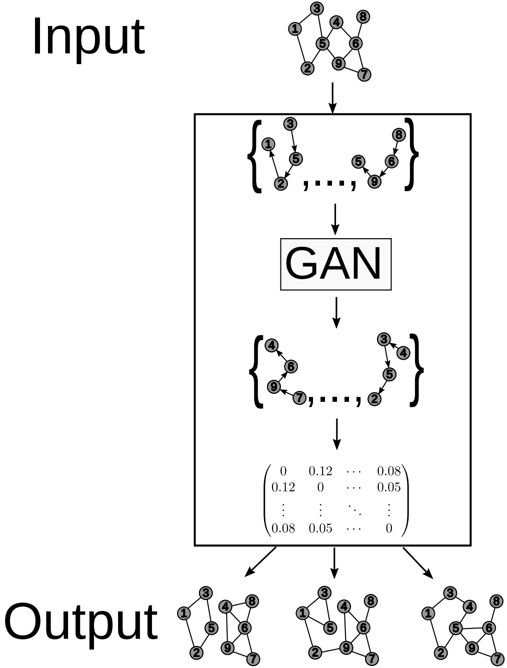
NetGAN without GAN: From Random Walks to Low-Rank Approximations

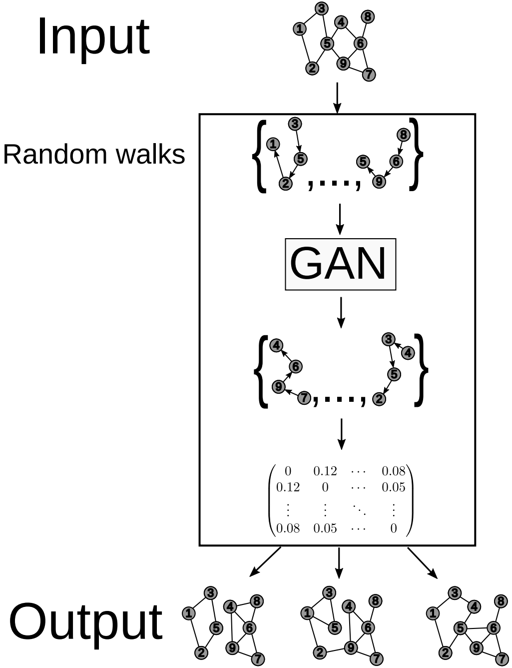
Luca Rendsburg, Holger Heidrich, Ulrike von Luxburg

ICML 2020

Input

Output


Input
GAN
0 0.12 ⋯ 0.08
0.12 0 ⋯ 0.05
0.08 0.05 ⋯ 0
Output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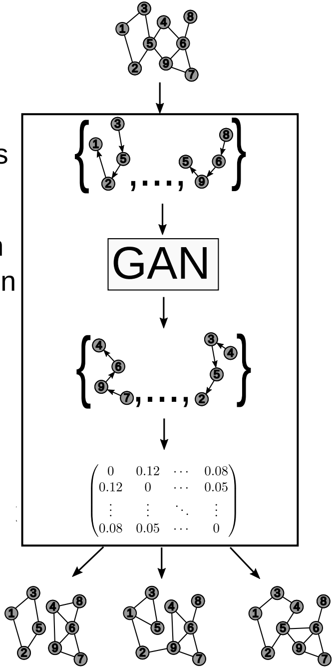
Random walks

Learn random walk distribution



Output

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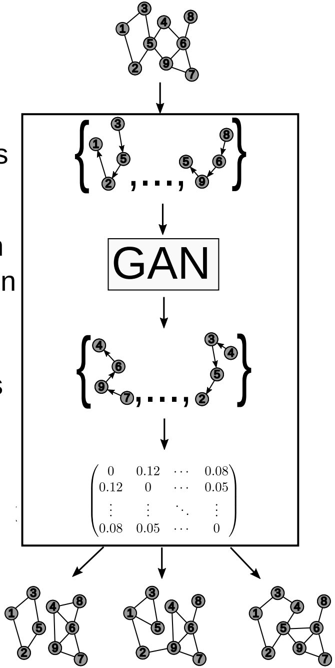
Synthetic random walks



