ILLINOIS
Electrical and Computer Engineering Department
Bioengineering Department
GRAINGER COLLEGE OF ENGINEERING

Prior Image-Constrained Reconstruction using Style-based Generative Models

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Images from Incomplete Measurements

Medical imaging

Image credits (L to R):
Canon Global

Images from Incomplete Measurements

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Astronomy

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ADAS

and many more …

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Aim: Estimate **f**.

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Case of interest : $m < n$ ill-posed, need prior knowledge of **f**.

Aim: Estimate **f**.

Compressed sensing

Traditional Compressed Sensing

[Candes *et al.*, 2008]

- Sparsity: $\mathbf{f}$ is k sparse.
  (In general, $\mathbf{z} = \Phi\mathbf{f}$ is k sparse)

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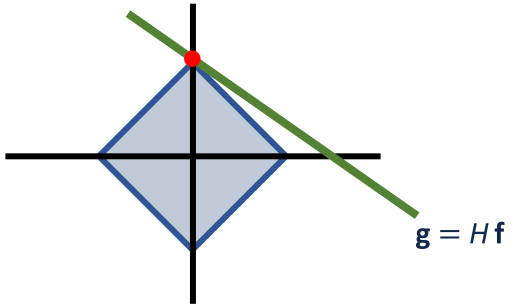

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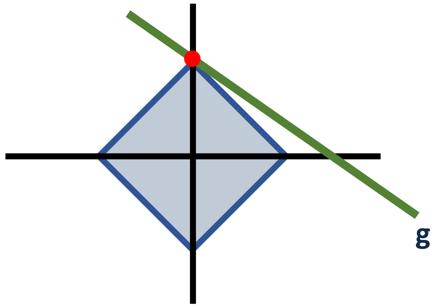

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Compressed sensing with Generative models

[Bora *et al.*, 2017]

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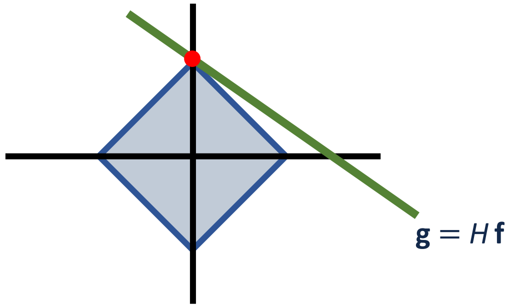

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$$\|H\mathbf{f}_1 - H\mathbf{f}_2\| \geq \gamma\|\mathbf{f}_1 - \mathbf{f}_2\| - \delta; \quad \mathbf{f}_1, \mathbf{f}_2 \in \mathcal{R}(G)$$

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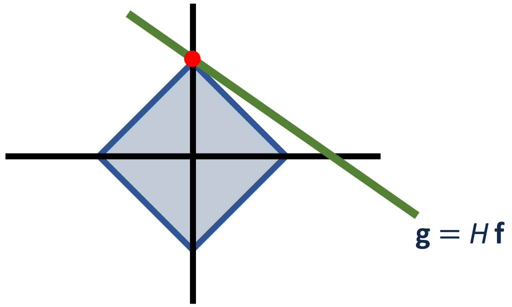

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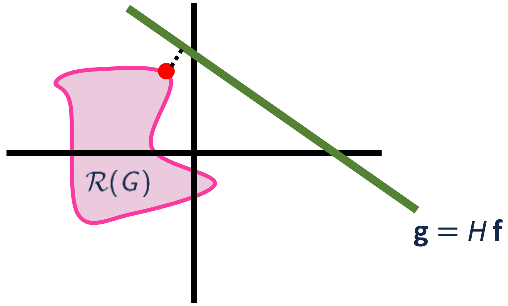
- $O(k\log(L))$ measurements needed.
  L: Lipschitz constant of G



Advances in GANs

StyleGAN2
[Karras, *et al.*, 2020]

DCGAN
[Radford, *et al.*, 2016]

