RNN with Particle Flow for Probabilistic Spatio-temporal Forecasting

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July 17, 2021

McGill HUAWEI Computer Networks Research Laboratory

Introduction

- Exploit underlying graph structure for time series forecasting

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- Applications: road traffic, wireless networks

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https://www.tomtom.com/blog/traffic-and-travel-information/road-traffic-prediction/

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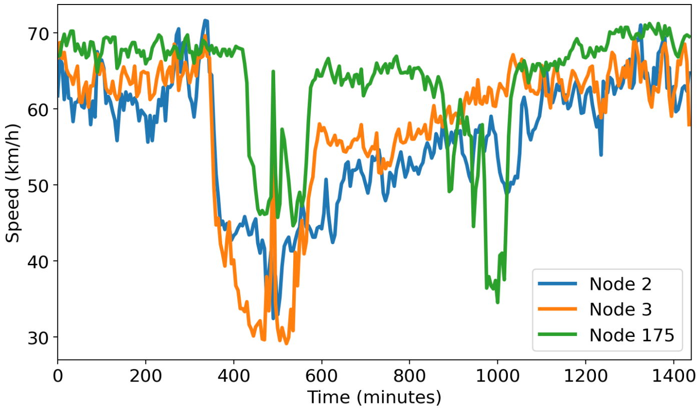

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  - Graph convolution + recurrent networks[1]
  - Temporal convolution[2]
  - Attention mechanism[3]

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Problem formulation

State-space model

Initial state distribution: $\mathrm{x}_1 \sim p_1(\cdot, \mathrm{z}_1, \rho)$,

State transition model: $\mathrm{x}_t = g_{\mathcal{G},\psi}(\mathrm{x}_{t-1}, \mathrm{y}_{t-1}, \mathrm{z}_t, \mathrm{v}_t)$, for $t > 1$,

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- Unknown model parameters: $\Theta = \{\rho, \psi, \sigma, \phi, \gamma\}$

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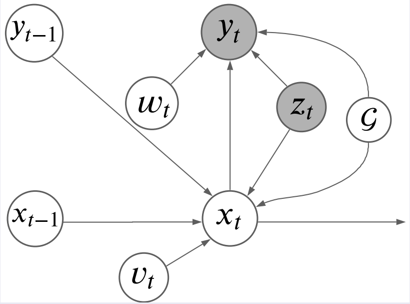

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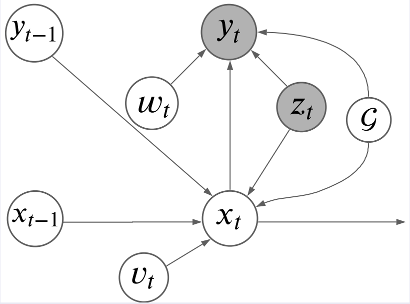

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Predict $\mathrm{y}_{t_0+P+1:t_0+P+Q}$ based on $\mathrm{y}_{t_0+1:t_0+P}$, $\mathrm{z}_{t_0+1:t_0+P+Q}$, and (possibly) $\mathcal{G}$

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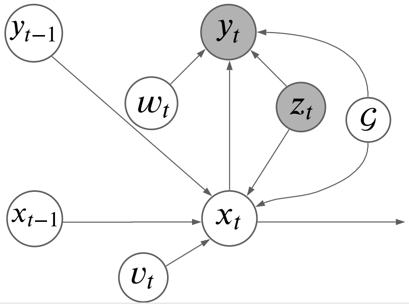

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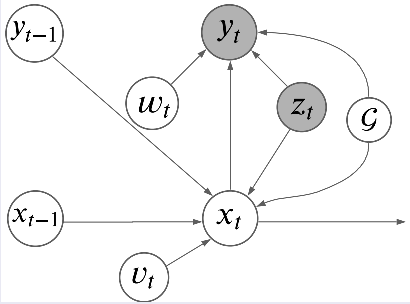

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- Train the model to learn Θ
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Computing forecast distribution

$$p_\Theta(y_{P+1:P+Q} | y_{1:P}, z_{1:P+Q}) = \int \prod_{t=P+1}^{P+Q} \Big(p_{\phi,\gamma}(y_t | x_t, z_t) \, p_{\psi,\sigma}(x_t | x_{t-1}, y_{t-1}, z_t) \Big) \, p_\Theta(x_P | y_{1:P}, z_{1:P}) dx_{P:P+Q} \, .$$

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Particle filter: weight degeneracy

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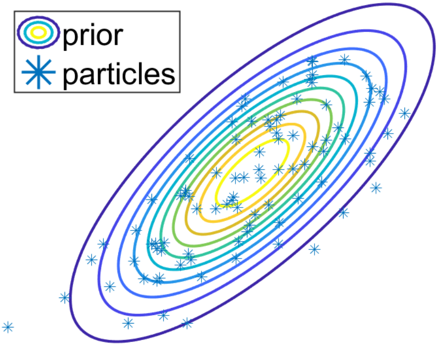