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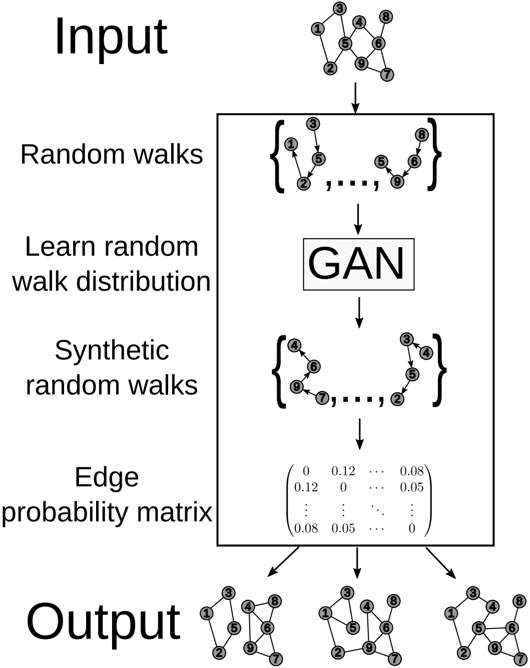
Input
Random walks
Learn random
walk distribution
GAN
Synthetic
random walks
Edge
probability matrix
Output

Our contribution

Conceptual analysis

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Conceptual analysis

- Inductive bias of NetGAN

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Simplified version (no GAN, no sampling): "Cross-Entropy Low-rank Logits (CELL)"

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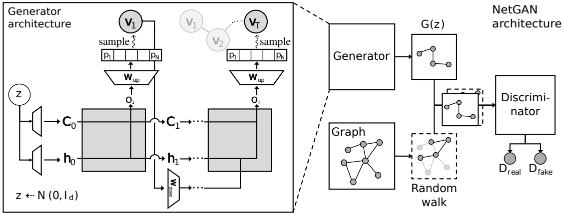
Simplified version (no GAN, no sampling): "Cross-Entropy Low-rank Logits (CELL)"

- Higher transparency
- Comparable generalization performance
- Huge speedup

NetGAN: learning step

- **Goal:** learn random walk distribution

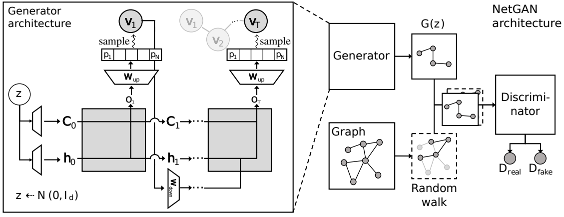


(a) Generator architecture (b) NetGAN architecture

Figure from Bojchevski et al. [2018]

NetGAN: learning step

- **Goal:** learn random walk distribution
- **Training set:** unbiased random walks $\{(v_0^{(i)}, \ldots, v_T^{(i)})\}_i$ of length T over input graph



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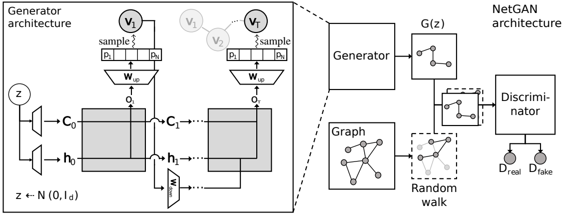

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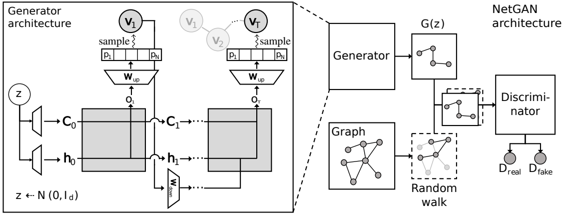
- **Discriminator:** distinguish synthetic from real random walks



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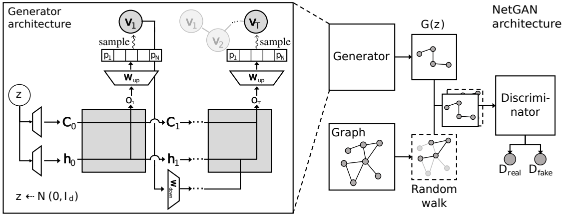
- **Architecture:** LSTMs with Wasserstein loss



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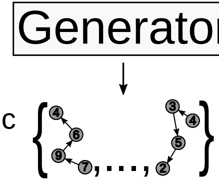
NetGAN: reconstruction step

Generator

NetGAN: reconstruction step

Generate *many* synthetic random walks



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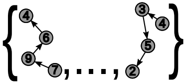

Count transitions in *score matrix* S

$$\begin{pmatrix} 0 & 360 & \cdots & 212 \\ 307 & 0 & \cdots & 150 \\ \vdots & \vdots & \ddots & \vdots \\ 240 & 143 & \cdots & 0 \end{pmatrix}$$

$S_{i,j} = \#\text{ transitions from } i \text{ to } j$

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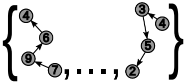

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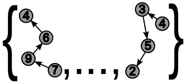