Image source: Bora, et. Al. 2017

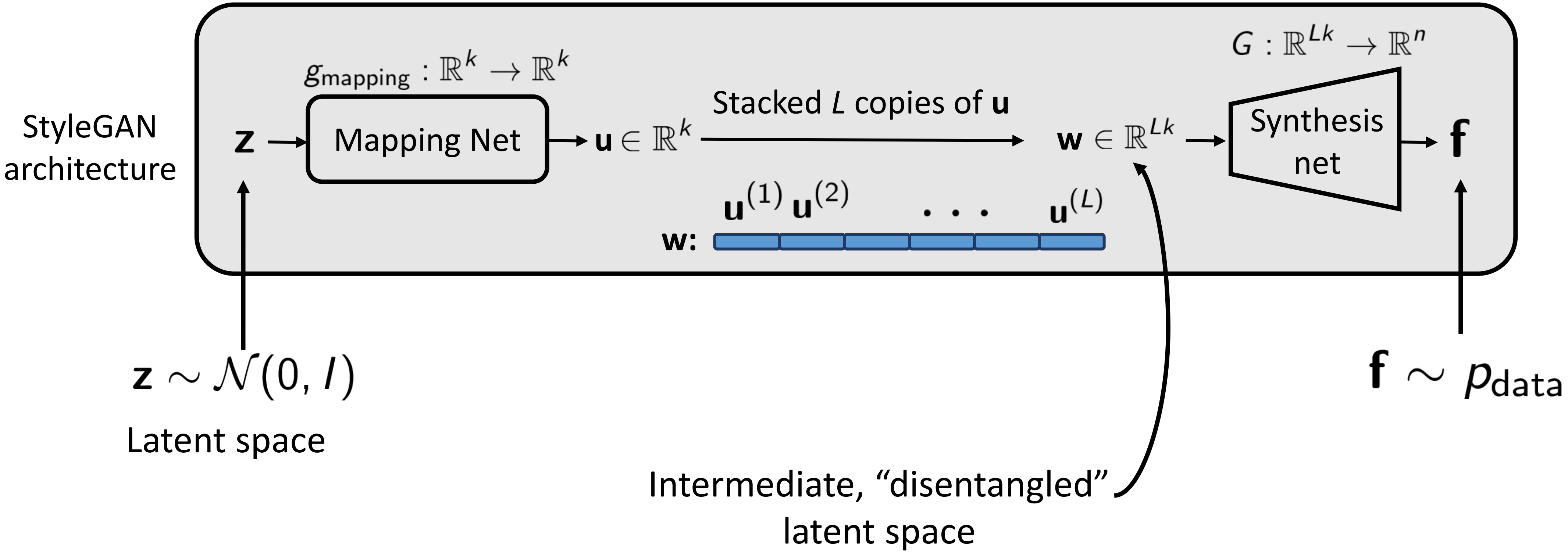
Progressively Growing GAN
[Karras, *et al.*, 2018]

StyleGAN
[Karras, *et al.*, 2019]

- Huge improvements in diversity, invertibility and controllability of GANs
- CSGM benefits from many of these.

StyleGAN: Controlling individual semantic features

An intermediate "disentangled" latent space controls features at different scales.



