Segmenting Hybrid Trajectories Using Latent ODEs

Ruian (Ian) Shi

University of Toronto

Vector Institute

Quaid Morris

Memorial Sloan Kettering Cancer Center

Vector Institute

University of Toronto

Computer Science UNIVERSITY OF TORONTO

VECTOR INSTITUTE

Memorial Sloan Kettering Cancer Center

LatSegODE: Reconstructing and Segmenting Hybrid Trajectories

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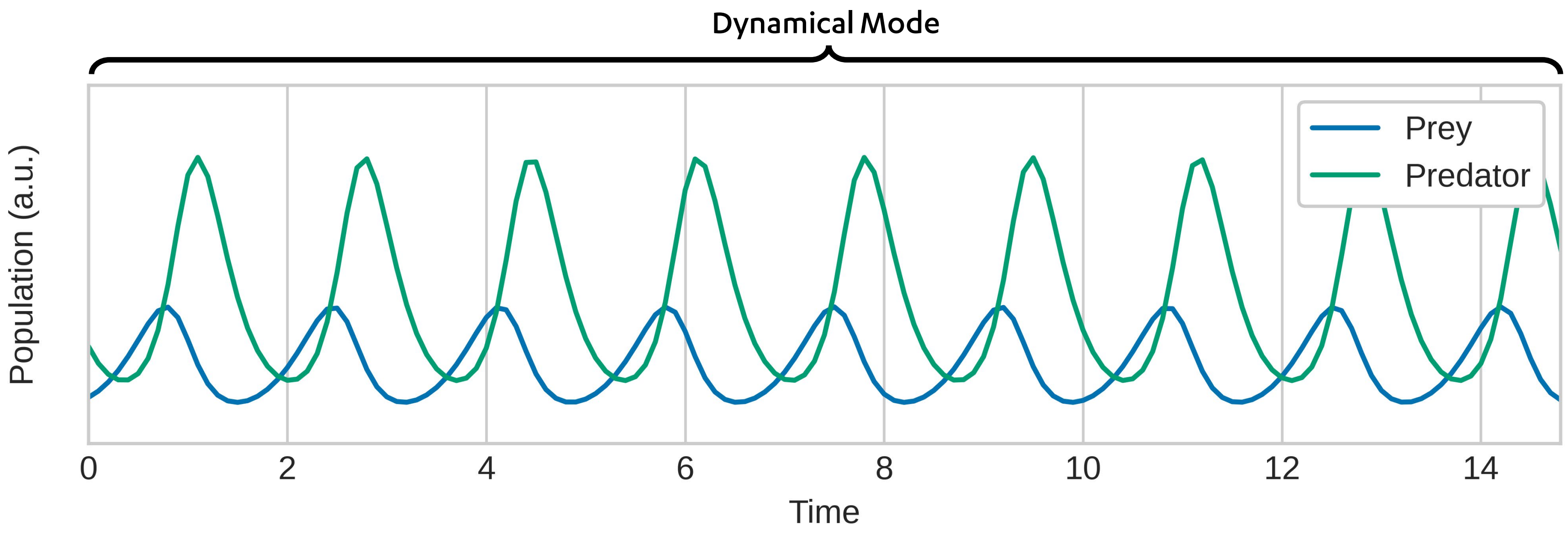

