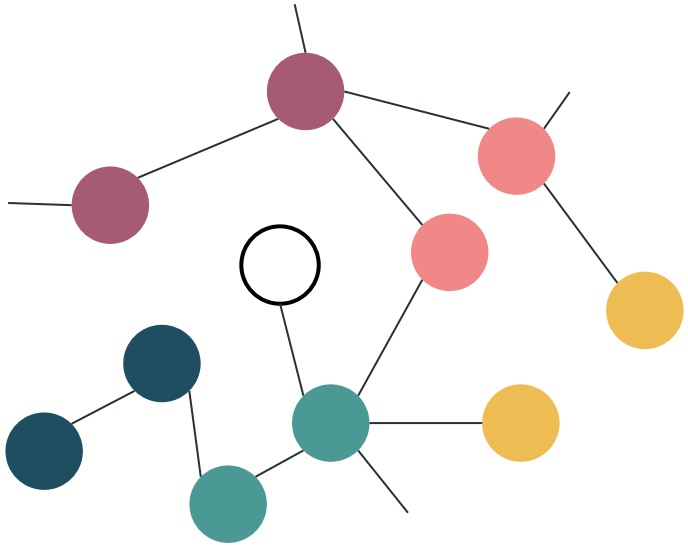
Directed Graph Embeddings in Pseudo-Riemannian Manifolds

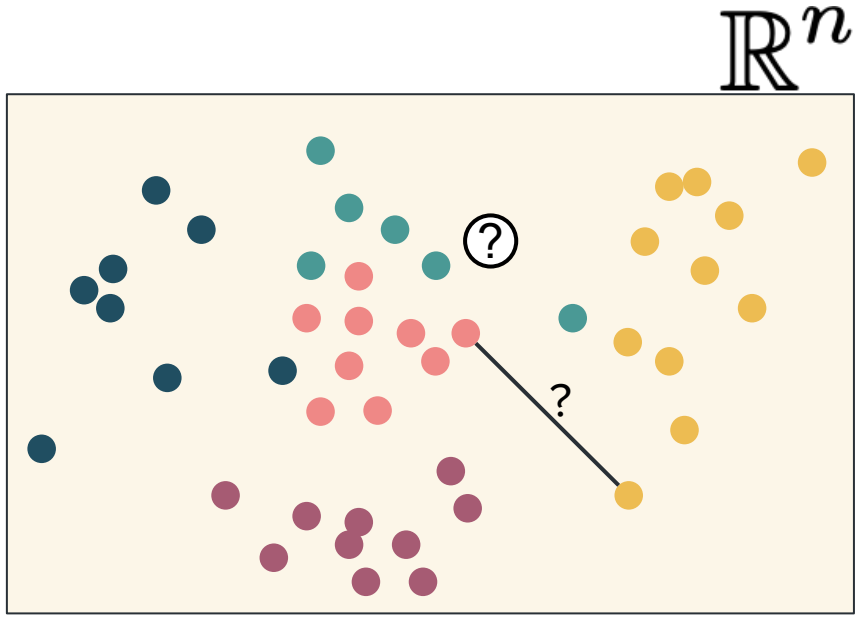
Aaron Sim Maciej Wiatrak Angus Brayne Páidí Creed Saee Paliwal

Benevolent AI

The Graph Embedding Problem:

Finding a vector representation for each node in a graph



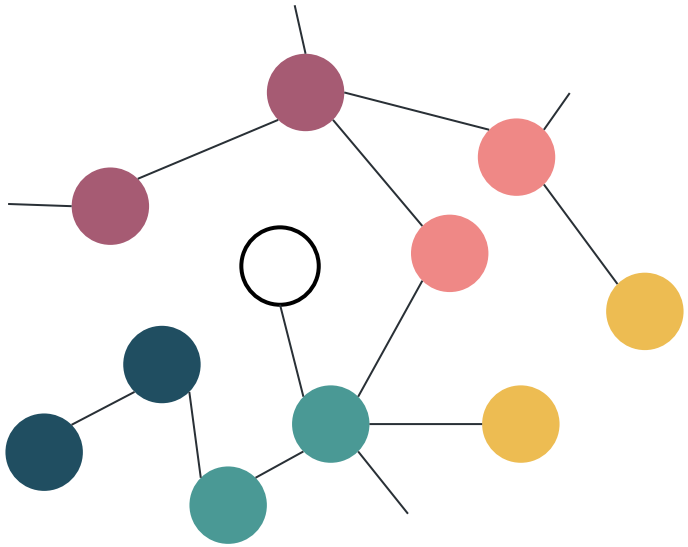


- Small social networks (**10^2 nodes)**
- Gene regulatory networks (**10^5 nodes)**
- Knowledge graphs (**10^7 nodes)**