Contours of the prior distribution

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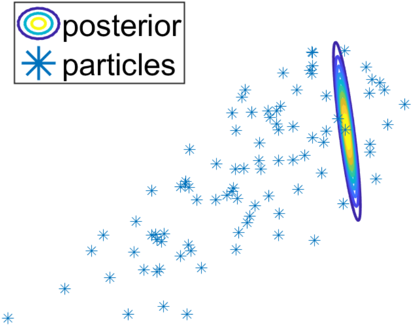

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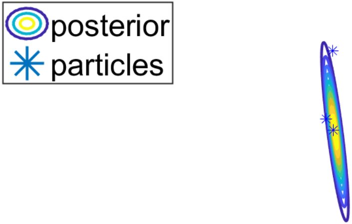

Resampling of the particles

Particle flow

Particles flow[7] migrates particles from the prior to the posterior distribution.

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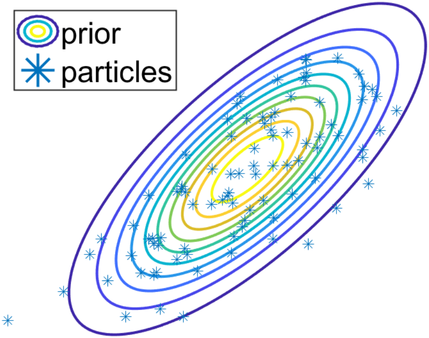

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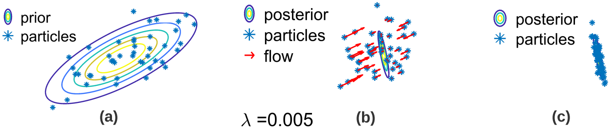
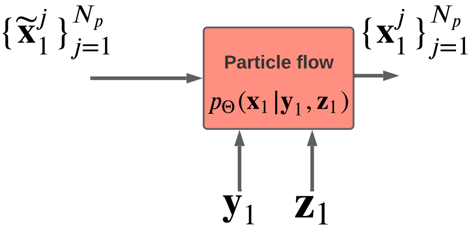


(a) Samples (asterisk) from the prior distribution
(b) Contours of the posterior distribution and the direction of flow for the particles at an intermediate step
(c) Particles after the flow, approximately distributed according to the posterior distribution

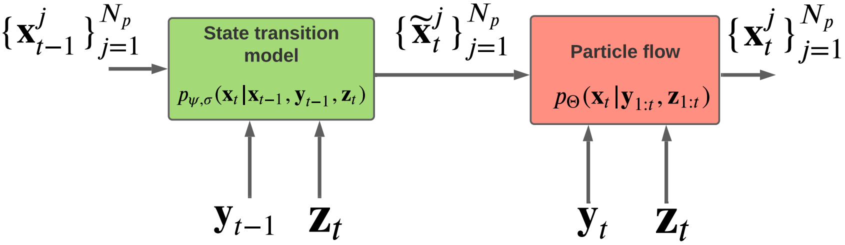
$$t = 1$$

$$\{\widetilde{\mathbf{x}}_1^j\}_{j=1}^{N_p} \sim p_1(\cdot, \mathbf{z}_1, \rho)$$



Computing forecast distribution

$$2 \leqslant t \leqslant P$$



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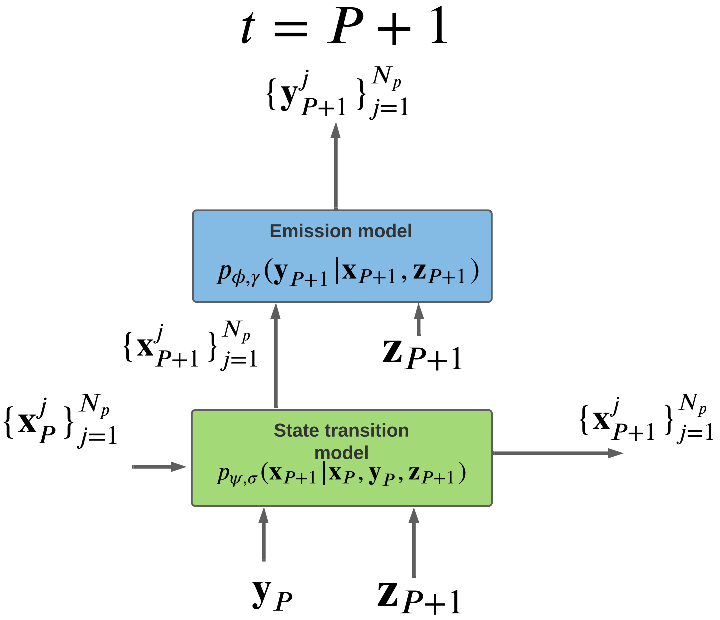