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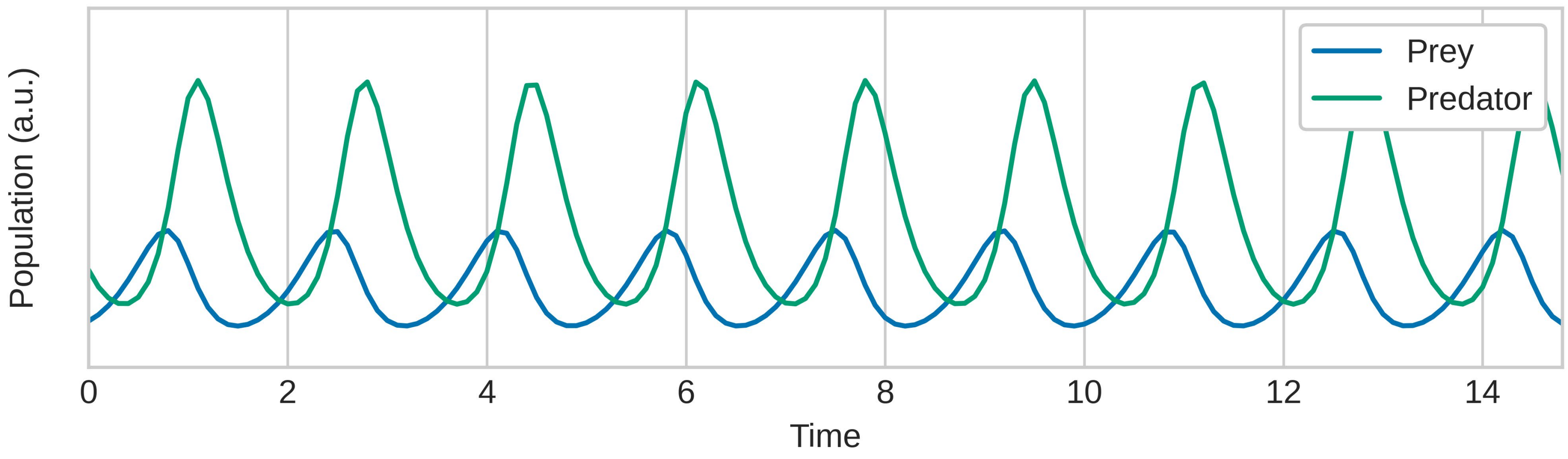
Dynamical Coefficients

Dynamical Mode



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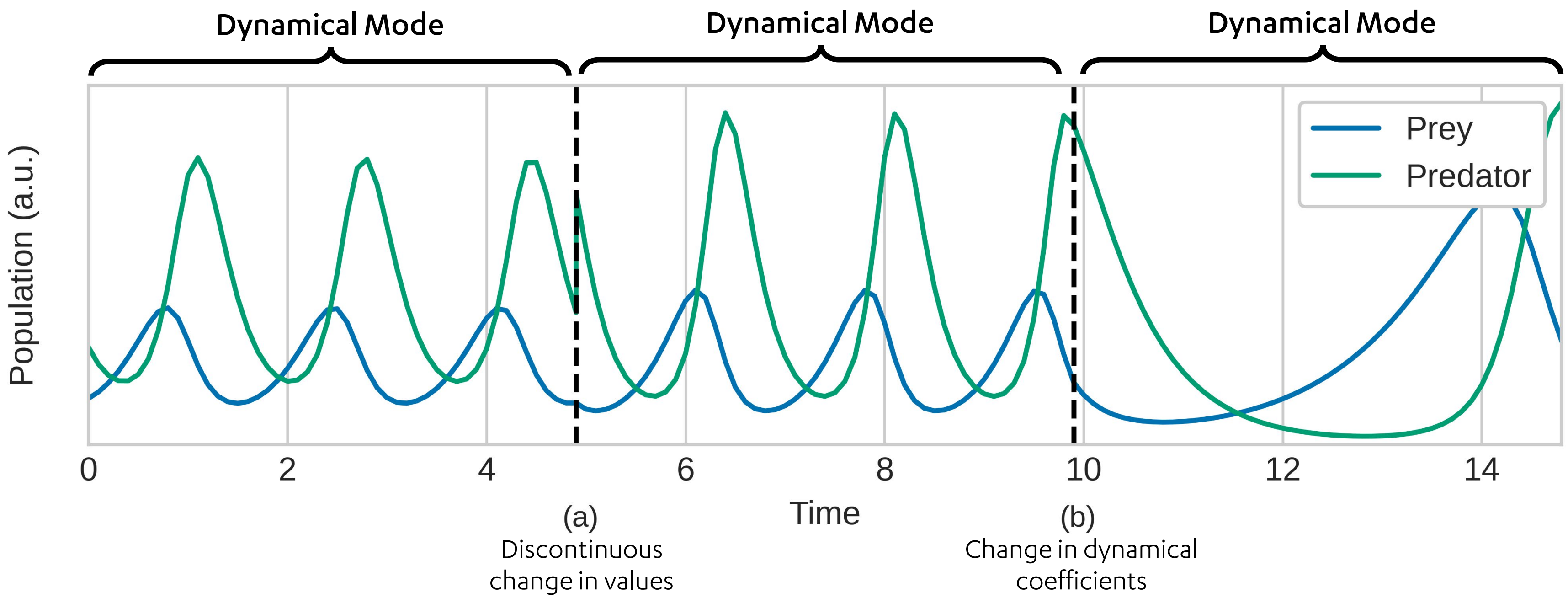
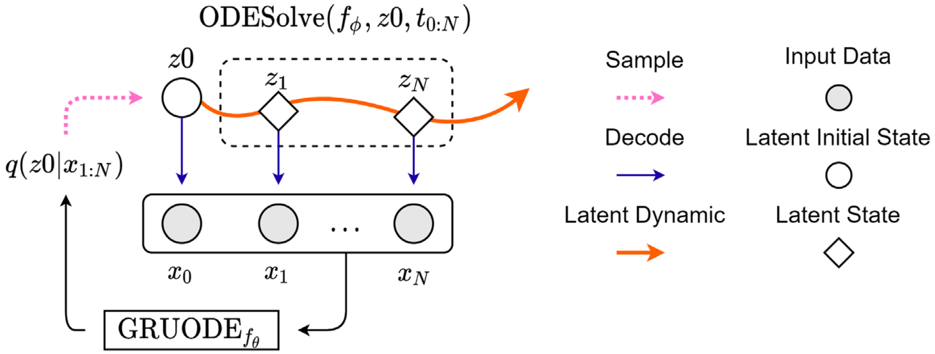