- Uncover hidden data features
- Node Classification, Link Prediction

The Graph Embedding Problem:

Finding a vector representation for each node in a graph



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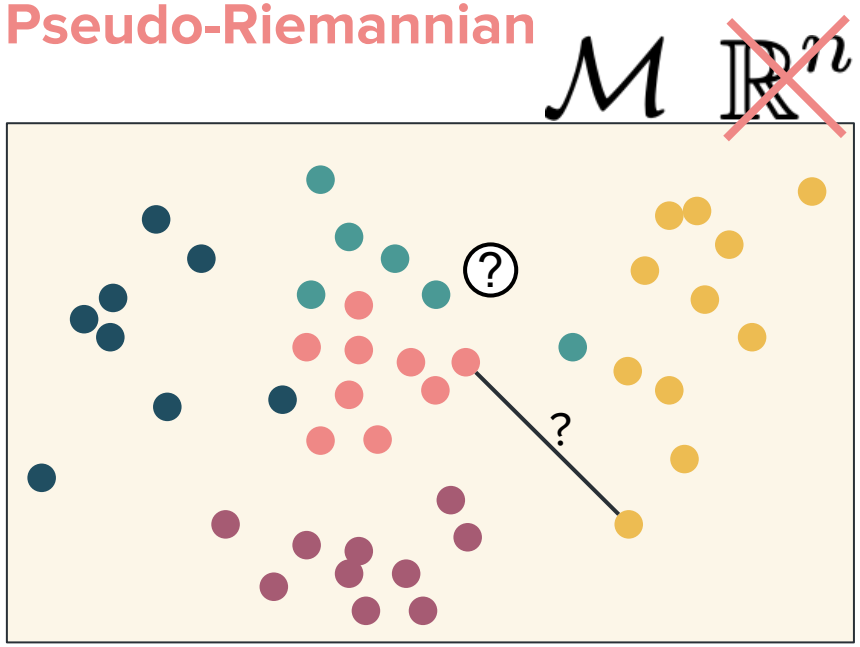
Pseudo-Riemannian



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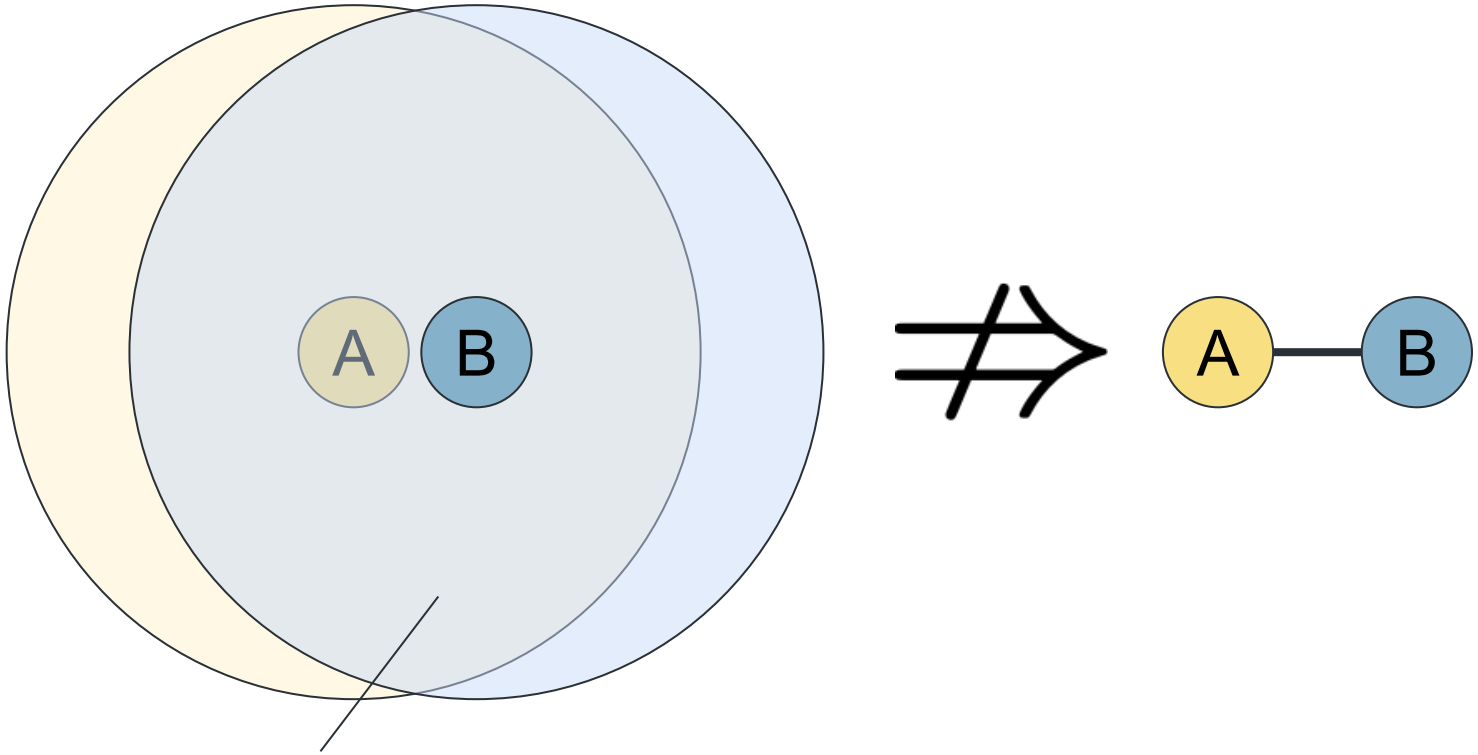
Semantic Similarity vs. Graph Edges



