POSTER #124

Differentially Private Learning of Geometric Concepts

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joint work with

Haim Kaplan, Yishay Mansour, and Yossi Matias

Privately Learning Union of Polygons

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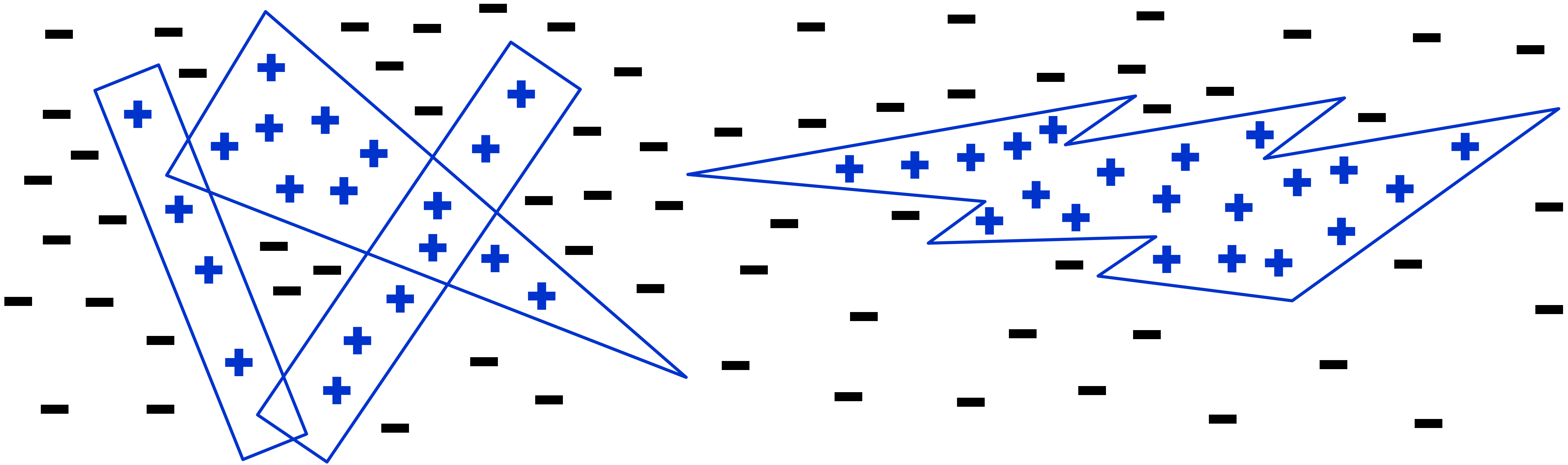
Given: $\boldsymbol{n}$ points in $\mathbb{R}^2$ with binary labels: $\{(\boldsymbol{x_i}, \boldsymbol{y_i})\}_{i=1}^{n}$

Assume: ∃collection of polygons $\{\boldsymbol{P_1}, \dots, \boldsymbol{P_t}\}$ with a total of al most $\boldsymbol{k}$ edges s.t. $\forall \boldsymbol{i} \in [\boldsymbol{n}]$: $\boldsymbol{x_i} \in \bigcup_j \boldsymbol{P_j} \Leftrightarrow \boldsymbol{y_i} = \mathbf{1}$

Find: Hypothesis $\boldsymbol{h}$: $\mathbb{R}^2 \rightarrow \{\mathbf{0}, \mathbf{1}\}$ with small error