Baseline Approaches

- The Latent ODE [1] is recent powerful model which combines Neural ODE in VAE setup.



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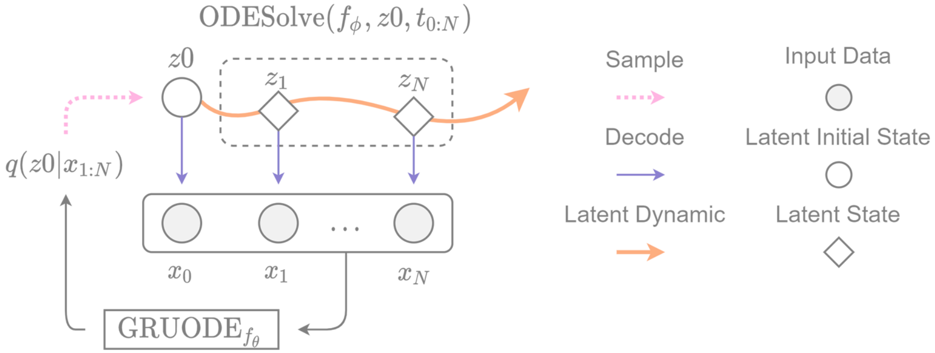

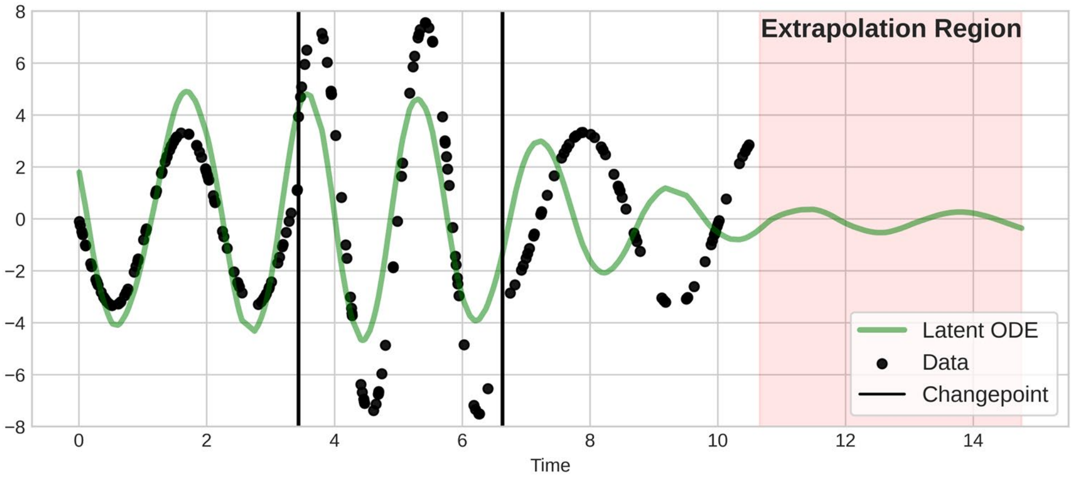

