

# Upper Confidence Reinforcement Learning with Value Targeted Regression

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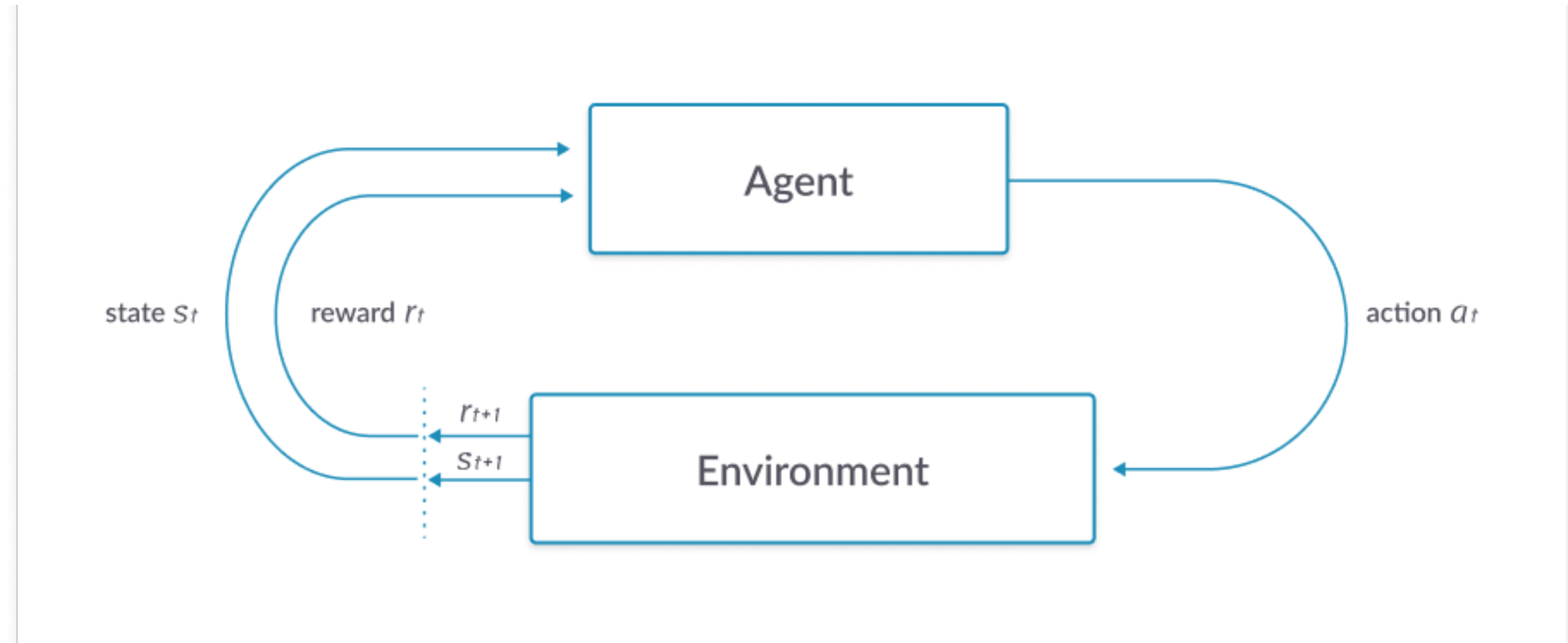


