FILTRA: Rethinking Steerable CNN by Filter Transform

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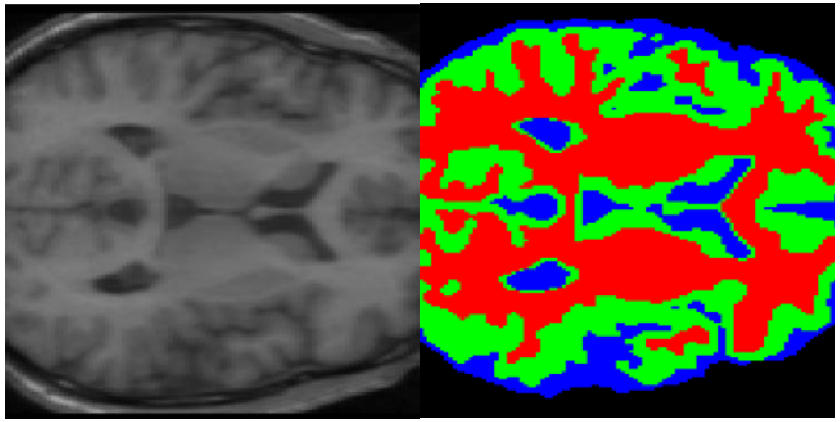
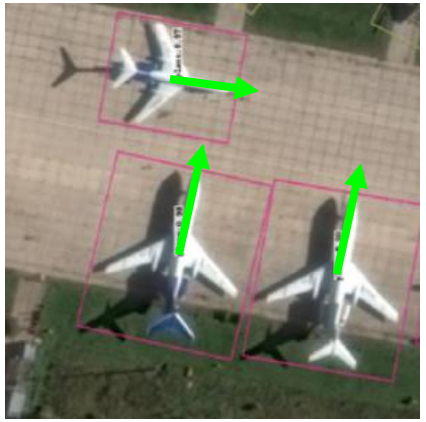
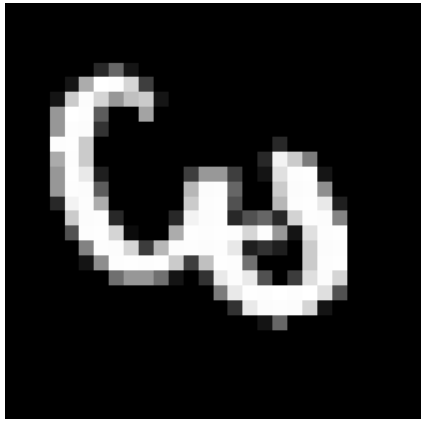
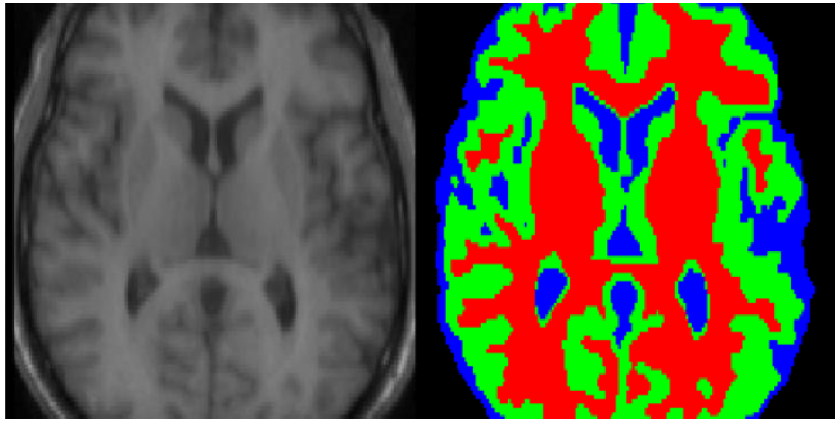
Practical Demand for Steerable CNN

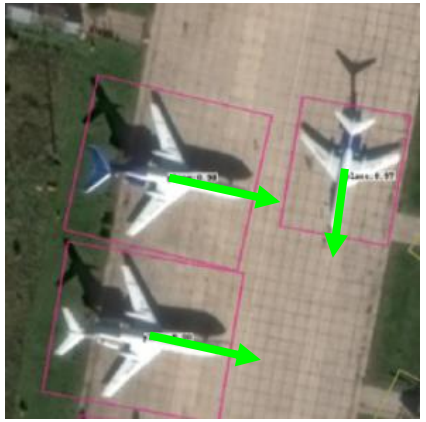
Rotation











Classification

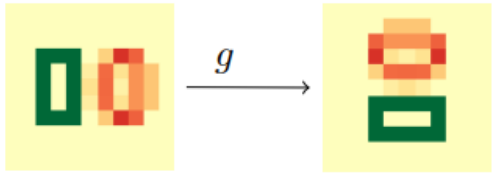
Segmentation

Regression

Group Representation for a Feature Map

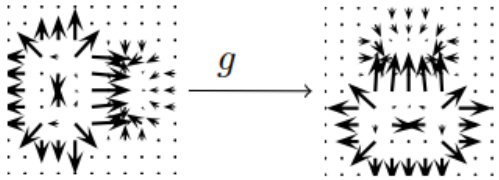
Trivial repr
RGB, gray, segmentation images

$\rho \equiv 1$



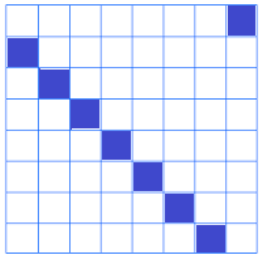
Irreducible repr
Oriented bboxes, direction vectors

