# MetaFun: Meta-Learning with Iterative Functional Updates

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# Supervised Meta-Learning



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What is learning?







What is meta-learning? (in encoder-decoder approaches like CNP<sup>[1]</sup>)



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Both parameterised by NNs

### Incorporating Inductive Biases into Deep Learning Models



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### **Functional Representation**

### **Encoders with Iterative Structure**



(permutation of data points should not change set representation)

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Encoder(	) = <b>\sum h( \lefter b)</b> )	[1][2][7]
	for all	
	in the context	

Fixed dimensional representation can be limiting for large set size<sup>[4]</sup>, and often lead to underfitting<sup>[3]</sup>.

### **Functional Representation**

Permutation invariance

Flexible capacity

### **Encoders with Iterative Structure**



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Self-attention modules<sup>[6]</sup> or relation network<sup>[9]</sup> can model interaction within the context, but not context-target interaction

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Within-context and context-target interaction

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**Euclidean Space** 

**Function Space** 

(e.g. Hilbert Space)















#### **Gradient Descent**

solve

$$\argmin_{\theta} L(\theta)$$

by iterative optimisation

$$\theta_{t+1} = \theta_t - \alpha \nabla_\theta L(\theta_t)$$



#### Gradient Descent

#### **Functional Gradient Descent**

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 $\theta_{t+1} = \theta_t - \alpha \nabla_\theta L(\theta_t)$ 

by iterative optimisation

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 $\operatorname*{arg\,min}_{f} L(f)$ 

 $f_{t+1} = f_t - \alpha \nabla_f L(f_t)$ 

For supervised learning problems, the objective function often has this form:

$$L(f_t) = \frac{1}{|C|} \sum_{i \in C} l(f_t(\mathbf{x}_i), \mathbf{y}_i)$$



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#### **Functional Gradient Descent**

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solve  $\arg \min L(\theta)$ 

by iterative optimisation

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$$r^{(t)}(\mathbf{x}_i), \forall i \in C$$





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$$\Delta r^{(t)}(\cdot) = \operatorname{FunPooling}(\{(\mathbf{x}_i, \mathbf{u}_i)\}_{i \in C}) = \sum_{i \in C} k(\cdot, \mathbf{x}_i) \mathbf{u}_i$$





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MetaFun Iteration

Local update funcion:

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Functional pooling:

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Apply functional updates:

 $r^{(t+1)}(\cdot) = r^{(t)}(\cdot) - \alpha \Delta r^{(t)}(\cdot)$ 

 $r^{(T)}(\cdot)$  will be the final representation after T iterations

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Within-context and context-target interaction

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Both the within-context interaction and the interaction between context and target are considered when updating the representation at each iteration.





Deep kernels or attention modules



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$$\frac{\partial L}{\partial f^{(t)}(\mathbf{x}_i)} \longrightarrow \left[ \frac{\partial (\text{cross entropy loss})}{\partial (\text{predictive logit } k)} \right]_{k=1:K}^{\top}$$

Regression:	MetaFun Iteration
MLP on concatenation of inputs	Local update funcion:
Classification:	$ \mathbf{u}_i = u(\mathbf{x}_i, \mathbf{y}_i, r^{(t)}(\mathbf{x}_i))$
With structure similar to	Functional pooling:
$\lceil \partial \text{ (cross entropy loss)}  ceil^ op$	$\Delta r^{(t)}(\cdot) =  ext{FunPooling}(\{(\mathbf{x}_i, \mathbf{u}_i)\}_{i \in C}) = \sum k(\cdot, \mathbf{x}_i) \mathbf{u}_i$
$\left\lfloor \overline{\partial \text{ (predictive logit } k)} \right\rfloor_{k=1:K}$	Apply functional updates:
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Naturally integrate within-class and between-class interaction

Deep kernels or attention modules

Model Agnostic Meta-Learning (MAML)<sup>[8]</sup>

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Model Agnostic Meta-Learning (MAML)<sup>[8]</sup>

During meta-training phase, MAML finds a good initialisation from related tasks.

During test time, MAML runs a few gradient descent steps from the learned initialisation on the context of a new task.

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SumPooling Local updates (permutation-invariant) (following gradient)

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(parameterised by NNs)



1D Sinusoid Regression Tasks



Smooth updates and match the ground truth very well across the whole period.



Non-smooth updates and not as good predictions especially on the left side where there is no context points.

MAML:



1D Sinusoid Regression Tasks

MAML: Non-smooth updates and not as good predictions especially on the left side where there is no context points.

### Large-Scale Few-shot Classification

### minilmageNet

(without data augmentation)

Model	1-shot	5-shot
LEO <sup>[9]</sup>	61.76 ± 0.08%	77.59 ± 0.12%
MetaFun (deep kernel version)	61.16 ± 0.15%	78.20 ± 0.16%
MetaFun (attention version)	62.12 ± 0.30%	77.78 ± 0.12%

(with data augmentation)

Model	1-shot	5-shot
LEO	63.97 ± 0.20%	79.49 ± 0.70%
MetaOptNet-SVM <sup>[10]</sup>	64.09 ± 0.62%	80.00 ± 0.45%
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#### tieredImageNet

(without data augmentation)

Model	1-shot	5-shot
LEO	66.33 ± 0.05%	81.44 ± 0.09%
MetaOptNet-SVM	65.81 ± 0.74%	81.75 ± 0.58%
MetaFun (deep kernel version)	67.27 ± 0.20%	83.28 ± 0.12%
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### Large-Scale Few-shot Classification

We demonstrates that encoder-decoder style meta-learning methods like conditional neural processes can also also achieves SOTA on large-scale few-shot classification benchmarks.

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We demonstrates that encoder-decoder style meta-learning methods like conditional neural processes can also also achieves SOTA on large-scale few-shot classification benchmarks.



# Thank you!



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@jinxu06 (code available here)



### References

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