Large overlapping neighborhood set

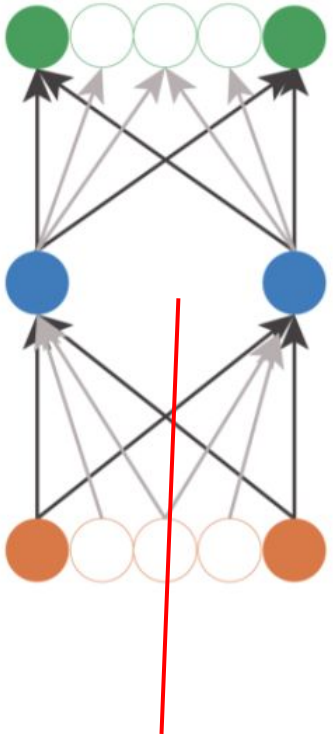
Social networks
- Colleagues

Competitive sports
- Siblings

Gene regulatory networks
- Correlated expression vs. No direct causal link

Common feature in real world graphs

Riemannian manifolds cannot disambiguate semantic similarity and edges



No edge

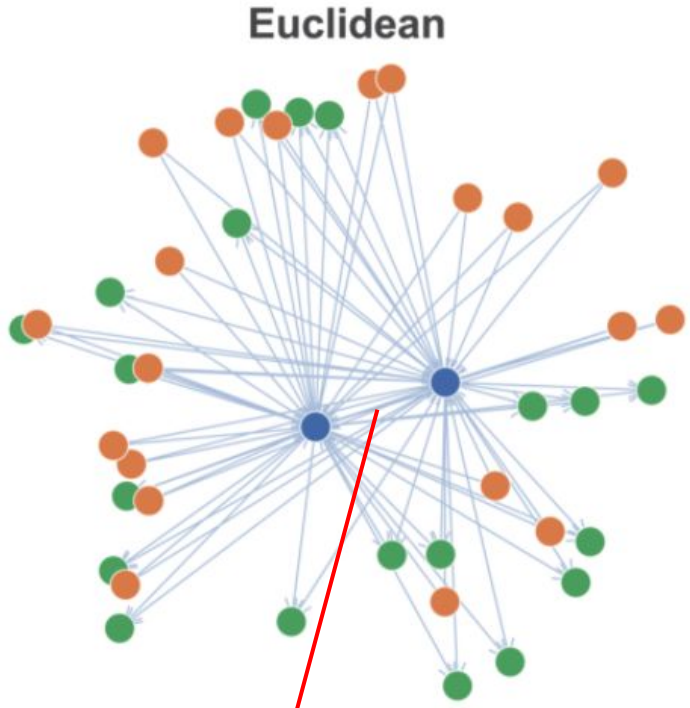
Embed in Euclidean space (SGD)