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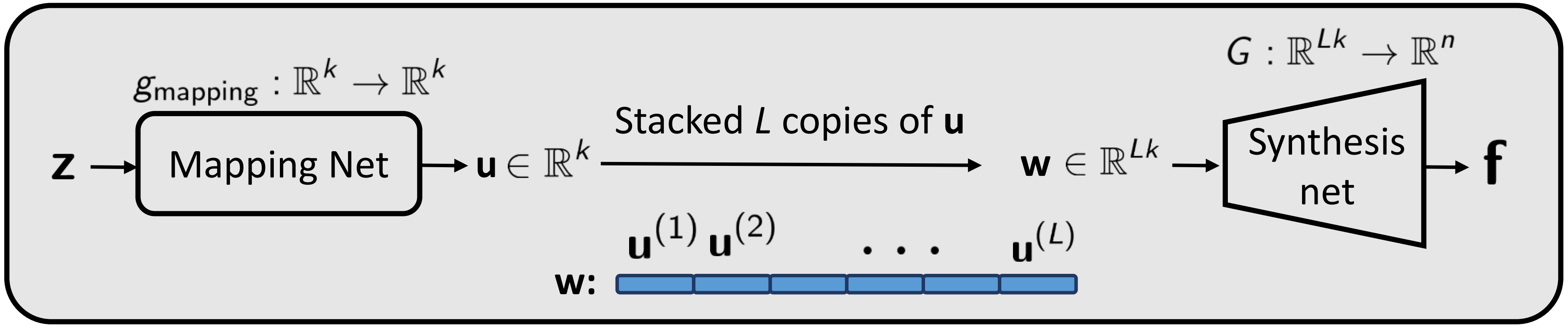
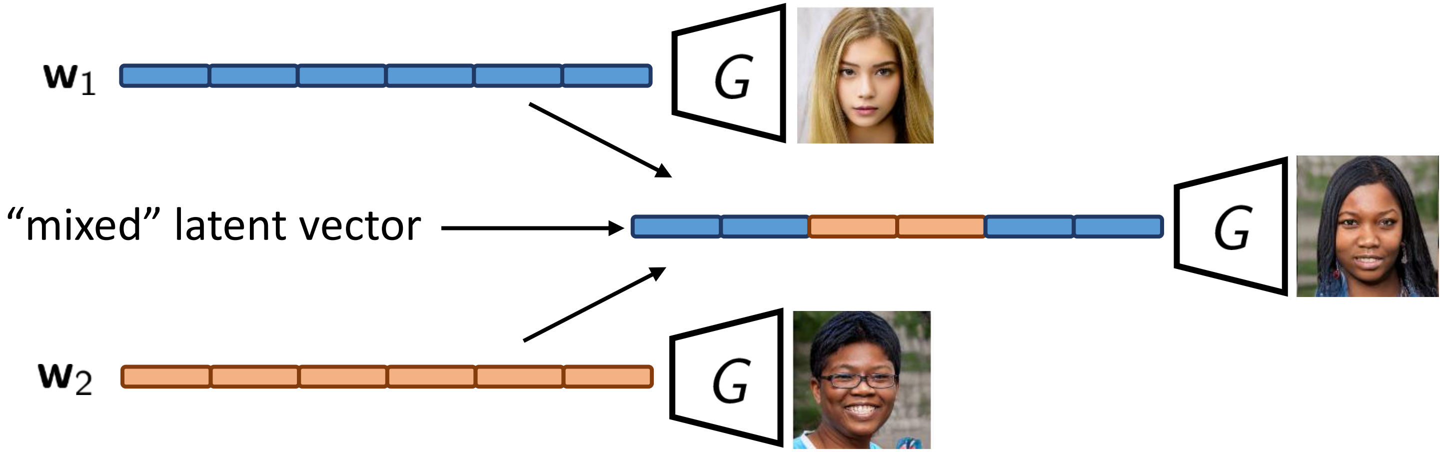
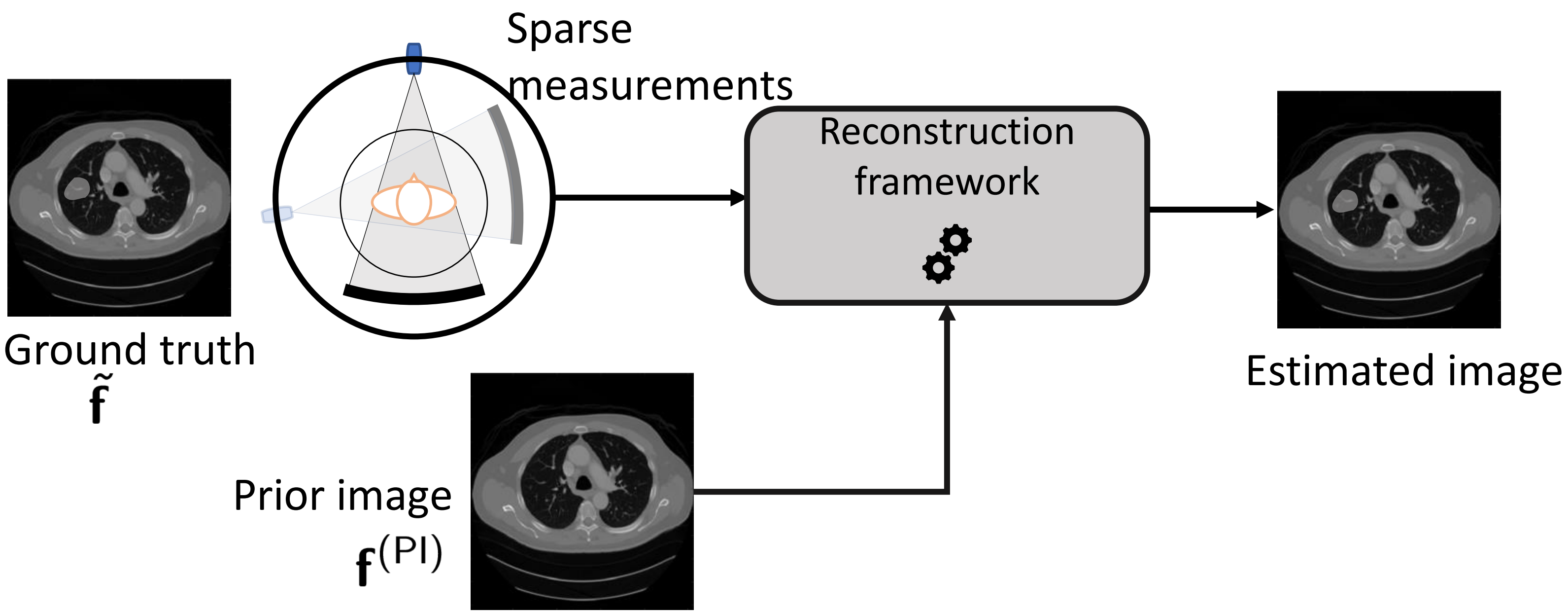
StyleGAN architecture