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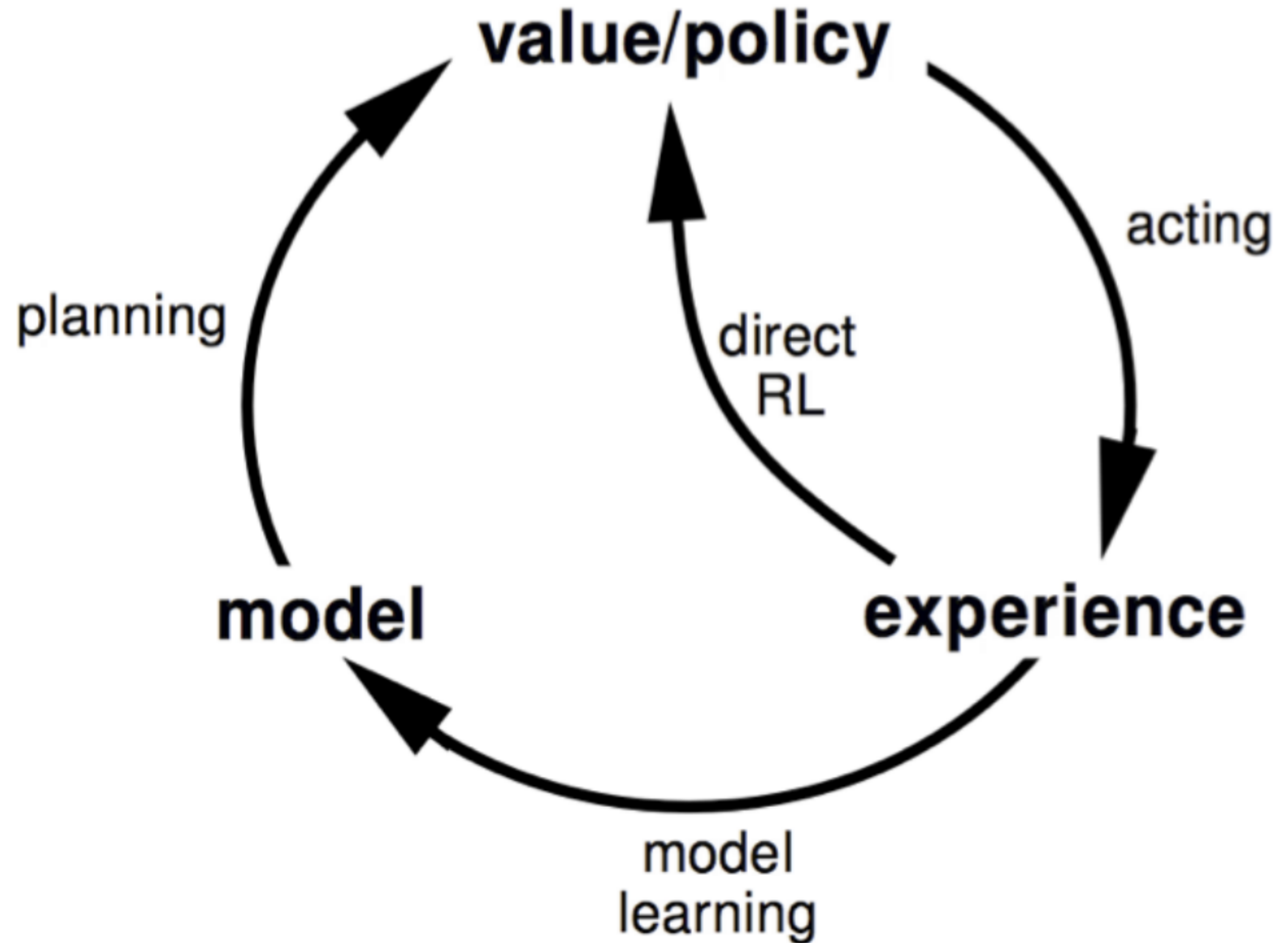
Lin F. Yang  
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# The Reinforcement Learning (RL) Problem

