On the Computation and Communication Complexity of Parallel SGD with Dynamic Batch Sizes for Stochastic Non-Convex Optimization

Poster @ Pacific Ballroom #103

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Stochastic Non-Convex Optimization

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- Singe node training:
  - Larger *B* can improve the utilization of computing hardware
- Data-parallel training:
  - Multiple nodes form a bigger “mini-batch” by aggregating individual mini-batch gradients at each step.
  - Given a budget of gradient access, larger batch size yields fewer update/comm steps

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- Recall B=1 means poor hardware utilization and huge communication cost

Dynamic BS: reduce communication without sacrificing SFO convergence

- Motivating result:
  For strongly convex stochastic opt, [Friedlander&Schmidt'12] and [Bottou et.al.'18] show that SGD with exponentially increasing BS can achieve the same $O(1/T)$ SFO convergence as SGD with fixed small BS

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  - do not sacrifice SFO convergence in (***parallel***) SGD. Recall (N node parallel) SGD with (B=1) has $O(1/\sqrt{\boxed{N}T})$ SFO convergence

T: SFO access budge at each node

Linear speedup w.r.t. # of nodes; computation power perfectly scaled out

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  - reduce communication complexity (# of used batches) in ***parallel*** SGD

Non-Convex under PL condition

- PL condition: $\frac{1}{2}\|\nabla f(x)\|^2 \geq \mu(f(x) - f^*), \forall x$
  - Milder than strong convexity: strong convexity implies PL condition.
  - Non-convex fun under PL is typically as nice as strong convex fun.

Algorithm 1 CR-PSGD$(f, N, T, \mathbf{x}_1, B_1, \rho, \gamma)$

Input: N, T, $\mathbf{x}_1 \in \mathbb{R}^m$, γ , B_1 and $\rho > 1$.
Initialize $t = 1$
budge of SFO access at each worker
while $\sum_{\tau=1}^{t} B_\tau \leq \boxed{T}$ **do**
  Each worker calculates batch gradient average $\bar{\mathbf{g}}_{t,i} = \frac{1}{B_t}\sum_{j=1}^{B_t} F(\mathbf{x}_t; \zeta_{i,j})$.
  Each worker aggregates gradient average $\bar{\mathbf{g}}_t = \frac{1}{N}\sum_{i=1}^{N} \bar{\mathbf{g}}_{t,i}$.
  Each worker updates in parallel via: $\mathbf{x}_{t+1} = \mathbf{x}_t - \gamma\bar{\mathbf{g}}_t$.
  Set batch size $B_{t+1} = \lfloor \rho^t B_1 \rfloor$.
  Update $t \leftarrow t + 1$.
end while
Return: $\mathbf{x}_t$

Non-Convex under PL condition

- Under PL, we show using exponentially increasing batch sizes in PSGD with *N* workers has $O(\frac{1}{NT})$ SFO convergence with $O(\log T)$ comm rounds
    - SoA $O(\frac{1}{NT})$ SFO convergence with $O(\sqrt{NT})$ inter-worker comm rounds attained by local SGD in [Stich'18] for strongly convex opt only

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- How about general non-convex without PL?
- Inspiration from "**catalyst acceleration**" developed in [Lin et al.'15][Paquette et al.'18]
    - Instead of solving original problem directly, it repeatedly solves "strongly convex" proximal minimization

General Non-Convex Opt

- A new catalyst-like parallel SGD method

Algorithm 2 CR-PSGD-Catalyst$(f, N, T, \mathbf{y}_0, B_1, \rho, \gamma)$

Input: N, T, θ, $\mathbf{y}_0 \in \mathbb{R}^m$, γ , B_1 and $\rho > 1$.
Initialize $\mathbf{y}^{(0)} = \mathbf{y}_0$ and $k = 1$.
while $k \leq \lfloor \sqrt{NT} \rfloor$ **do**

strongly convex fun whose unbiased stochastic gradient is easily estimated

  Define $\boxed{h_\theta(\mathbf{x}; \mathbf{y}^{(k-1)}) \overset{\Delta}{=} f(\mathbf{x}) + \frac{\theta}{2}\|\mathbf{x} - \mathbf{y}^{(k-1)}\|^2}$.
  Update $\mathbf{y}^{(k)}$ via