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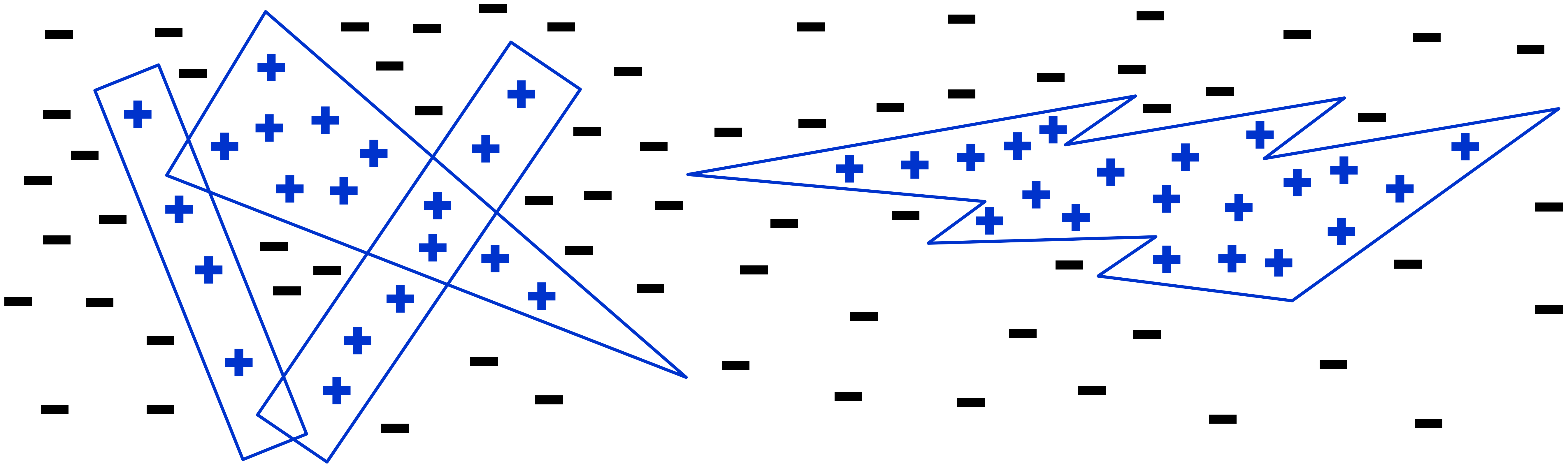

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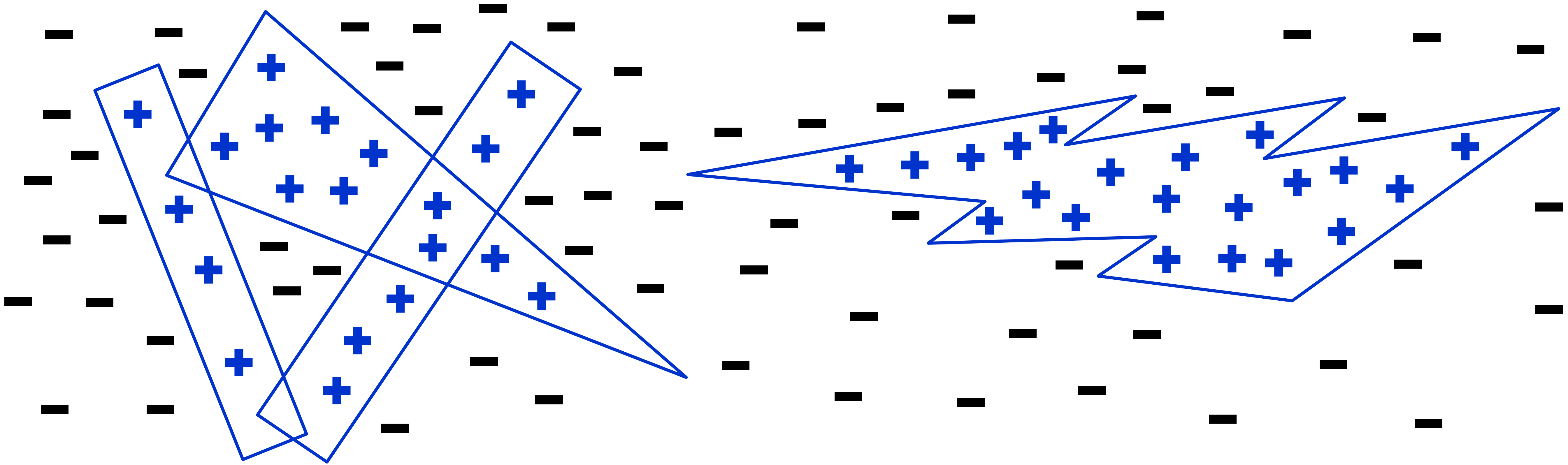
- ✓ Every labeled example represents the (private) information of one individual
- ✓ **Goal:** the output hypothesis does not reveal information that is specific to any single individual