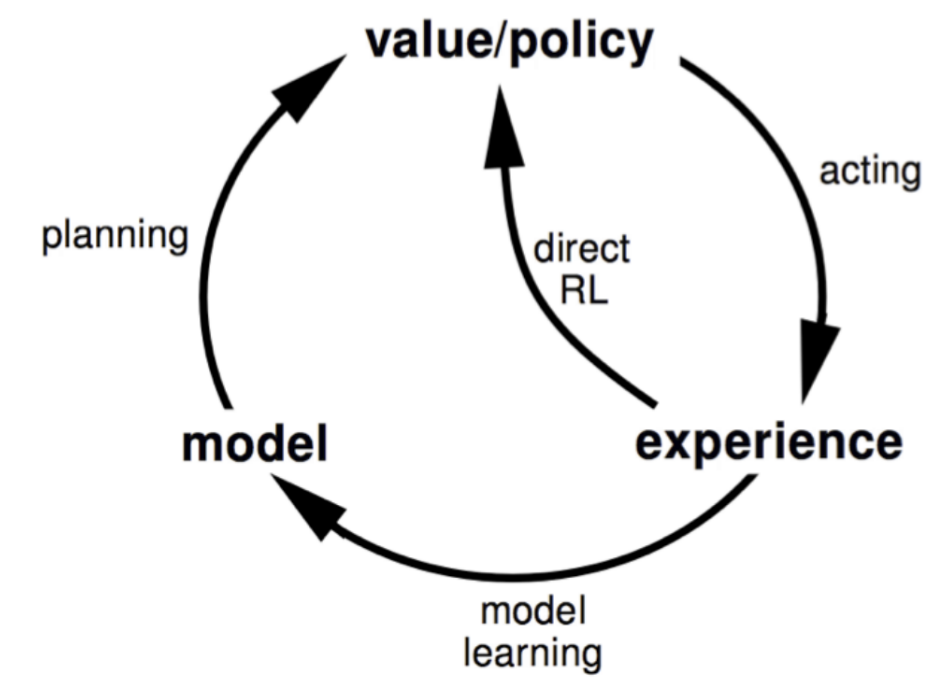


# Model-Based RL (MBRL)

- We fit a model to  $(s_t, a_t, s_{t+1}, r_{t+1})$



# How to Learn a Transition Model ?

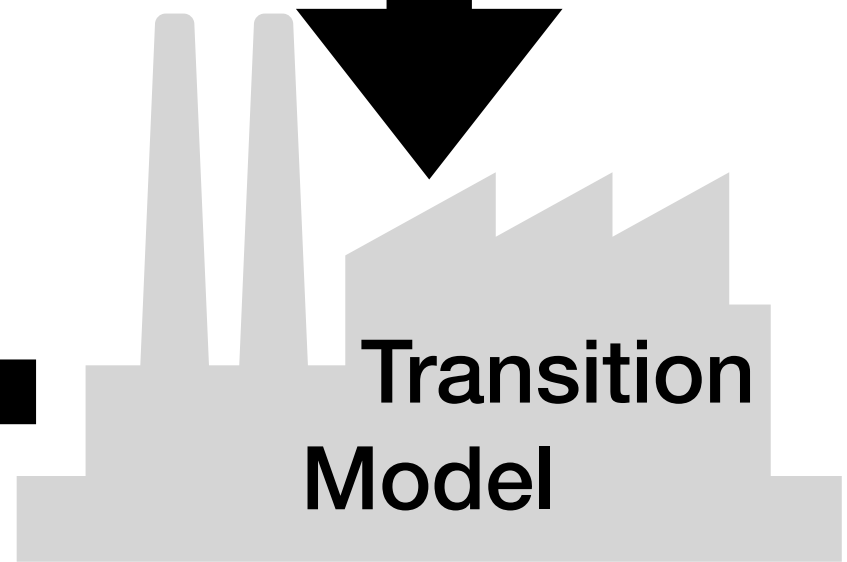
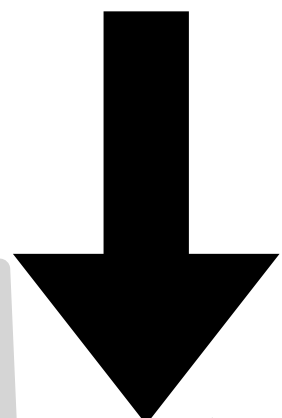


Sutton and Barto (2018)

Canonical

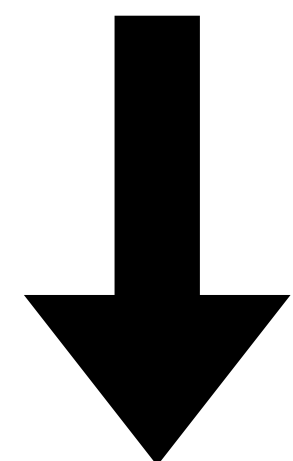
Input:

$(s_t, a_t)$



Planning

Transition Model



predicts

$s_{t+1}$

Value Target

For each:

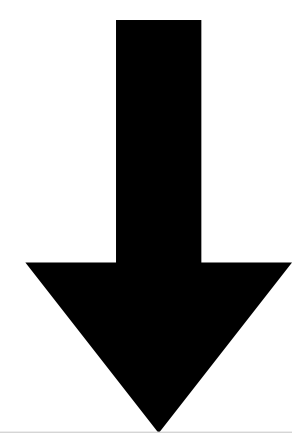
$(s_t, a_t)$

Input:

$V$

Planning

Transition Model

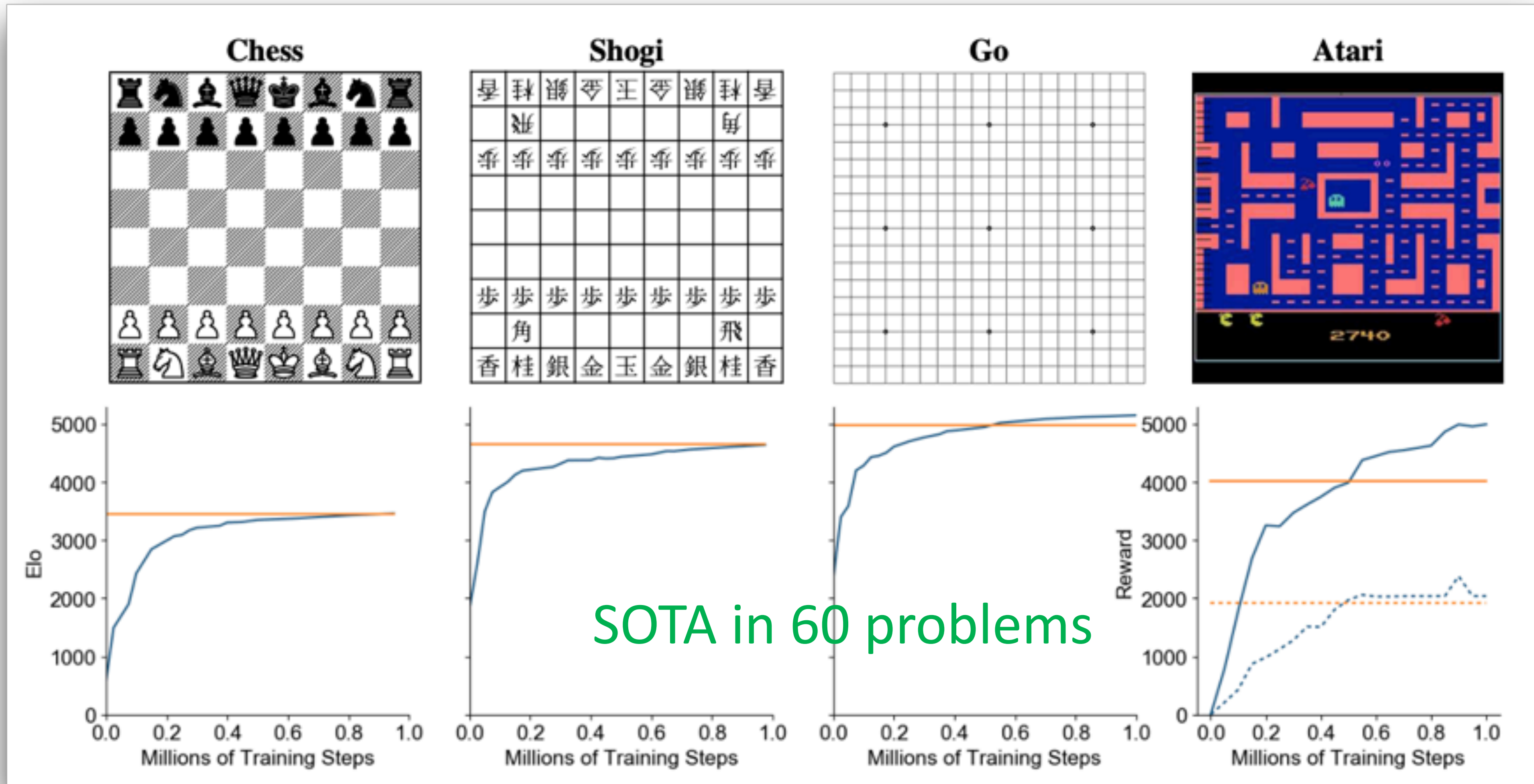


predicts

$V(s_{t+1})$

# Using neural nets: MuZero!

Objective:  
Keep  $\|(P - \tilde{P})\tilde{V}\|_{n,2}^2$  small



*Is value-targeted learning sufficient and efficient for model-based online RL?*

# Episodic Markov Decision Process (MDP)

- MDP:  $M = (\mathcal{S}, \mathcal{A}, P, r, H, s_o)$

- The Value Function of policy  $\pi$  is defined:  $V_h^\pi(s) = \mathbb{E}_\pi \left[ \sum_{i=h}^H r(s_i, \pi(s_i)) \mid s_h = s \right]$

- The goal is to minimize the total regret:

- $R(T) = \sum_{k=1}^K V_1^*(s_1^k) - \sum_{k=1}^K \sum_{h=1}^H r(s_h^k, a_h^k)$ , where  $T = KH$ .



# Assumptions about our Problem Setting

- Assumption 1 (Known Transition Model Family)

- $P \in \mathcal{P}$

- $\mathcal{P}$  is known

- Definition 1 (Linear Mixture Models)

- $$P(s' | s, a) = \sum_{j=1}^d \theta_j P_j(s' | s, a)$$

- where  $\theta_1, \dots, \theta_d$  are unknown.

# Model-Based Optimistic Planning

- We want  $P = \arg \max_{P' \in B} V_{P',1}^* (s_1)$
- We compute the optimal policy,  $\pi_P^*$ , according to  $P$ .
- Then we follow  $\pi_P^*$  in the current episode.
- How to construct  $B$ ?