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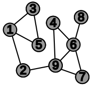
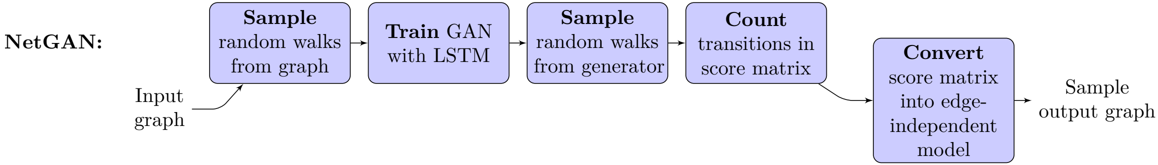
Sample edges without replacement



Overview of simplifications



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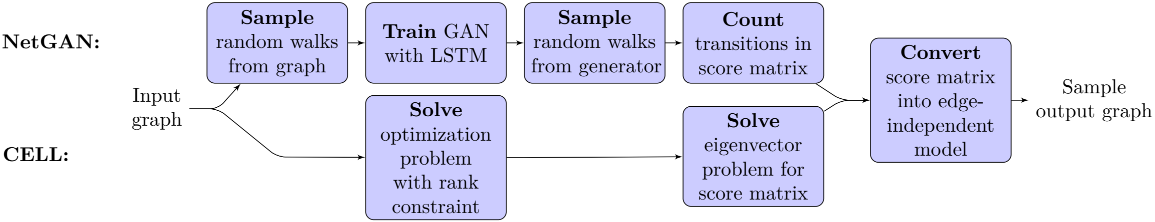

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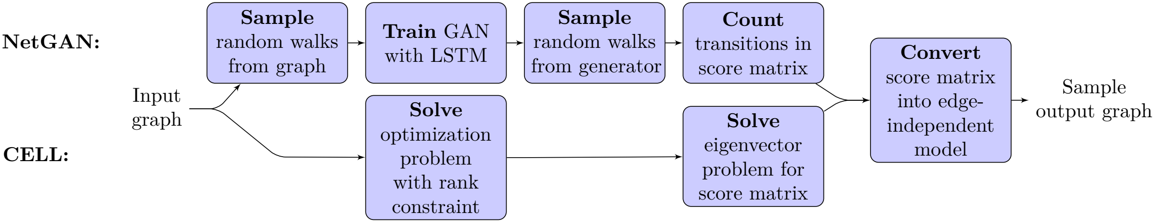

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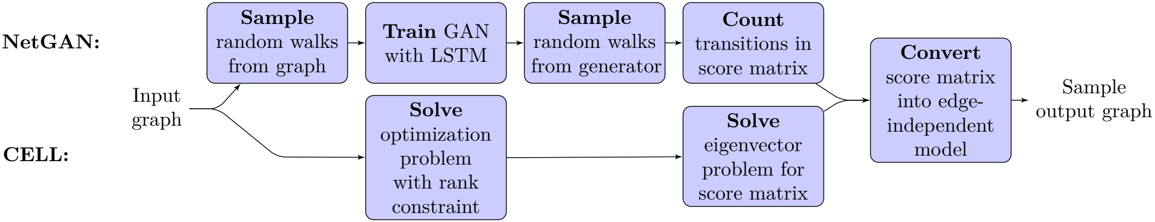
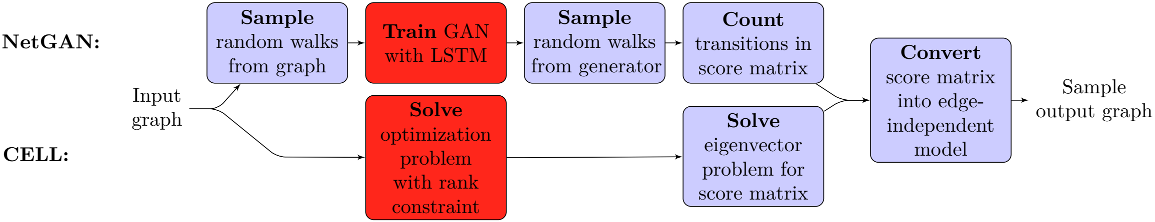


1. Replace GAN with rank-constrained optimization problem
2. Bypass random walk sampling

Replacing the GAN (1)



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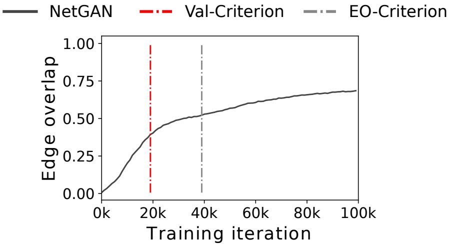

Figure from Bojchevski et al. [2018]

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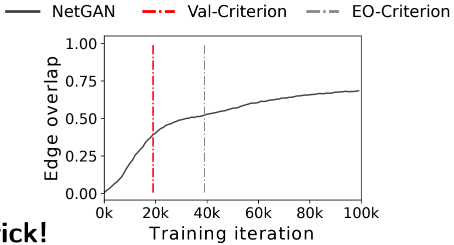

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Computational trick!