$$p(A \to B) = \frac{1}{e^{s^2} + 1}$$

Squared embedding distance

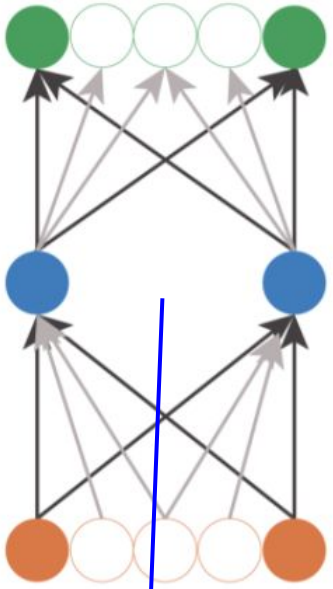
$$s^2 = 0 \quad \Rightarrow \quad p = \frac{1}{2}$$

$$s^2 \to \infty \quad \Rightarrow \quad p \to 0$$



$p = 0.50$

Proposal 1: Embed in pseudo-Riemannian manifolds



No edge

Embed in Minkowski spacetime (PR-SGD)

$$p(A \to B) = \frac{1}{e^{s^2} + 1}$$

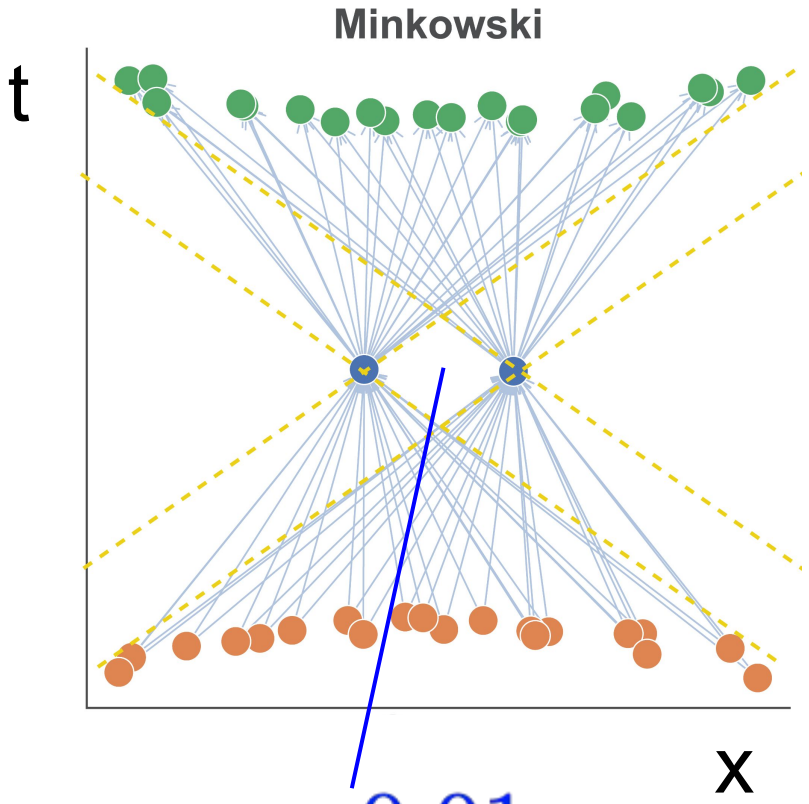