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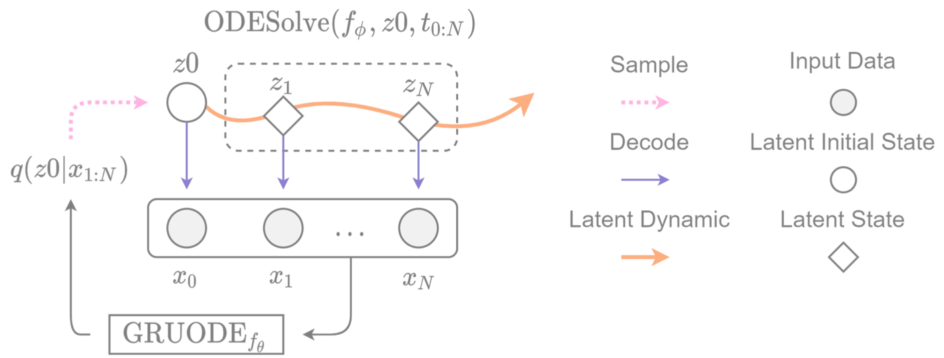

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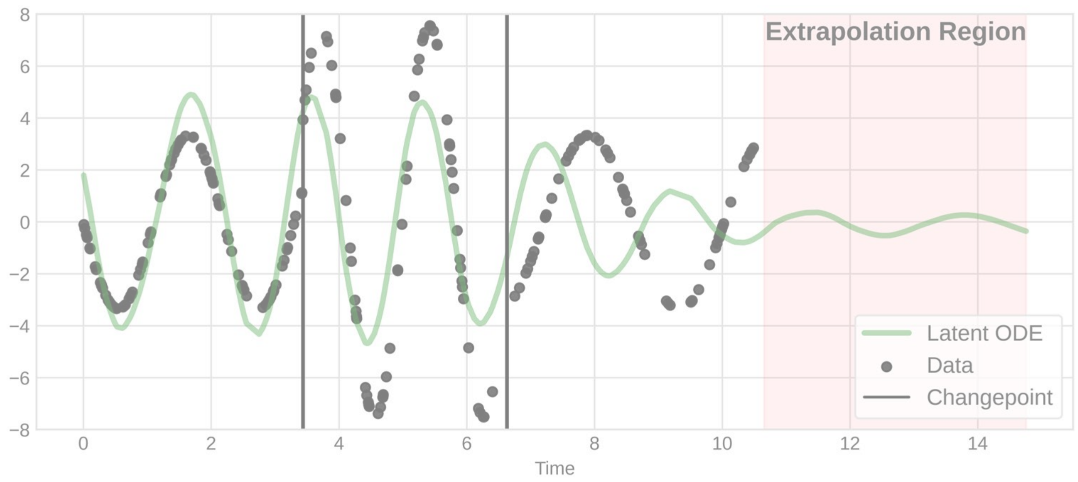
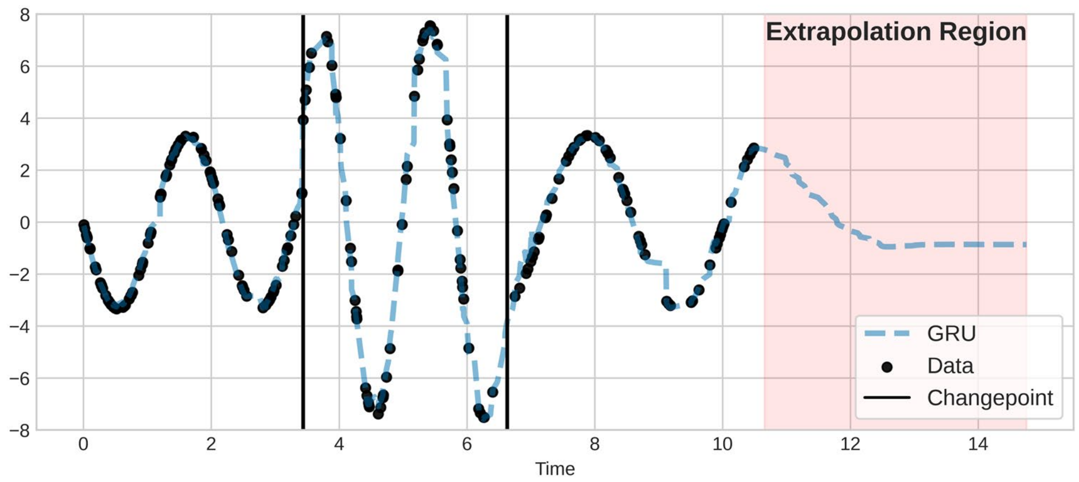
- Classical deep time series methods (GRU [2]) perform poorly when extrapolating