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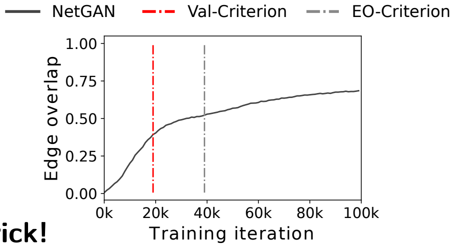

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✓ Low-dimensional bottleneck

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- Number of nodes N, set of random walks $\mathcal{R}$, low rank H, row-wise softmax σ_{rows}

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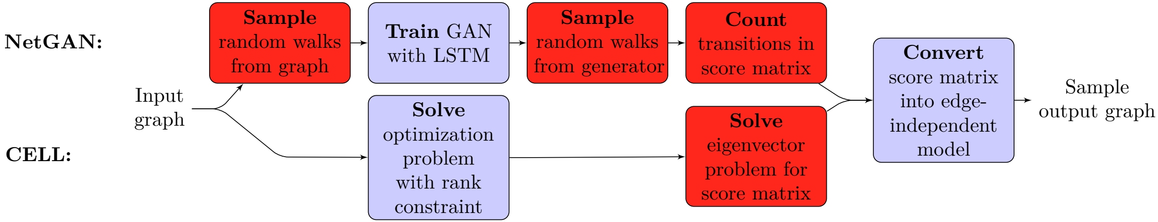
- Do this directly with **maximum likelihood estimation**

$$\boxed{\begin{aligned} &\min_{W\in\mathbb{R}^{N\times N}} -\sum_{(i,j)\in\mathcal{R}} \log \sigma_{\text{rows}}(W)_{i,j} \,, \\ &\text{s.t.} \quad \operatorname{rank}(W) \leqslant H \,. \end{aligned}}$$

Bypassing sampling (1)



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- Length of random walk T, amount of random walks n. Normalized score matrix converges

$$\frac{S}{nT} \xrightarrow[n,T\to\infty]{a.\,s.} \operatorname{diag}(\pi)P\,,$$

where P is corresponding random walk transition matrix and π its stationary distribution.

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Replace normalized score matrix with its limit

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2. Reconstruction step (with synthetic transition matrix P^*)

$$\boxed{\text{Compute } S \text{ by counting transitions of synthetic random walks}} \rightarrow \boxed{S := \operatorname{diag}(\pi^*)P^*}$$

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Bypass sampling in both steps

Experiments (1)

Does CELL generate the same type of graphs as NetGAN?

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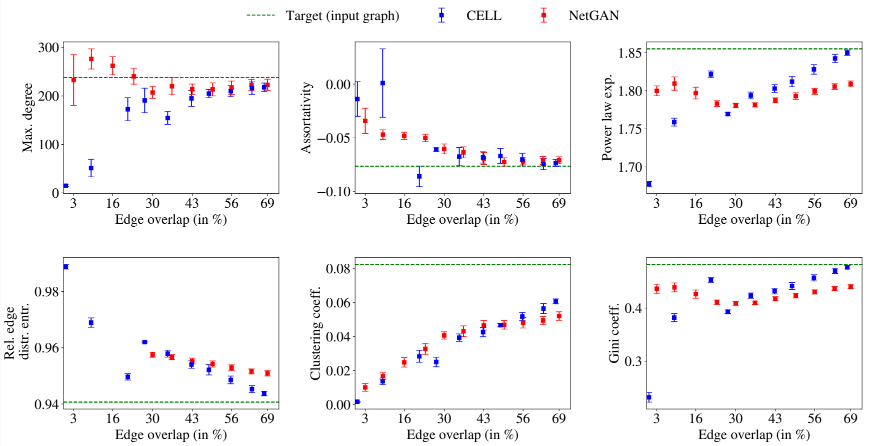
Graph: CORA-ML citation network (2,810/ 7,981)



Experiments (2)

CELL is significantly faster

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Table: Training time (in seconds) for NetGAN and CELL on a variety of networks. NetGAN requires a GPU, while CELL runs on a CPU.

Data set (Nodes/ Edges)		NetGAN	CELL
CORA-ML	(2,810/ 7,981)	7,478	21
Citeseer	(2,110/ 3,668)	4,654	10
PolBlogs	(1,222/ 16,779)	55,276	15
RT-GOP	(4,687/ 5,529)	14,800	23
Web-EDU	(3,031/ 6,474)	11,000	16

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