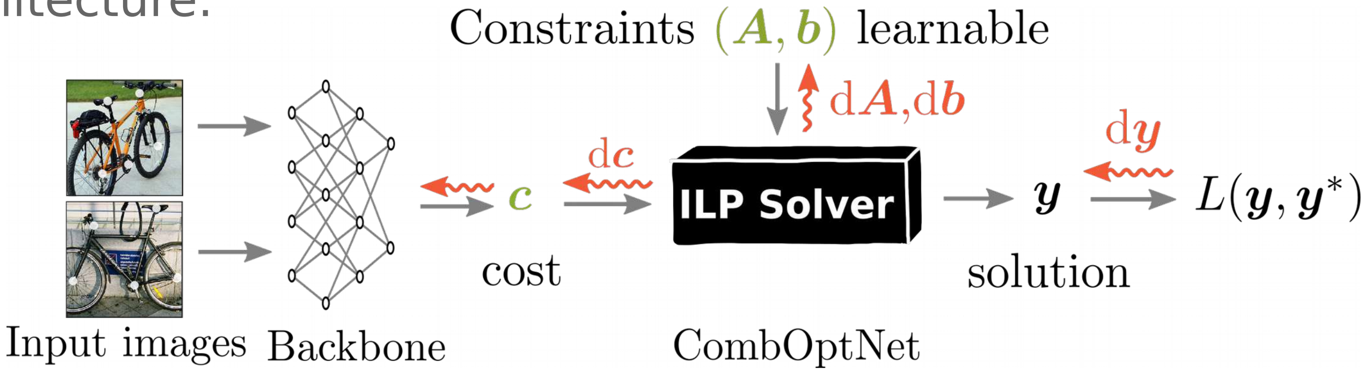
$$y(A, b, c) = \arg\min_{y \in Y} y \cdot c$$

$$\text{subject to } Ay \leq b$$

- Dataset:



- Architecture:

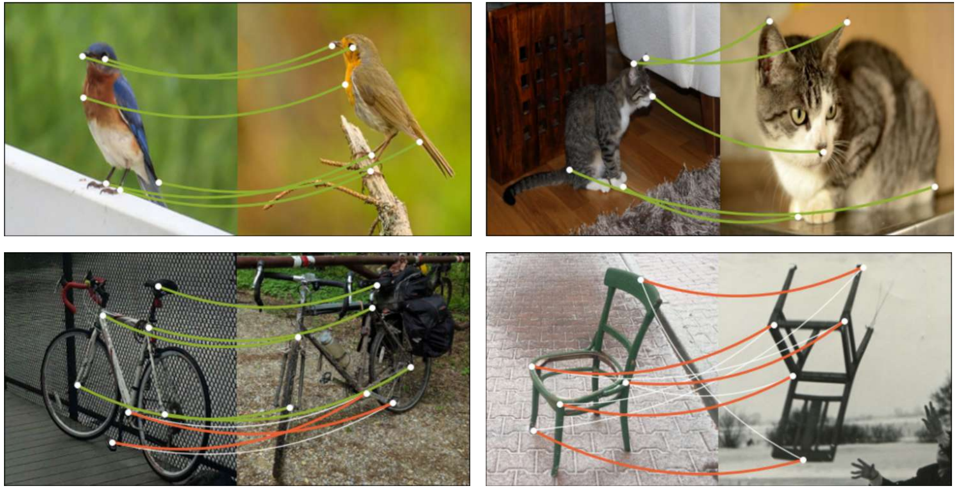


Experiments — Keypoint Matching

- Results:

Method	4×4	5×5	6×6	7×7
CombOptNet	83.1	80.7	78.6	76.1
BB-GM	84.3	82.9	80.5	79.8

- Examples:



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github.com/martius-lab/CombOptNet

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