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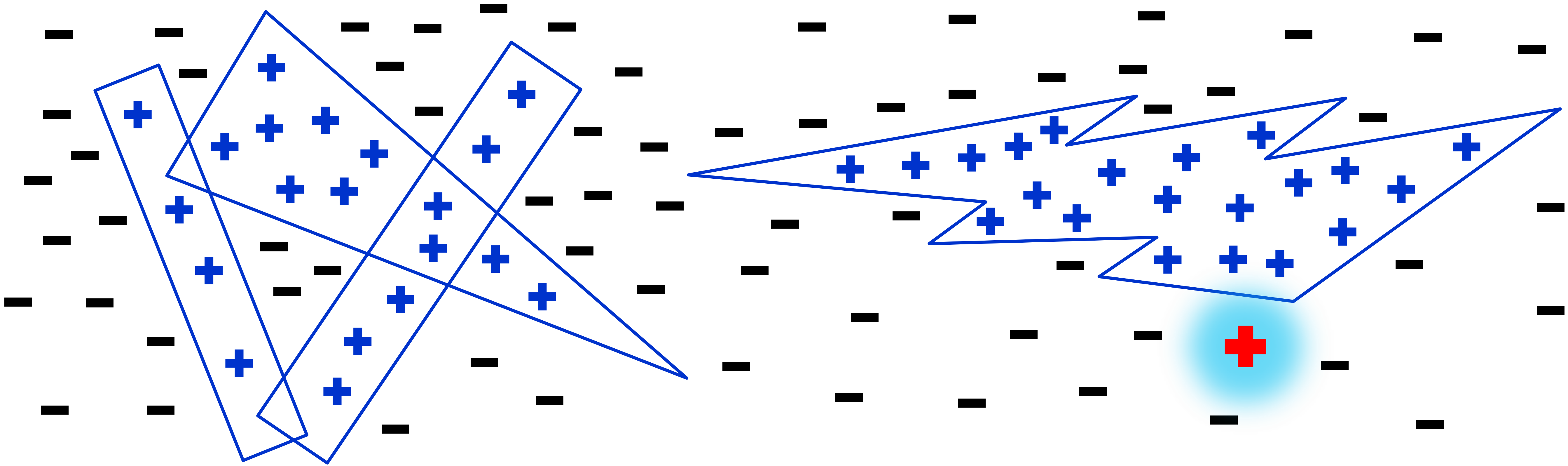

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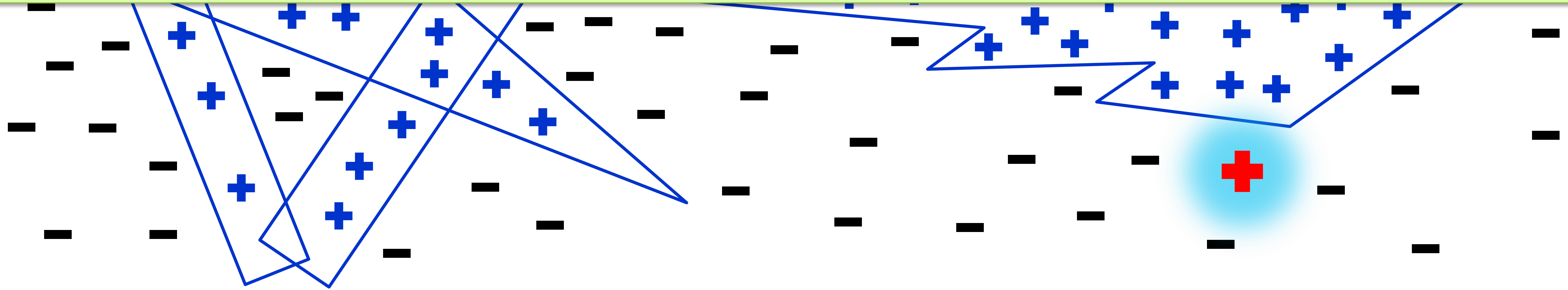
Why is that a good privacy definition?

Even if an observer knows all other data point but mine, and now she sees the outcome of the computation, then she still cannot learn "anything" on my data point



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Motivation: Analyzing Users' Location Reports

- ***Analyzing GPS navigation data***
- ***Learning the shape of a flood or a fire based on reports***
- ***Identifying regions with poor cellular reception based on reports***

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Differential Privacy and Discretization

- Impossibility results for differential privacy show that this problem (and even much simpler problems) cannot be solved over infinite domains

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- We assume that input points come from $[\boldsymbol{d}]^2 = \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{d}\} \times \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{d}\}$ for a discretization parameter $\boldsymbol{d}$

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- Furthermore, the sample complexity must grow with the size of the discretization

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Previous Result

Private learner with sample complexity

$\mathbf{O}(\boldsymbol{k} \cdot \mathbf{log}\, \boldsymbol{d})$ and runtime $\approx \boldsymbol{d}^{\boldsymbol{k}}$

(using a generic tool of MT'07)

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New Result

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$\tilde{\mathrm{O}}(k \cdot \log d)$ and runtime $\mathrm{poly}(k, \log d)$

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Summary

- ✓ New algorithm for privately learning union of polygons
- ✓ Efficient runtime and sample complexity
- ✓ Applications to privately analyzing users' location data