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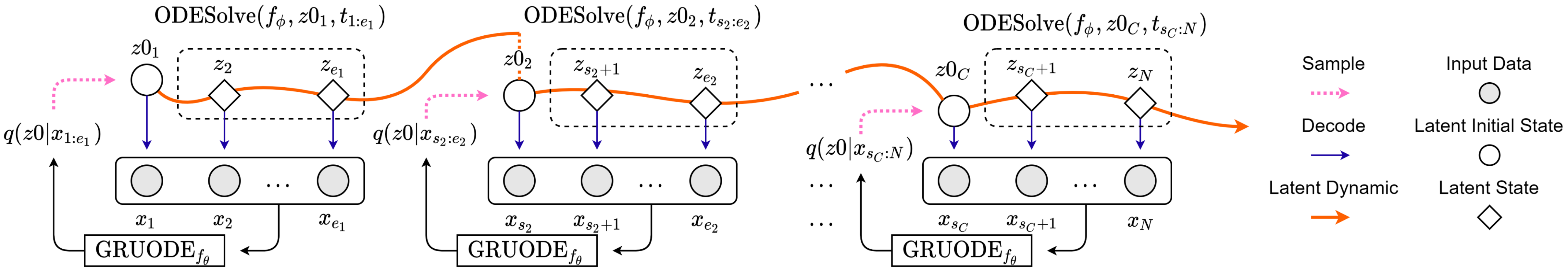
LatSegODE

- Represent complex hybrid trajectory as piece-wise sequence of simple Latent ODE dynamics