# Value Targeted Regression for Confidence Set Construction

- Confidence Set:  $B = \{P' \in \mathcal{P} \mid \tilde{L}(P') \leq \tilde{\beta}\}$
- where: 
$$\tilde{L}(P') = \sum_{k'=1}^k \sum_{h=1}^H \left( P'(\cdot \mid s_h^{k'}, a_h^{k'})^\top V_{h+1,k'} - y_{h,k'} \right)^2$$
- and  $y_{h,k'} = V_{h+1,k'}(s_{h+1}^{k'})$ ,  $h \in [H]$  and  $k' \in [k]$

# Theoretical Analysis

- Eluder Dimension - Length of the longest independent sequence
  - In the game “Battleship”, how long before you hit your opponents ship?
- $\mathcal{F} = \left\{ f \mid \exists P \in \mathcal{P} \text{ s.t. for any } (s, a, v) \in \mathcal{S} \times \mathcal{A} \times \mathcal{V}, f(s, a, v) = \int P(ds' | s, a)v(s') \right\} .$
- Regret for Known Transition Model Family (Assumption 1),
  - $R(K) \leq O \left( \text{poly}(d_E, d, H) / \sqrt{K} \right)$



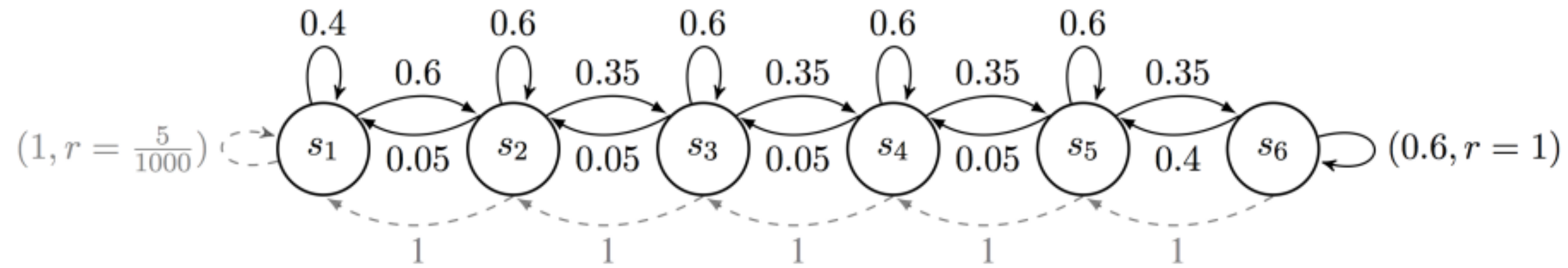
Eluder Dimension

- Regret for Linearly-Parameterized Transition Model (Defn 1)

- $R(K) = \tilde{O} \left( d \sqrt{H^3 K \log(1/\delta)} \right)$

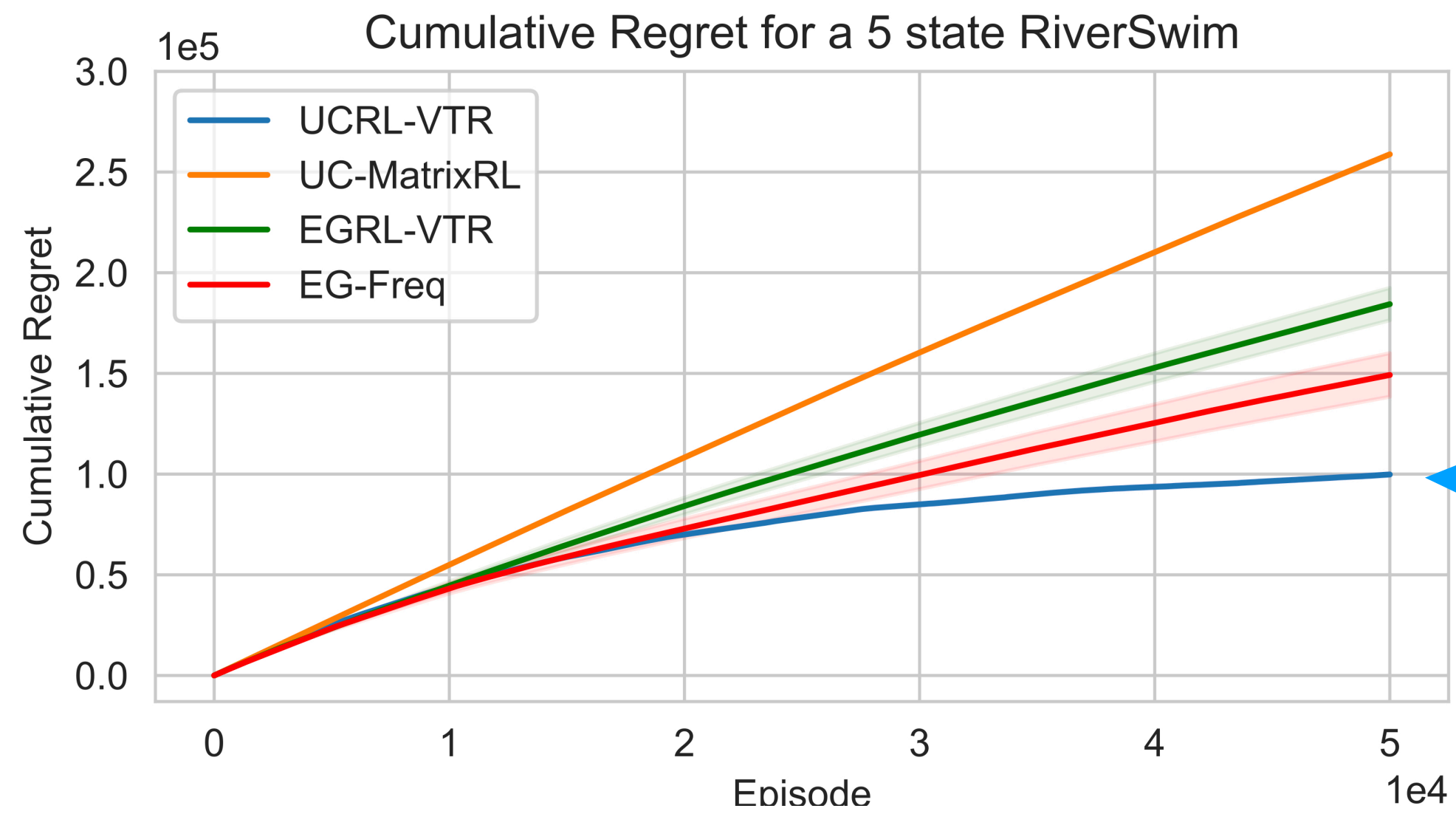
- Lower Bound:  $R(K) \geq \Omega \left( H \sqrt{dK} \right)$