Squared embedding distance

$$s^2 < 0 \quad \Rightarrow \quad \frac{1}{2} < p < 1$$

$$s^2 = 0 \quad \Rightarrow \quad p = \frac{1}{2}$$

$$s^2 \to \infty \quad \Rightarrow \quad p \to 0$$

$$s^2 = -(\Delta t)^2 + (\Delta x)^2$$

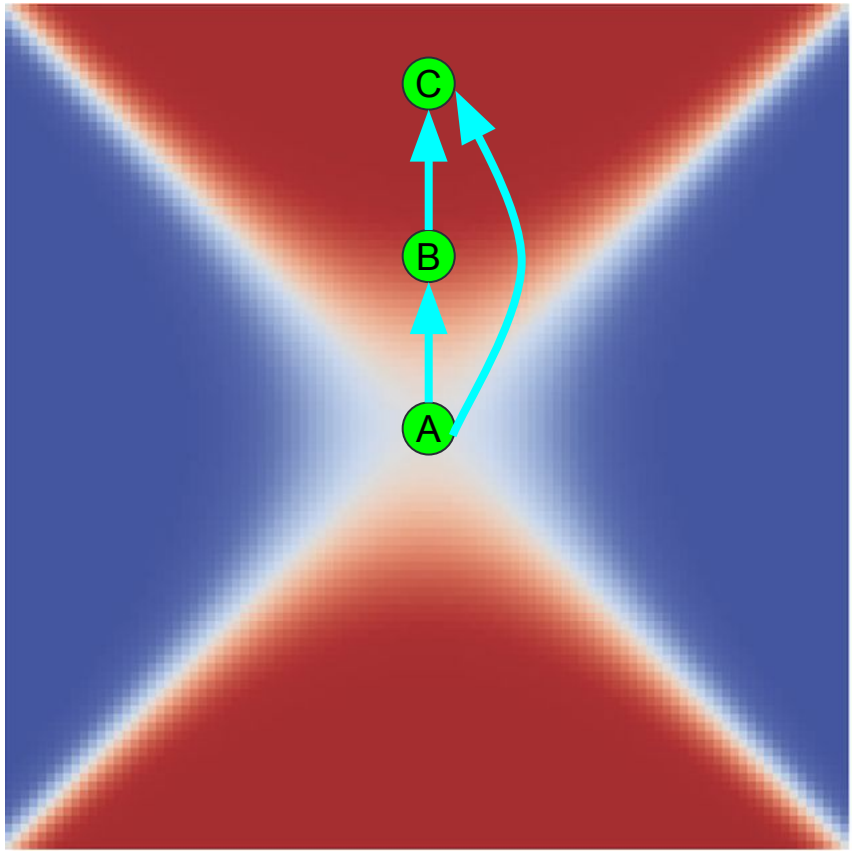


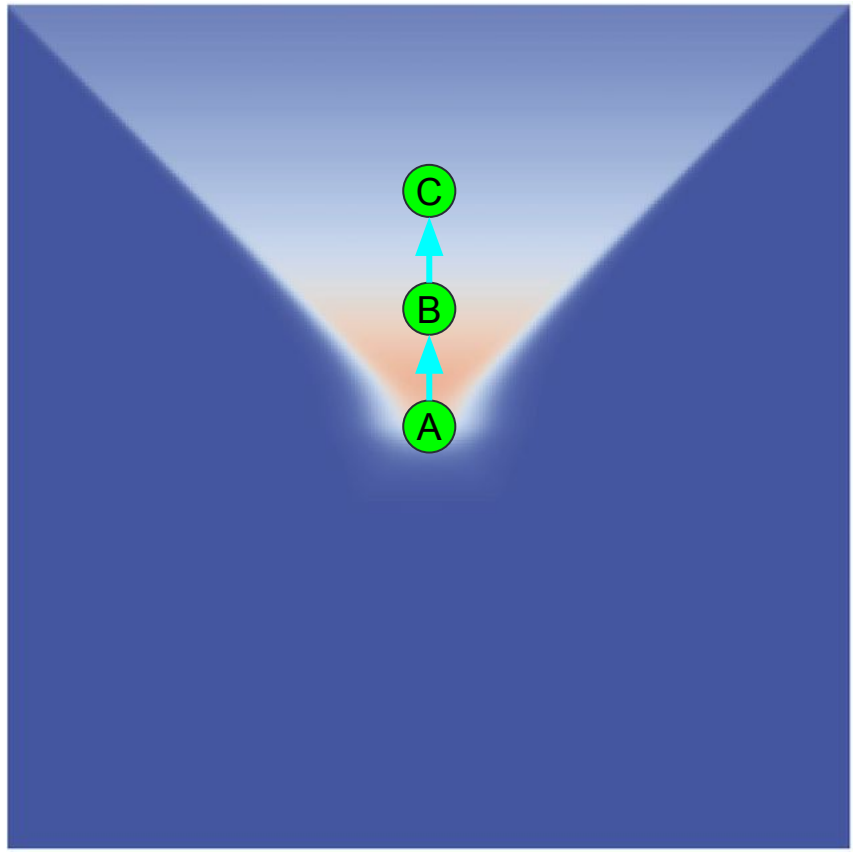
$p = 0.01$

Spacelike separated

Proposal 2: *Triple Fermi-Dirac* likelihood function

Time