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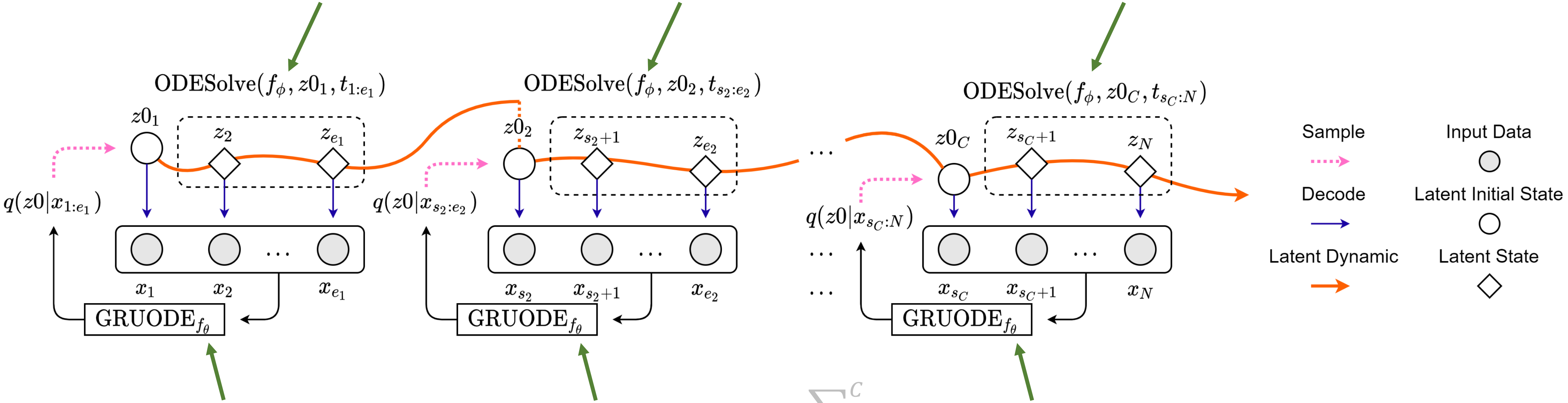
- Share Latent ODE parameters across all segments



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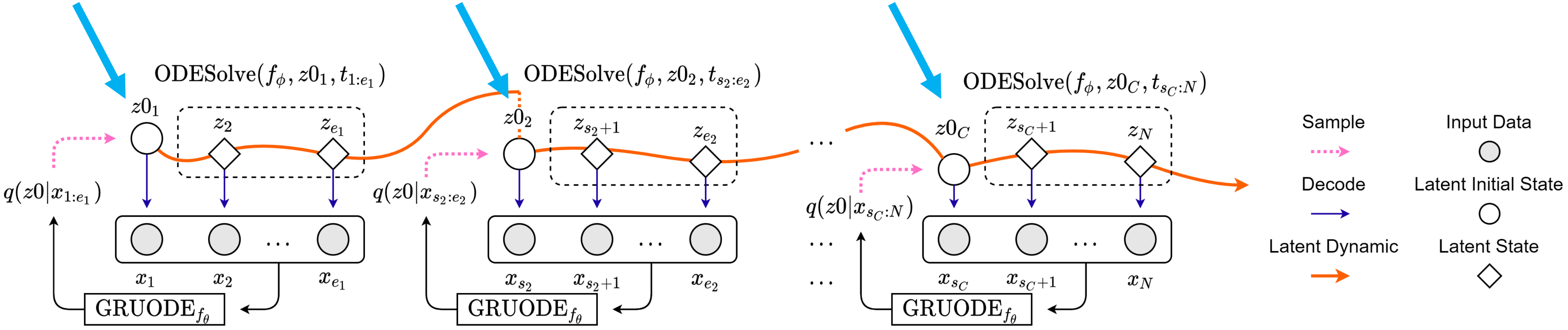
- Share Latent ODE parameters across all segments
- **Start at new latent initial position ($z0$) per segment**



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Segmentation Penalization with Bayesian Occam's Razor

- Use PELT [1] algorithm to search through exponential space of all possible changepoints

$$\mathcal{O}(2^N) \rightarrow O(N) \text{ to } O(N^2)$$

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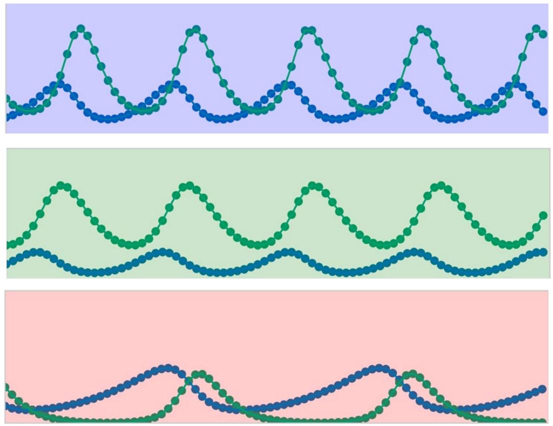
- Marginal likelihood automatically regularized through Bayesian Occam's Razor