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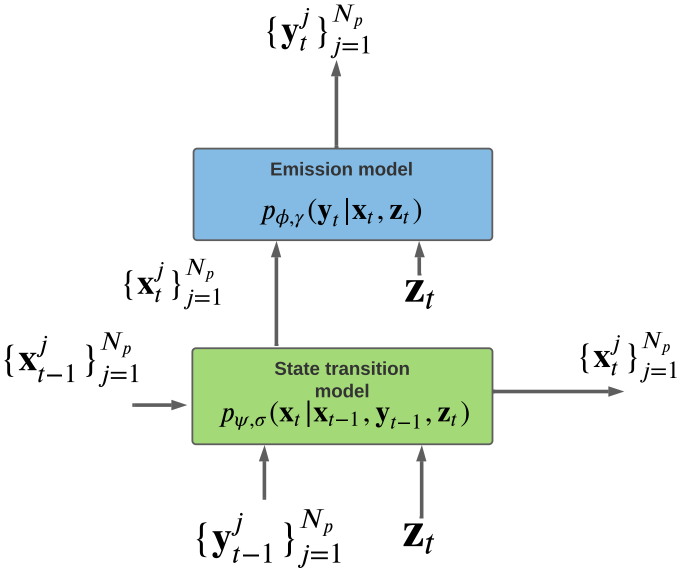
$$P + 2 \leqslant t \leqslant P + Q$$



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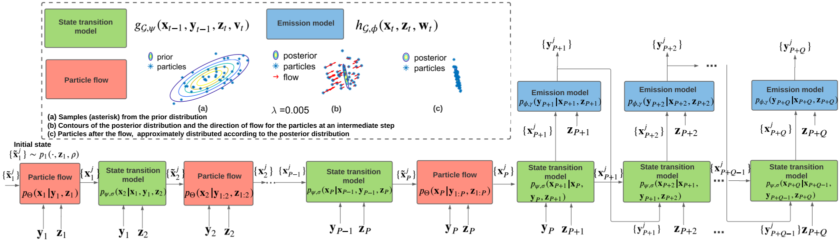

Approximation of the joint posterior distribution of the forecasts

Loss function

- For point forecasting: MAE, MSE
- For probabilistic forecasting: negative log posterior probability

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$$\mathcal{L}_{\text{prob}}(\Theta, \mathcal{D}) = -\frac{1}{|\mathcal{D}|} \sum_{n \in \mathcal{D}} \log p_{\Theta}(\mathrm{y}_{P+1:P+Q}^{(n)} | \mathrm{y}_{1:P}^{(n)}, \mathrm{z}_{1:P+Q}^{(n)}),$$

$$\widehat{p}_{\Theta}(\mathrm{y}_{P+1:P+Q} | \mathrm{y}_{1:P}, \mathrm{z}_{P+1:P+Q}) = \prod_{t=P+1}^{P+Q} \left[\frac{1}{N_p} \sum_{j=1}^{N_p} p_{\phi,\gamma}(\mathrm{y}_t | \mathrm{x}_t^j, \mathrm{z}_t) \right].$$

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- Performance metrics for probabilistic forecasting:
  - Continuous Ranked Probability Score (CRPS)[9]
  - P10, P50, and P90 Quantile Losses[10]

[8] Chen et al. 2000

[9] Gneiting & Raftery 2007

[10] Wang et al. 2019

Baselines

- Statistical and ML point forecast models:
  - HA, ARIMA[11], VAR[12], SVR[13], FNN, FC-LSTM[14]

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  - DeepAR[25], DeepFactors[26], MQRNN[27]