Prior image-constrained reconstruction

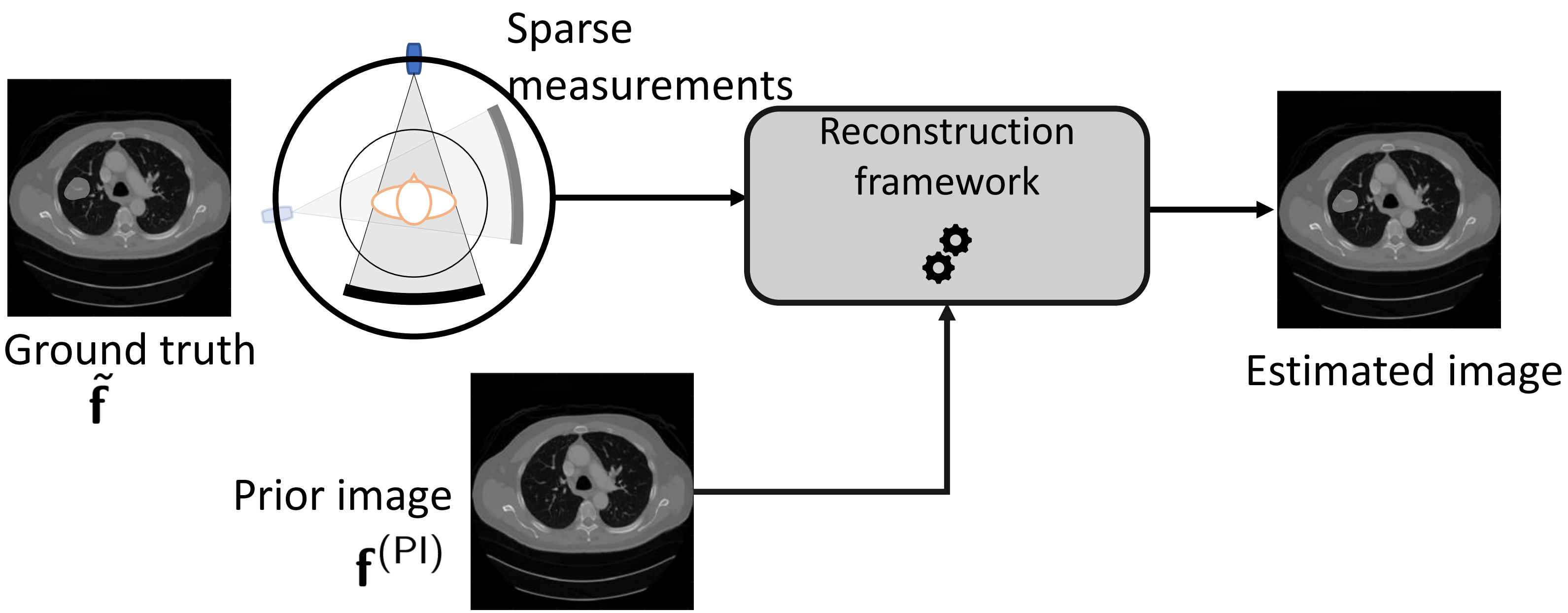
- Access to a previous, related image of the object.
- Arises in monitoring perfusion, tumor progression, sequential radar imaging.



- $\tilde{\mathbf{f}}$ and $\mathbf{f}^{(\mathrm{PI})}$ must be "similar" or "close" to each other.

Prior image-constrained reconstruction

- Access to a previous, related image of the object.
- How to incorporate info from the prior image?



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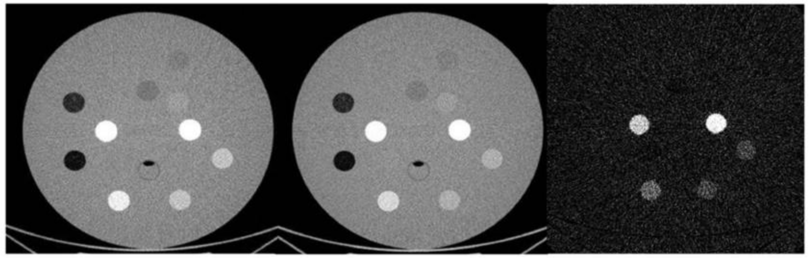
Prior image-constrained reconstruction

Classical approach

[Chen *et al.*, 2008]

Prior image-constrained compressed sensing (PICCS)

- Difference sparsity: Assume that $\tilde{\mathbf{f}} - \mathbf{f}^{(\mathrm{PI})}$ is sparse in some domain.
- Con: Cannot capture semantic differences between $\tilde{\mathbf{f}}$ and $\mathbf{f}^{(\mathrm{PI})}$.