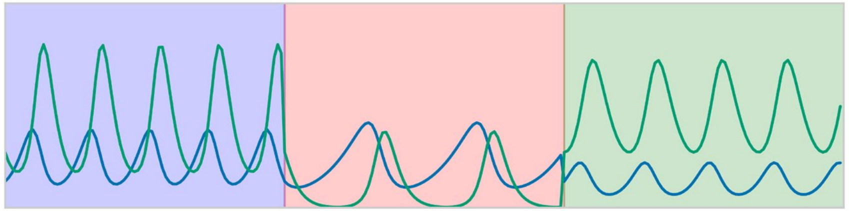
Training the LatSegODE Base Model

Simple Dynamical Primitives

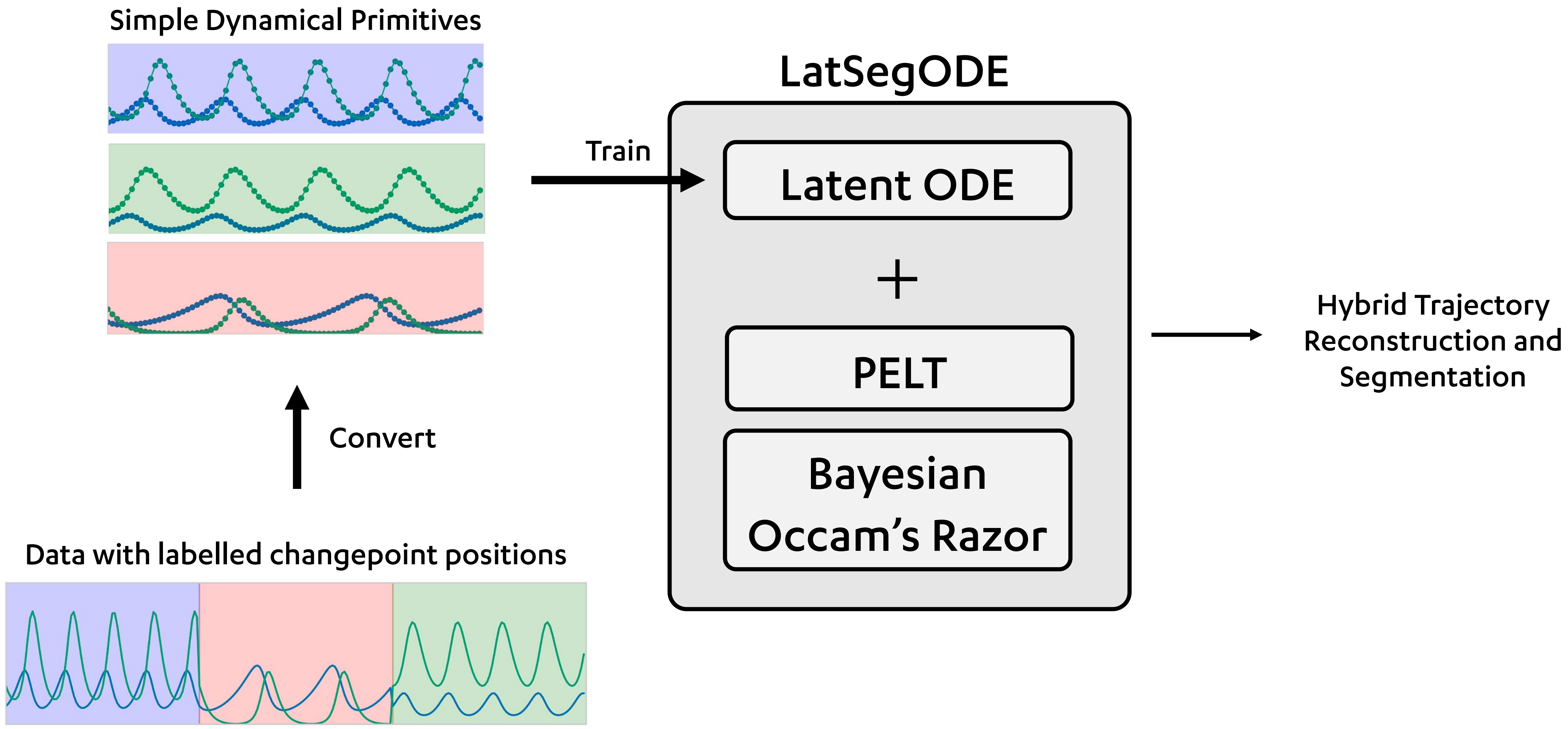


Convert

Data with labelled changepoint positions

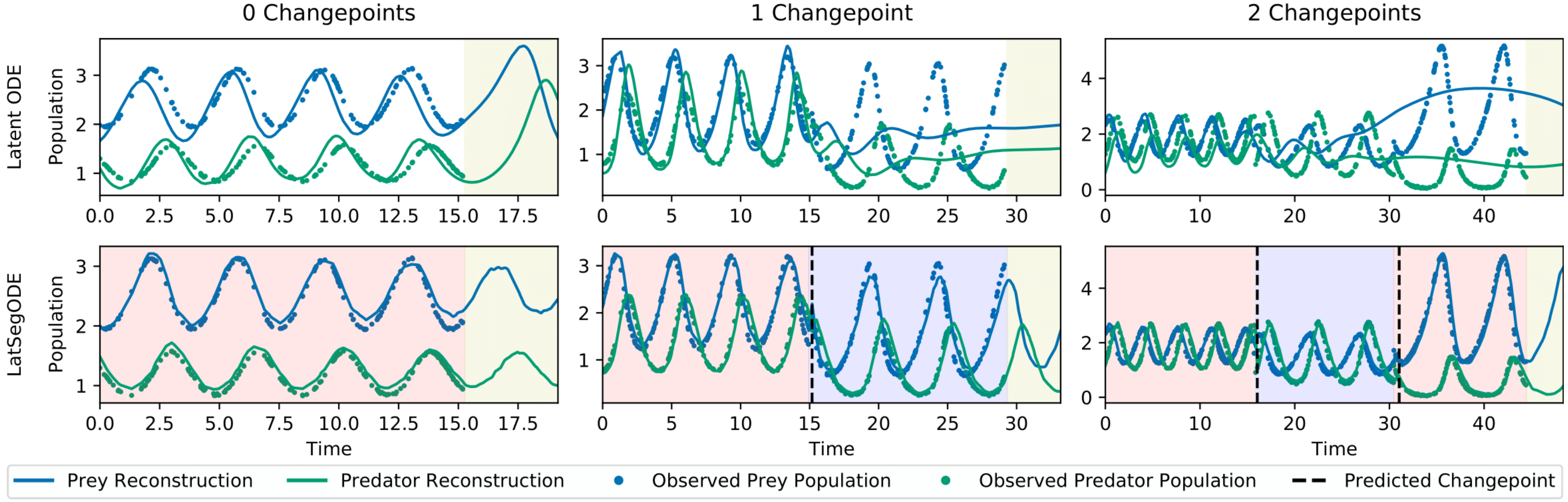


Training the LatSegODE Base Model



Results

- Experiment: Lotka-Volterra hybrid trajectories with up to 3 segments, randomly sampled coefficients.



Method	Test MSE	Rand Index	Hausdorff Metric
LatSegODE	**0.068**	**0.9464**	**47.67**
GRUΔt	0.1718	-	-
GRU-ODE	0.2747	-	-
Latent ODE	0.6155	-	-
RPT-RBF	-	0.7956	84.7
RPT-AR	-	0.6994	164.65
RPT-NORM	-	0.7693	105.92

- Latent ODE time series segmentation of hybrid trajectories
- High accuracy reconstruction and changepoint detection

Thanks!

Future Directions

- Semi-supervised or unsupervised training procedures
- Integration of alternative base models

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