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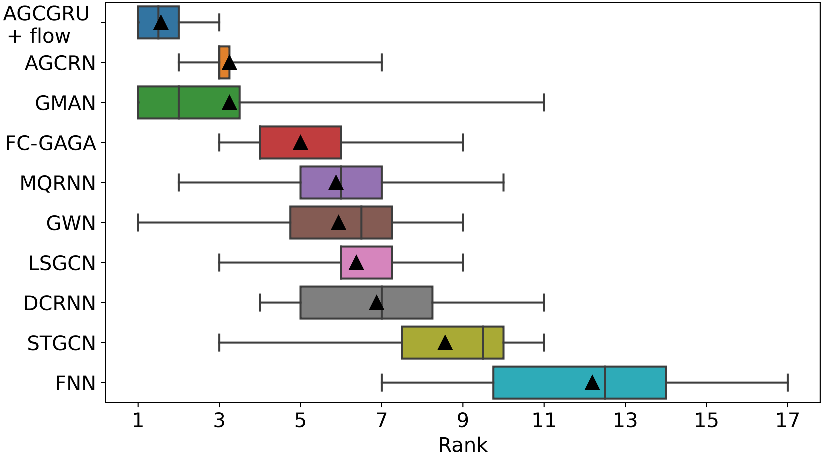
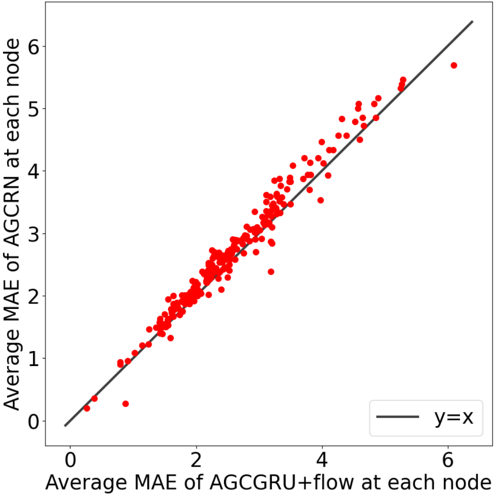
[25] Salinas et al. 2020, [26] Wang et al. 2019, [27] Wen et al. 2017

Experimental results: point forecasting



AGCGRU+flow achieves the best average rank.

Experimental results: node by node comparison



AGCGRU+flow outperforms AGCRN at majority of nodes in PeMSD7

Experimental results: probabilistic forecasting

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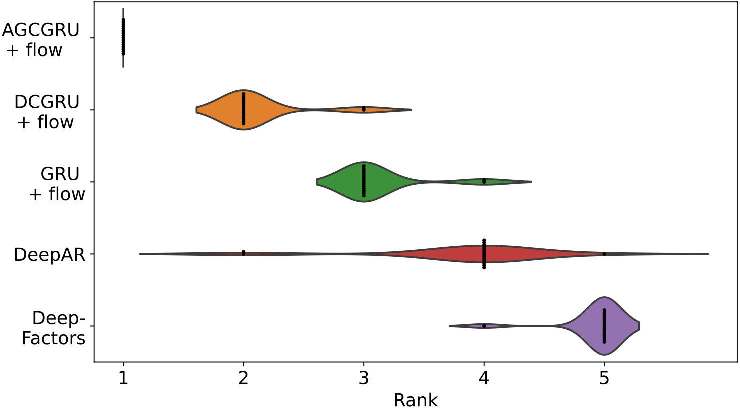

Our approaches obtain lower average CRPS.

Experimental results: quantile estimation

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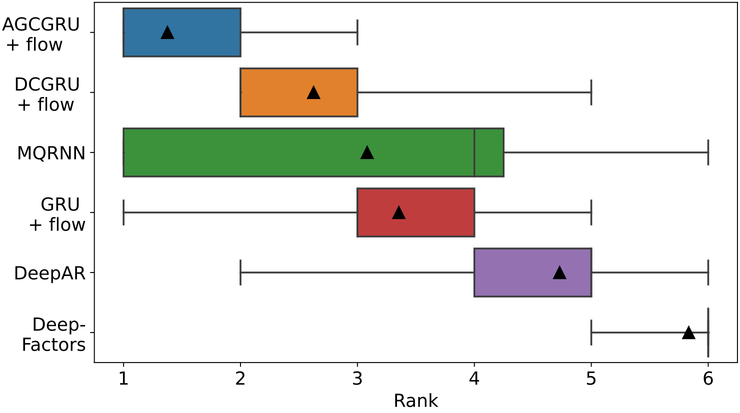
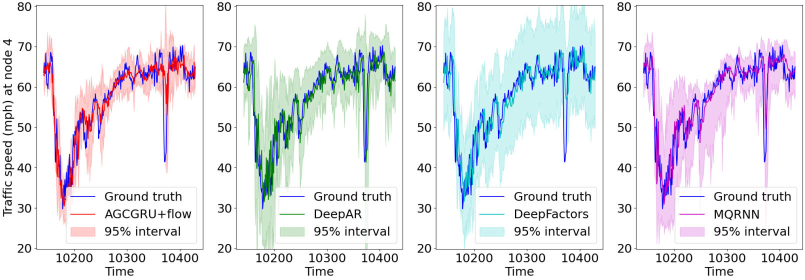


AGCGRU+flow has the lowest quantile error on average.

Experimental results: confidence intervals



Confidence intervals for 15 minutes ahead predictions at node 4 of PeMSD7 for the first day in the test set.

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- Code: `https://github.com/networkslab/rnn_flow`