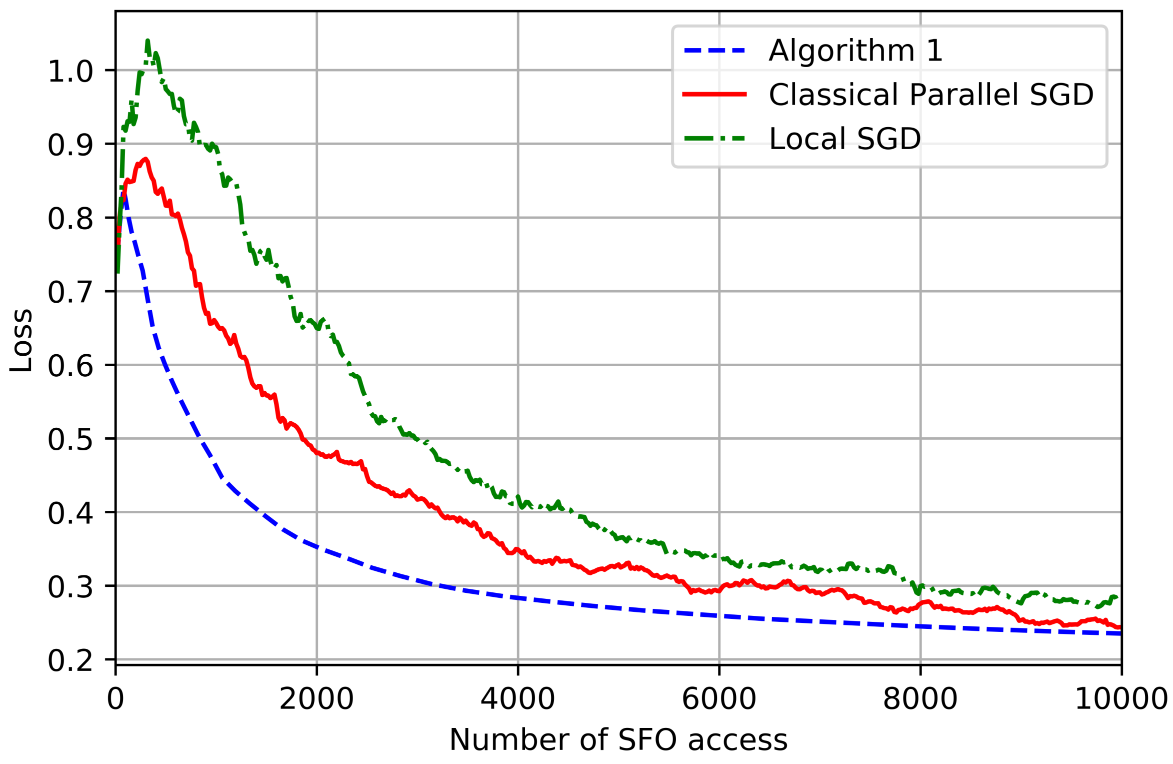
$$\mathbf{y}^{(k)} = \text{CR-PSGD}(h_\theta(\cdot; \mathbf{y}^{(k-1)}), N, \lfloor \sqrt{T/N} \rfloor, \mathbf{y}^{(k-1)}, B_1, \rho, \gamma)$$

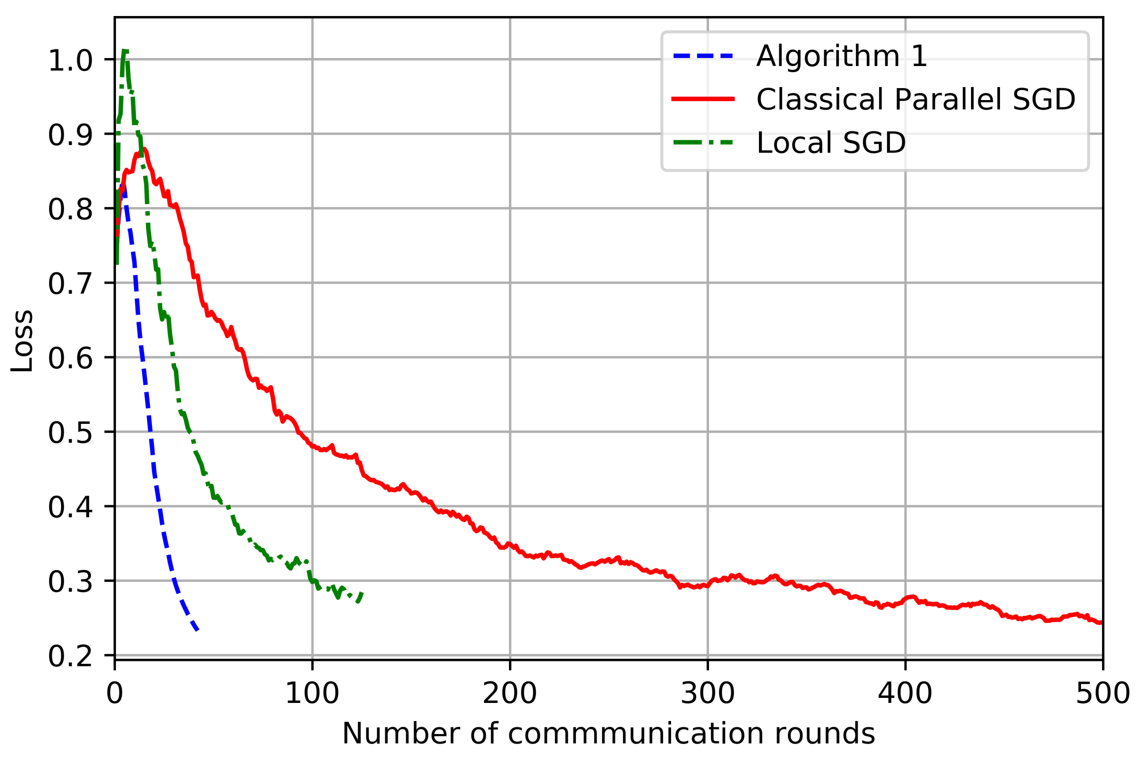
  Update $k \leftarrow k + 1$.
end while