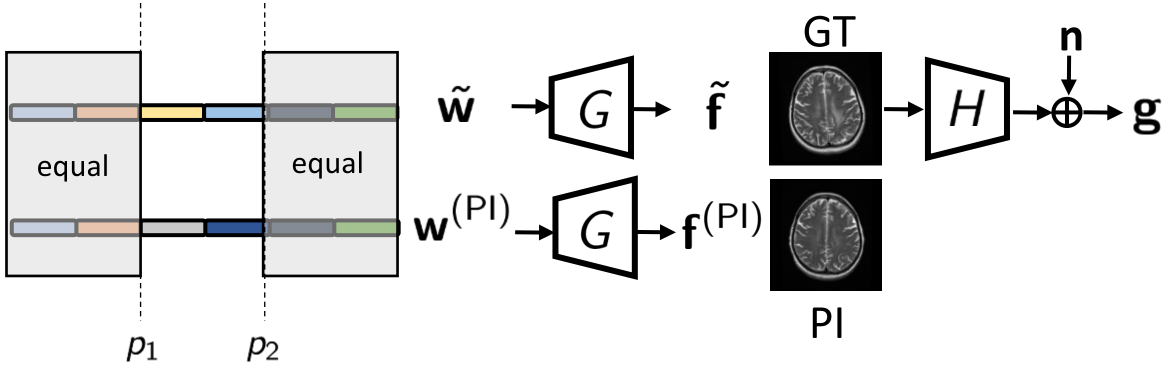
$\tilde{\mathbf{f}}$ $\quad$ $\tilde{\mathbf{f}} - \mathbf{f}^{(\mathrm{PI})}$

Using SyleGANs

[This work]

Prior image-constrained recon. using generative models (PICGM)

- For $\tilde{\mathbf{f}}$ and $\mathbf{f}^{(\mathrm{PI})}$ in $\mathcal{R}(G)$, assume that they differ by a few *styles.*



The PICGM inverse problem

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}} \|\mathbf{g} - HG(\mathbf{w})\|_2^2 + \phi(\mathbf{w}),$$

$$\hat{\mathbf{f}} = G(\hat{\mathbf{w}}).$$

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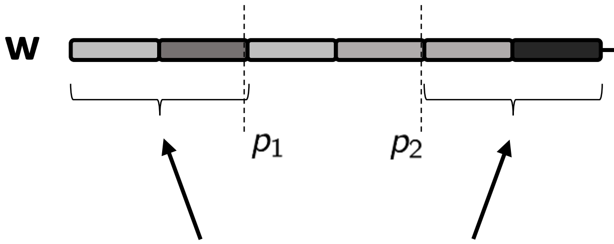

Kept these fixed to corresponding components of $\mathbf{w}^{(\mathrm{PI})}$

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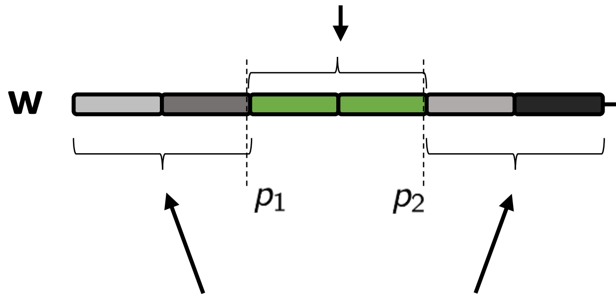

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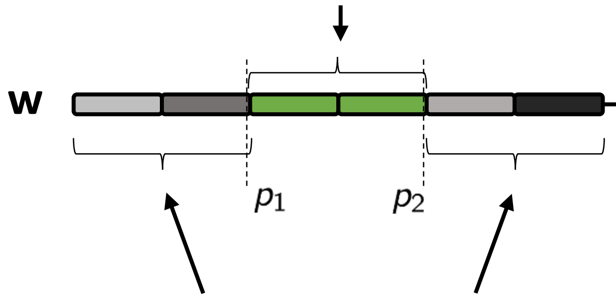

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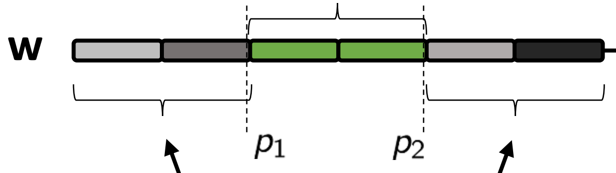

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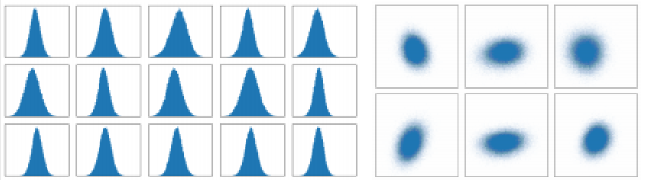
Based on a specific way to Gaussianized the latent space [Wulff and Torralba, 2020]

$$\mathbf{v} = \mathrm{LReL}_5(\mathbf{w}) \in \mathcal{V}$$

$$\mathbf{v} \sim \mathcal{N}(\bar{\mathbf{v}}, \Sigma)$$



Marginals in $\mathcal{V}$ | Pairwise distributions in $\mathcal{V}$

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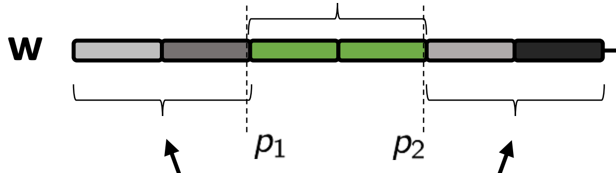