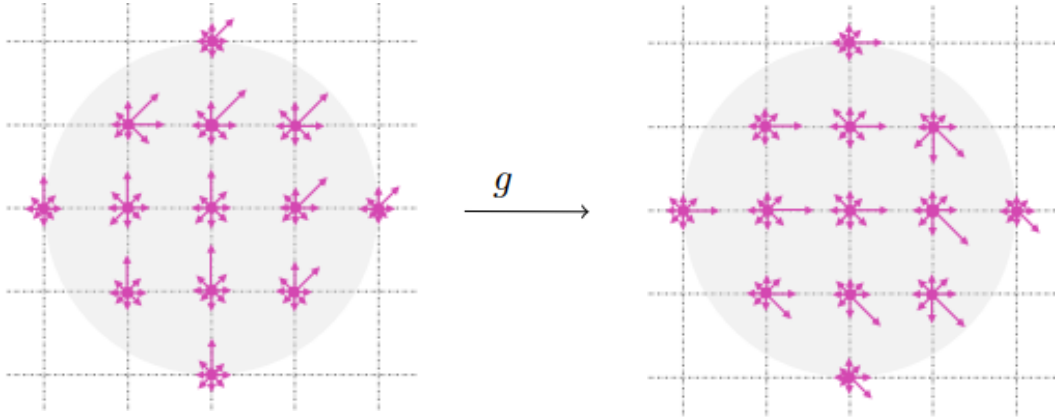
$$\rho = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$



Regular repr

$\rho =$





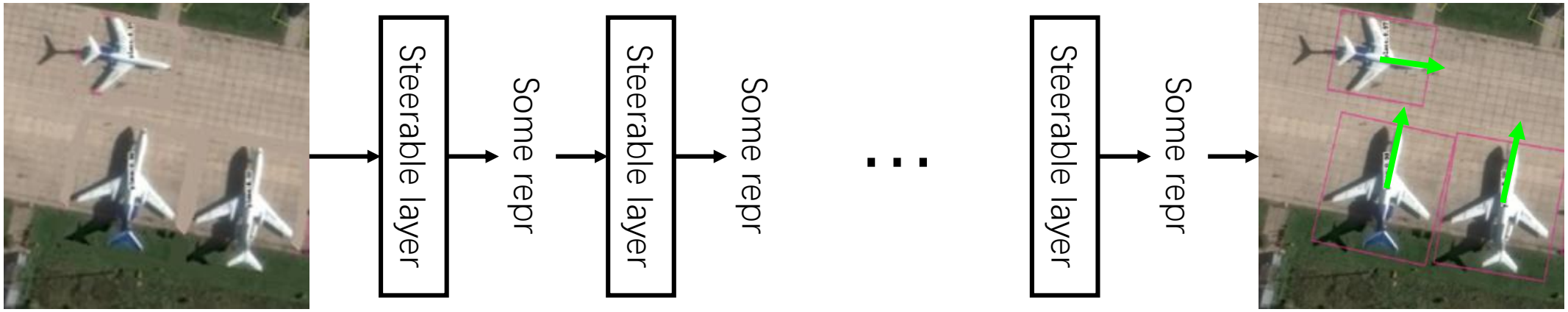
Steerable CNN Formulation

In the scenario of convolutional neural network, a convolution operator $f \mapsto \kappa \cdot f$ is considered as *steerable* if it satisfies

$$\kappa \cdot [\pi_1(g)f] = \pi_2(g)[\kappa \cdot f], \qquad (2)$$

In a word

Rotate then convolve = convolve then rotate



Trivial repr

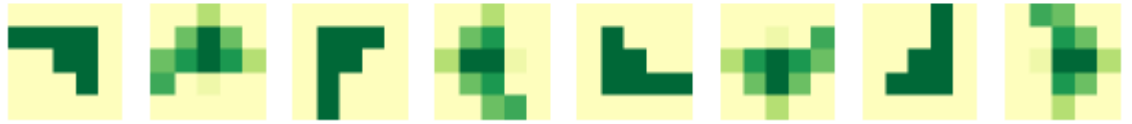
Inrreducible repr

Steerable Convolution between Reprs

A C_8 steerable filter from a nomal 5x5 filter

Trivial to regular 8x5x5x1

$$\mathrm{K}_{0\to\mathrm{reg}}^{\mathrm{C}_N} = \mathrm{K}$$



Trivial to regular 8x5x5x2

$$\mathrm{K}_{k\to\mathrm{reg}}^{\mathrm{C}_N}$$

$$\mathrm{diag}(\mathrm{K})\beta_k^0$$

$$\mathrm{diag}(\mathrm{K})\beta_k^1$$

A Steerable CNN with FILTRA Blocks

$$\mathsf{K}^{\mathrm{C}_N}_{0\to\mathrm{reg}} = \mathsf{K}.$$

$$\mathsf{K}^{\mathrm{C}_N}_{k\to\mathrm{reg}} = \mathrm{diag}(\mathsf{K})\beta_k,$$

$$\mathsf{K}^{\mathrm{C}_N}_{\mathrm{reg}\to 0} = \mathsf{K}.^{\top}$$

$$\mathsf{K}^{\mathrm{C}_N}_{\mathrm{reg}\to k} = \mathsf{K}^{\mathrm{C}_N}_{k\to\mathrm{reg}}{}^{\top}$$

$$\mathsf{K}^{\mathrm{C}_N}_{\mathrm{reg}\to\mathrm{reg}} = \left[\mathsf{K}^{\mathrm{C}_N}_{0\to\mathrm{reg}} \cdots \mathsf{K}^{\mathrm{C}_N}_{\lfloor\frac{N}{2}\rfloor\to\mathrm{reg}}\right] V^{-1}.$$