- We show this catalyst-like parallel SGD (with dynamic BS) has $O(1/\sqrt{NT})$ SFO convergence with $O(\sqrt{NT}\log(\frac{T}{N}))$ comm rounds
  - SoA is $O(1/\sqrt{NT})$ SFO convergence with $O(N^{3/4}T^{3/4})$ inter-worker comm rounds

Experiments

Distributed Logistic Regression: N=10





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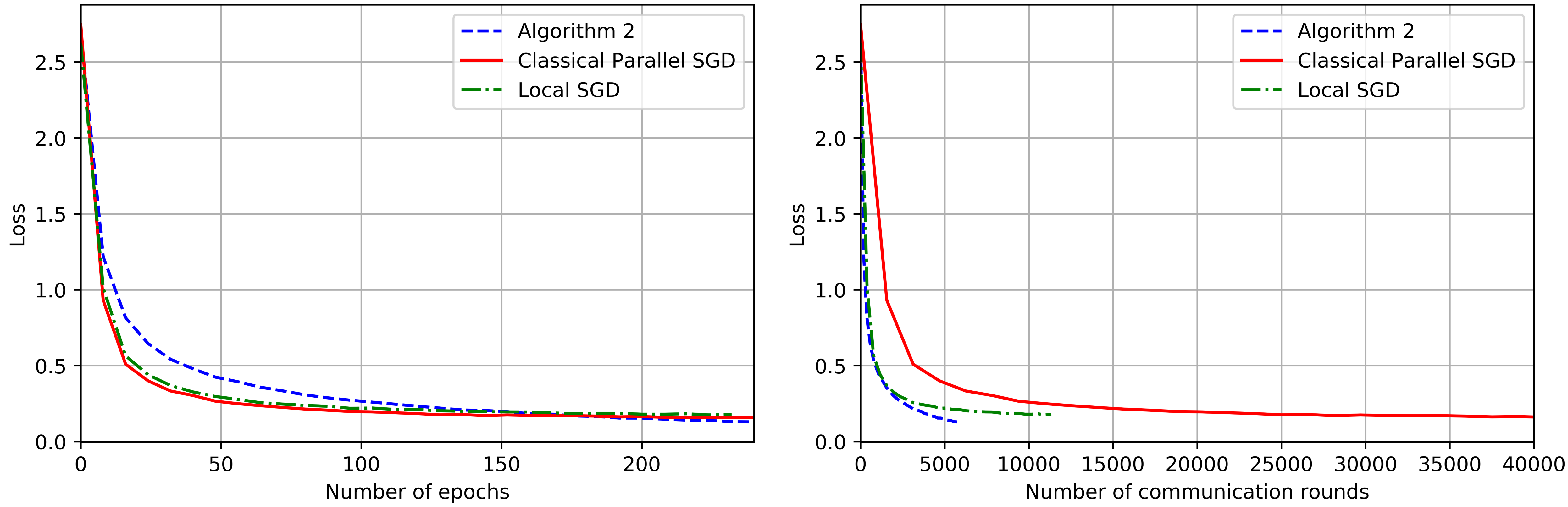
Training ResNet20 over Cifar10: N=8



Thanks!

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