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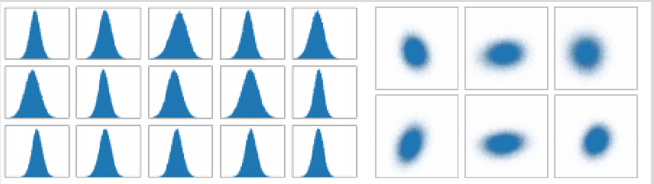


Marginals in $\mathcal{V}$ Pairwise distributions in $\mathcal{V}$

Theoretical Guarantees

Stable recovery up to $\delta + o(\delta)$ error for in-distribution objects from

$$\Omega\left((p_2 - p_1)\log(a\|\Sigma\|_F)/\delta\right)$$

measurements.

$(a = \mathbb{E}\|\mathrm{Jacobian}(G)\|_F)$

Numerical Studies: Face image study

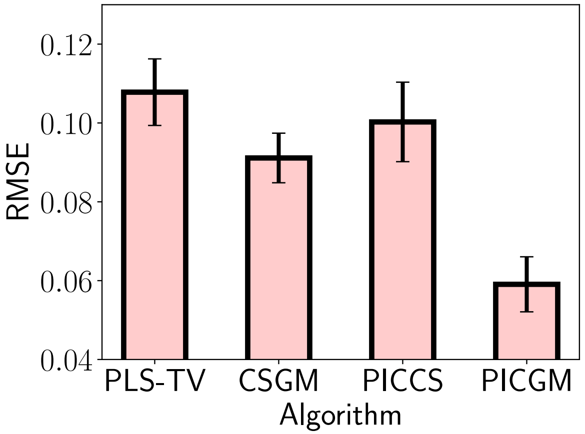
Setup:

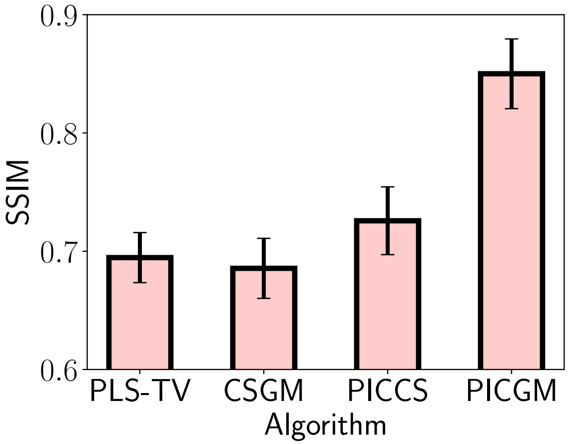
- Gaussian random forward model

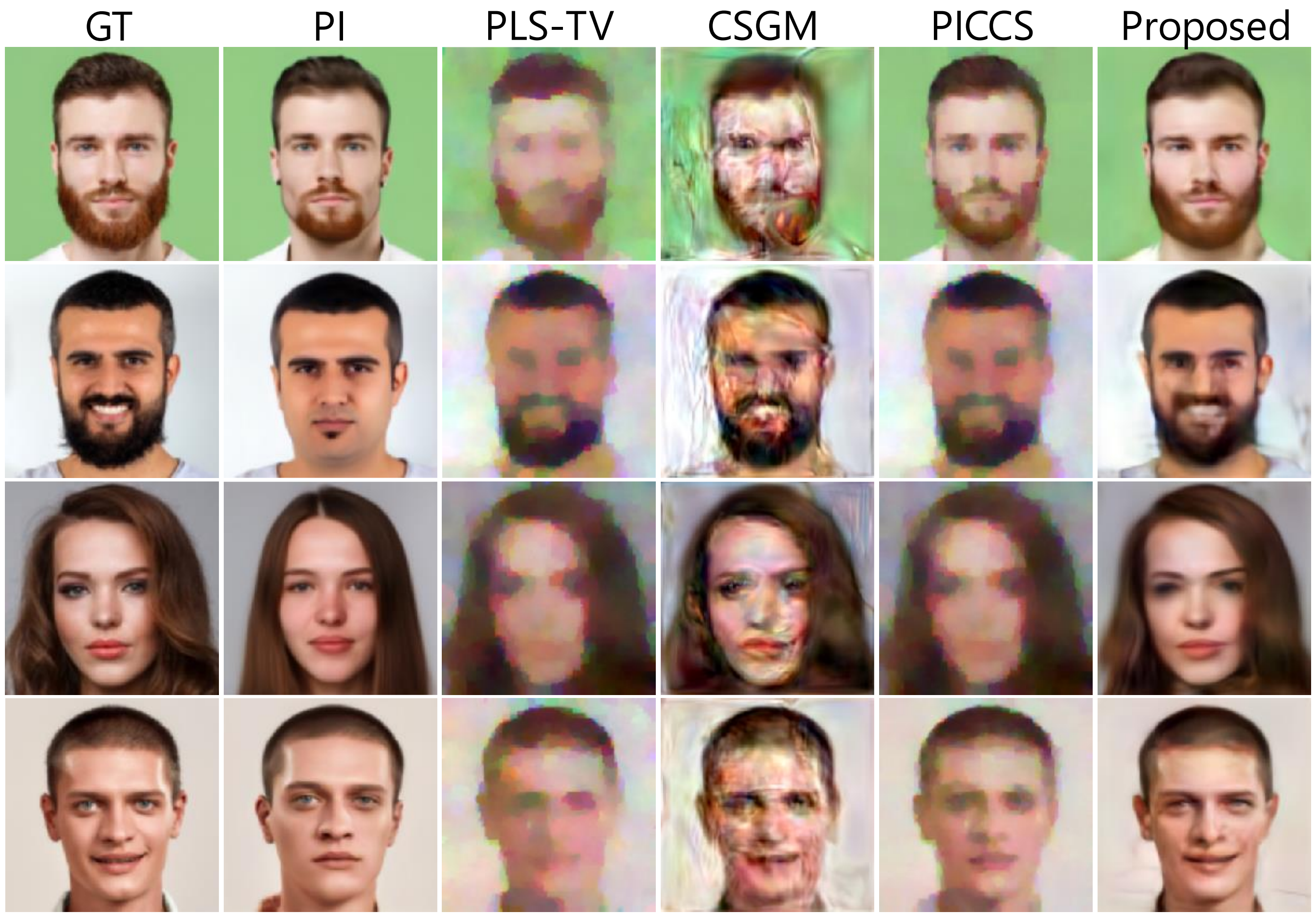
  50x subsampling.
- IID Gaussian noise with 20 dB SNR.

Baselines:

- *PLS-TV* – Penalized least-squares with TV regularization
- *CSGM* – Compressed sensing using generative models
- *PICCS* – Prior image constrained compressed sensing