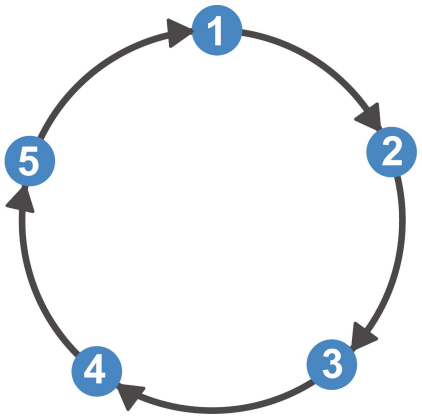




Symmetric
Transitive

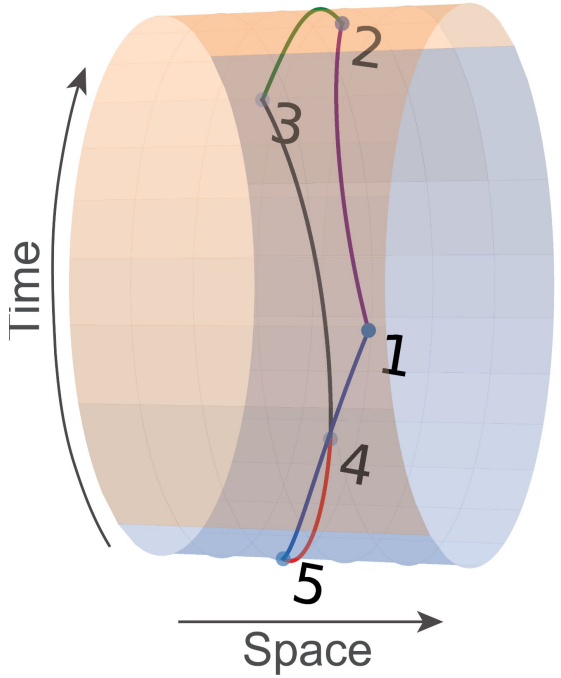
$$\frac{1}{e^{s^2}+1} \longrightarrow \left[\frac{1}{e^{s^2}+1}\cdot\frac{1}{e^{t}+1}\cdot\frac{1}{e^{-\alpha t}+1}\right]^{\frac{1}{3}}$$

Proposal 3: Compact time coordinate for graph cycles

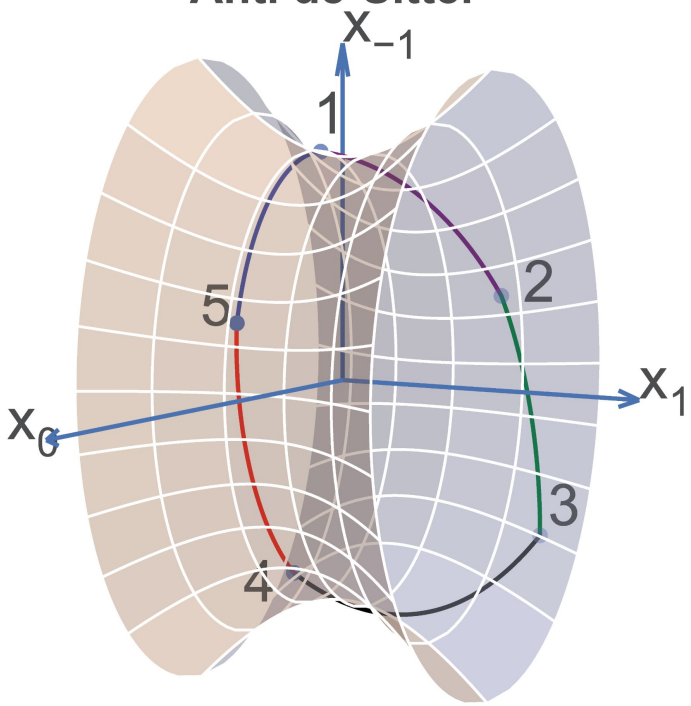


$$S^1 \times \mathbb{R}^n$$

Cylindrical Minkowski



Anti de-Sitter



Thank you