# Experimental Results



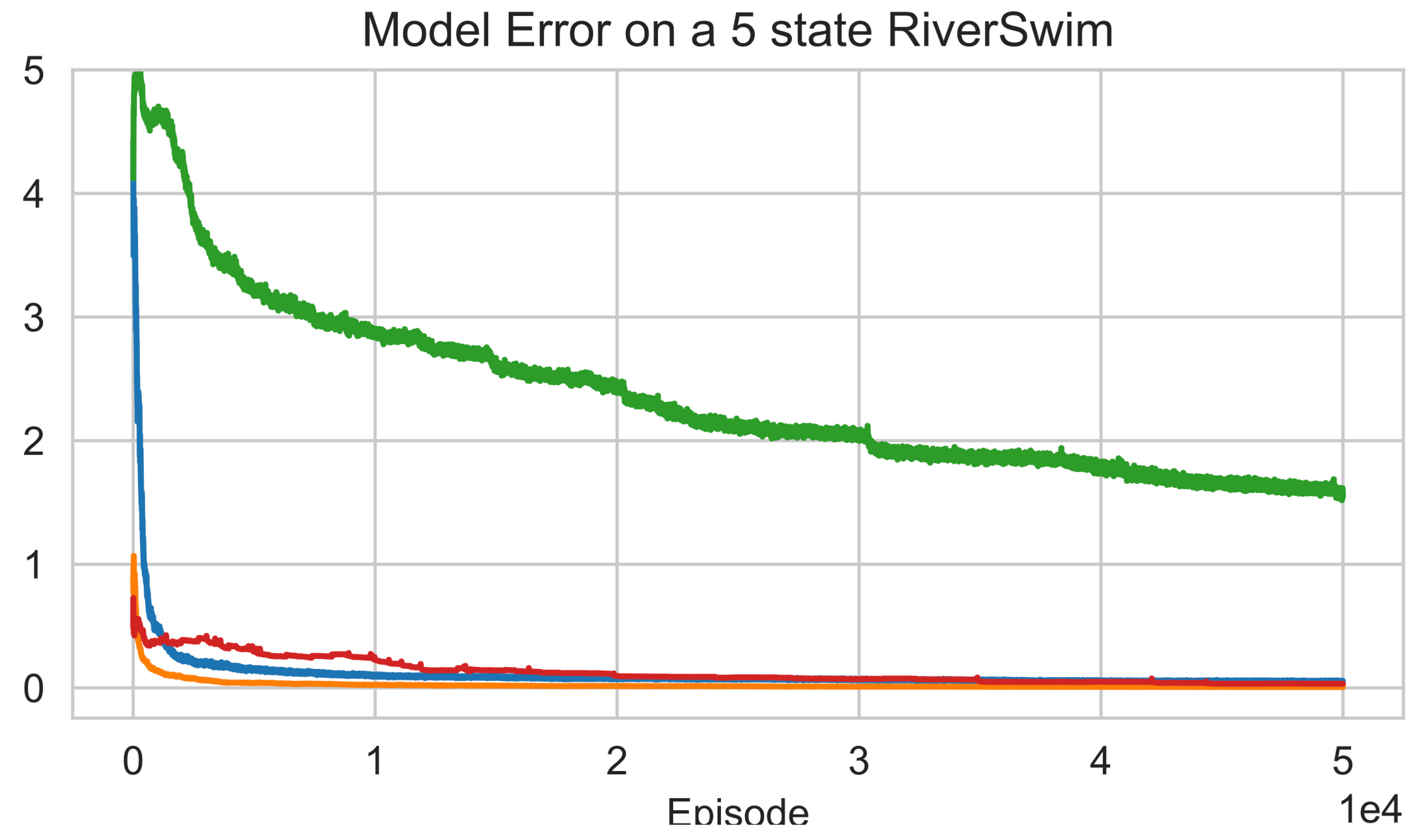
RiverSwim by Strehl & Littman

Exploration/ Targets	Optimism	Dithering
Next States	UC-MatrixRL	EG-Freq
Values	UCRL-VTR	EGRL-VTR