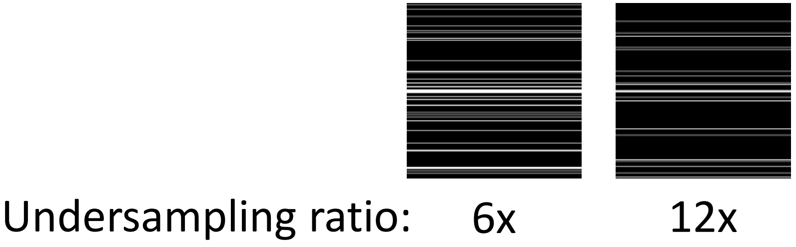


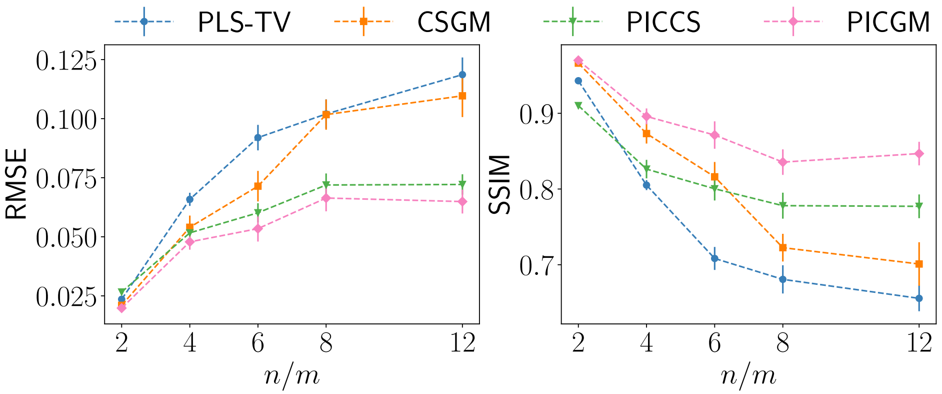


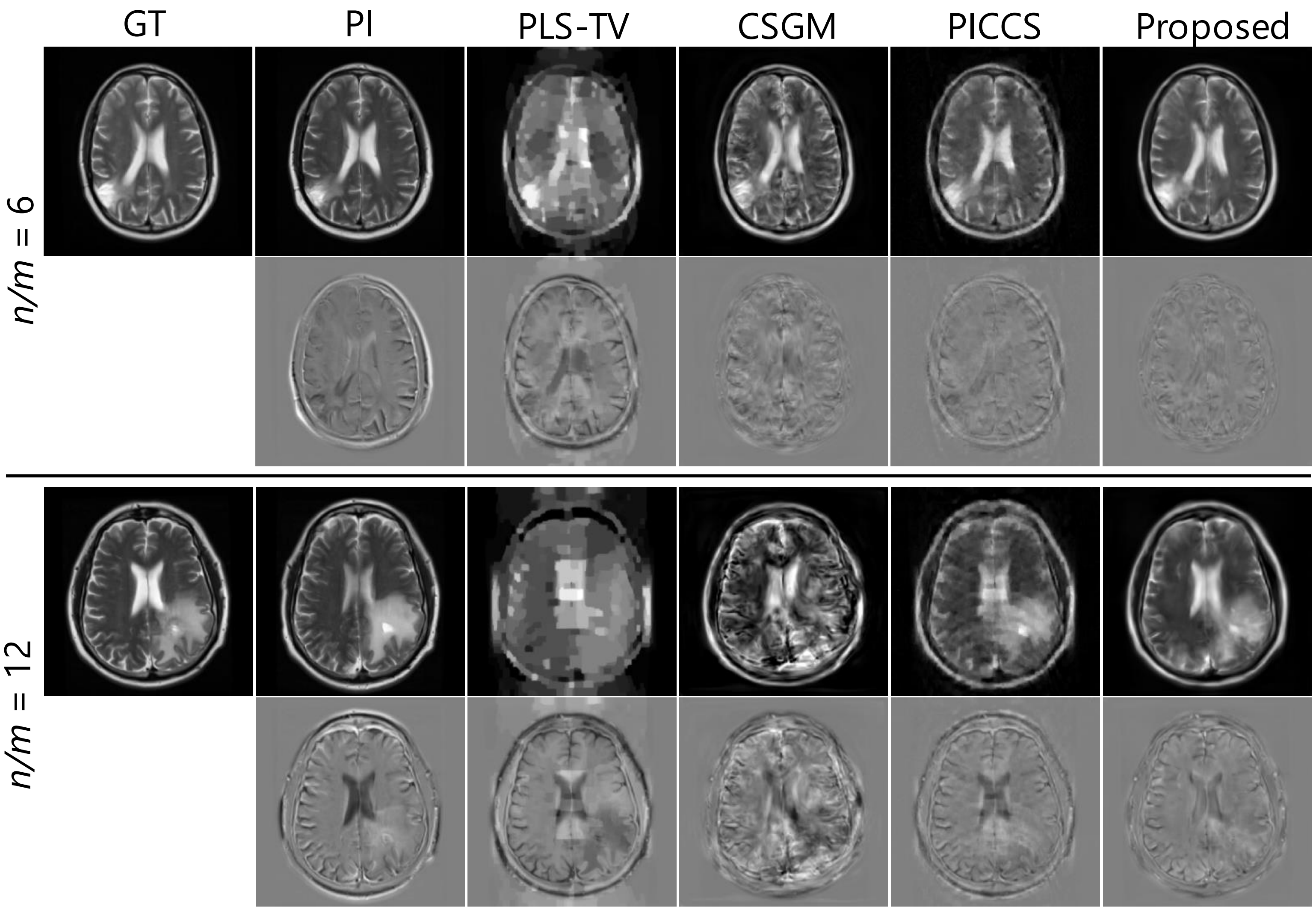
Numerical Studies: MR image study

Setup:

- Simulated MRI measurements with 6x and 12x Fourier undersampling
- Complex-valued iid Gaussian noise with 20 dB SNR.







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Promising numerical results on a realistic application: Proposed approach using StyleGAN and latent-space constraints outperforms classical approaches.

Thank you!

Computational Imaging Science Lab @UIUC

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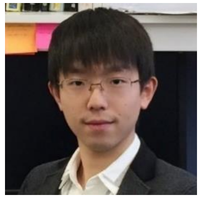
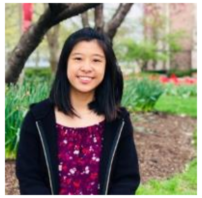
NIH R01EB028652


Mark Anastasio


Sayantan Bhadra


Weimin Zhou


Xiaohui Zhang


Frank Brooks

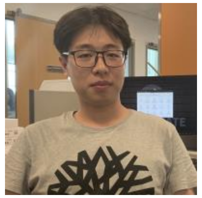

Varun Kelkar


Seonyeong Park


Aashiesh Avachat


Hua Li


Jason Granstedt

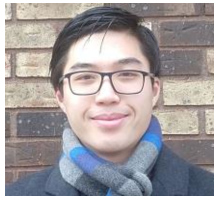

Kaiyan Li


Rucha Deshpande


Umberto Villa


Shenghua He


Fu Li


Joseph Kuo