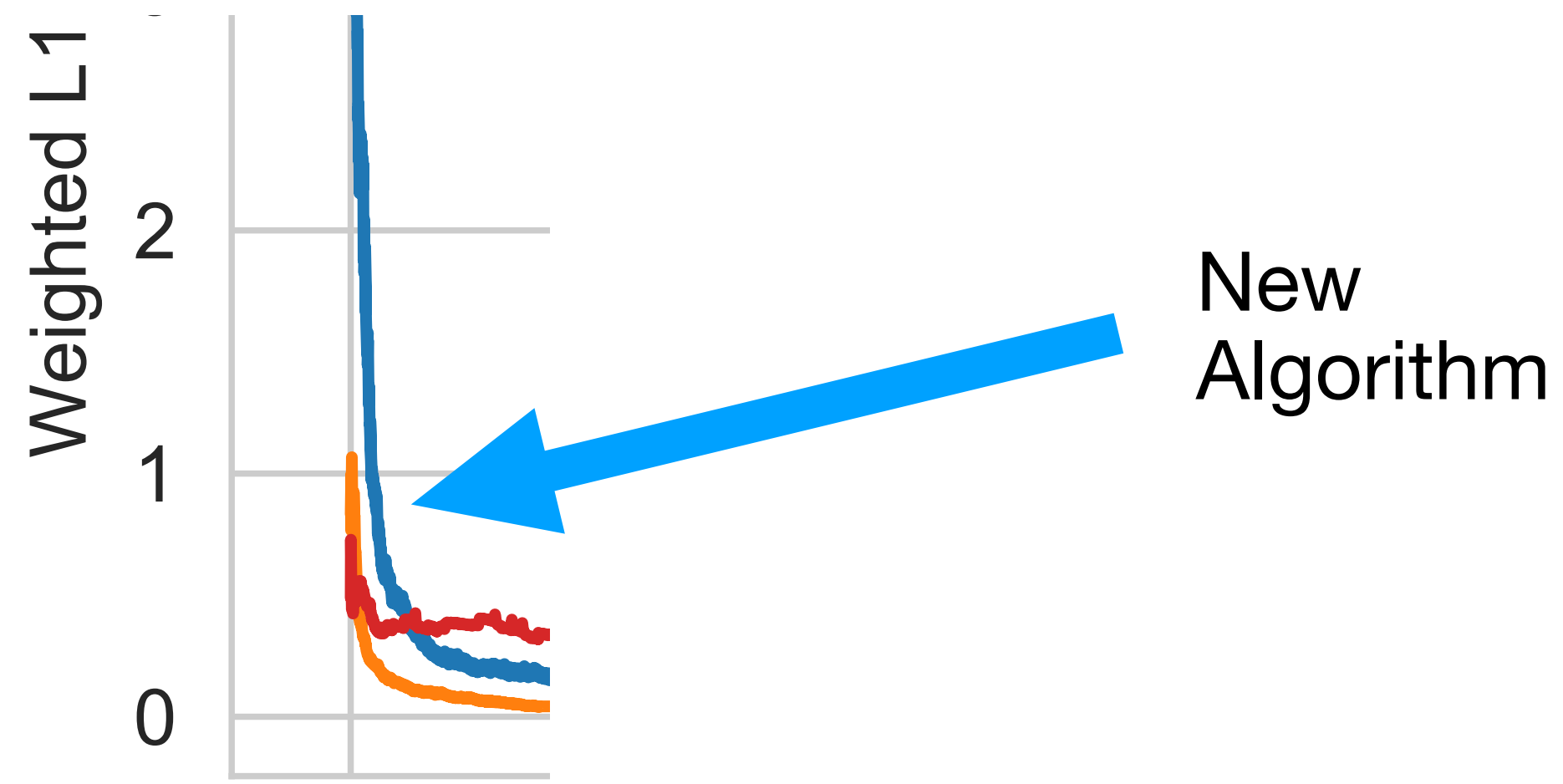


New  
Algorithm



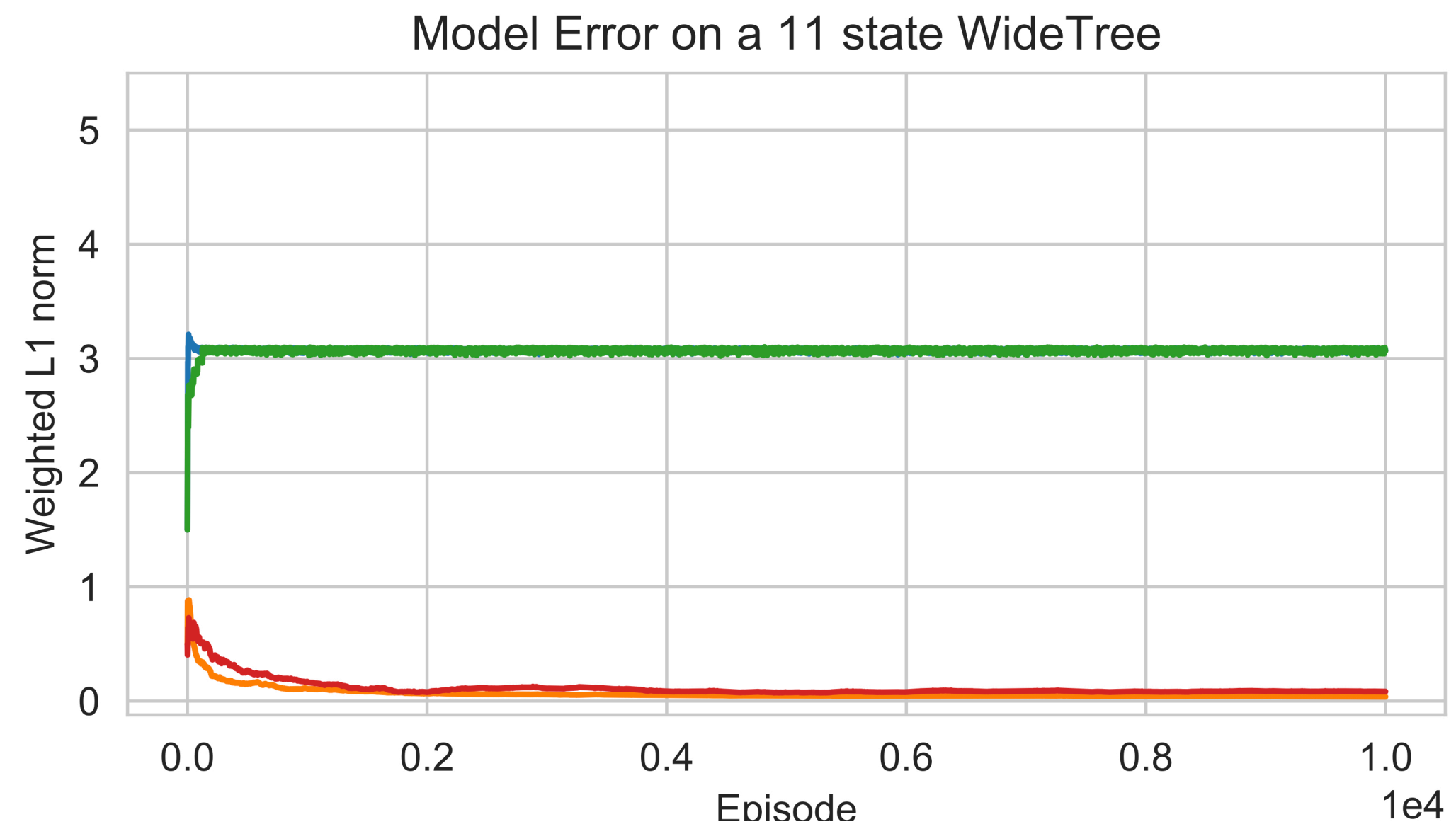


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# Conclusions

- Value-Targeted Regression is efficient/sufficient for MBRL.
- VTR outperforms canonical transition models both theoretically and experimentally
- Computation is expensive and future work is needed to come up with computationally feasible methods to compute the VTR model.