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in collaboration with
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ICML 2019

Descent Algorithms and Local Minima in Spiked Matrix-Tensor Models

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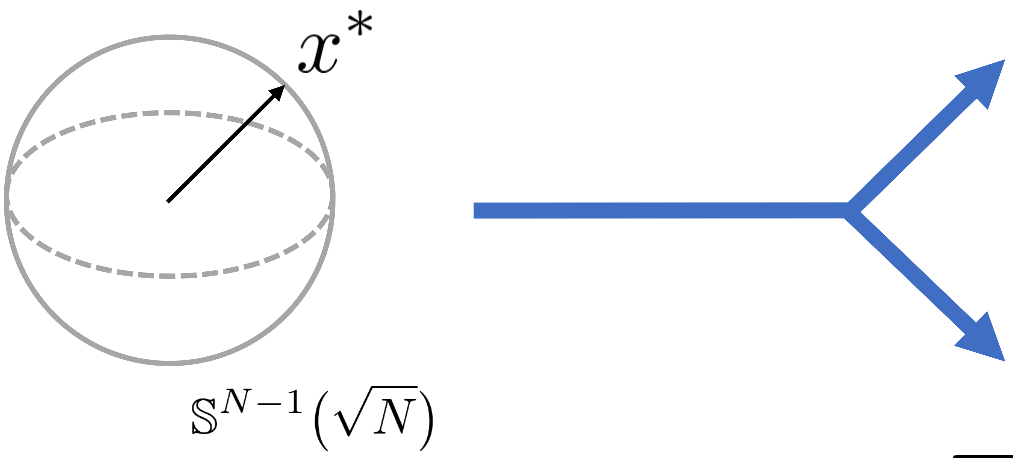
Spiked Matrix-Tensor Model

$$Y_{ij} = \frac{1}{\sqrt{N}} x_i^* x_j^* + \xi_{ij} \qquad \xi_{ij} \sim \mathcal{N}(0, \Delta_2)$$



$$T_{ijk} = \sqrt{\frac{(p-1)!}{N^{p-1}}} x_i^* x_j^* x_k^* + \xi_{ijk} \qquad \xi_{ijk} \sim \mathcal{N}(0, \Delta_p)$$

Spiked Matrix-Tensor Model

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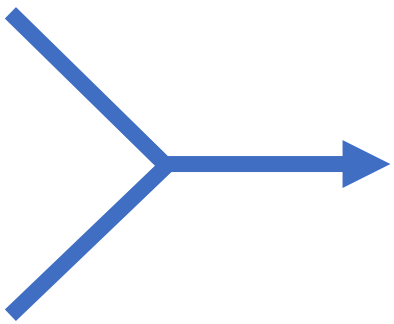

$$x^* = ?$$

$$\hat{x} = \arg\min_x \mathcal{L}(x)$$

minus log-likelihood

$$T_{ijk} = \sqrt{\frac{(p-1)!}{N^{p-1}}} x_i^* x_j^* x_k^* + \xi_{ijk}$$

Spiked Matrix-Tensor Model

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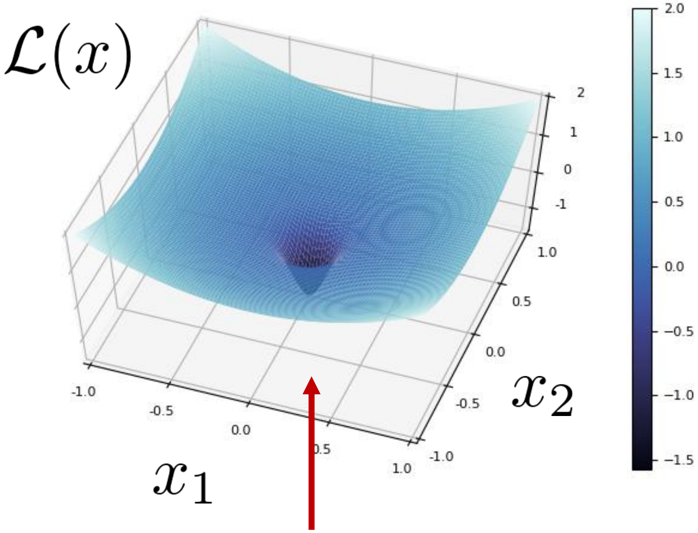

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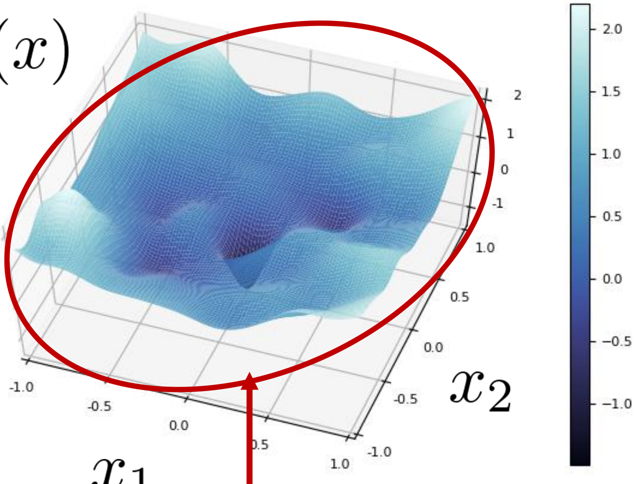

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Gradient Flow Analysis

Analytical analysis of **gradient flow dynamics**.

$$\frac{\partial}{\partial t}C(t,t') = 2R(t',t) - \mu(t)C(t,t') + Q'(m(t))m(t') +$$
$$+ \int_0^t dt'' R(t,t'')Q''(C(t,t''))C(t',t'') + \int_0^{t'} dt'' R(t',t'')Q'(C(t,t''))\,,$$

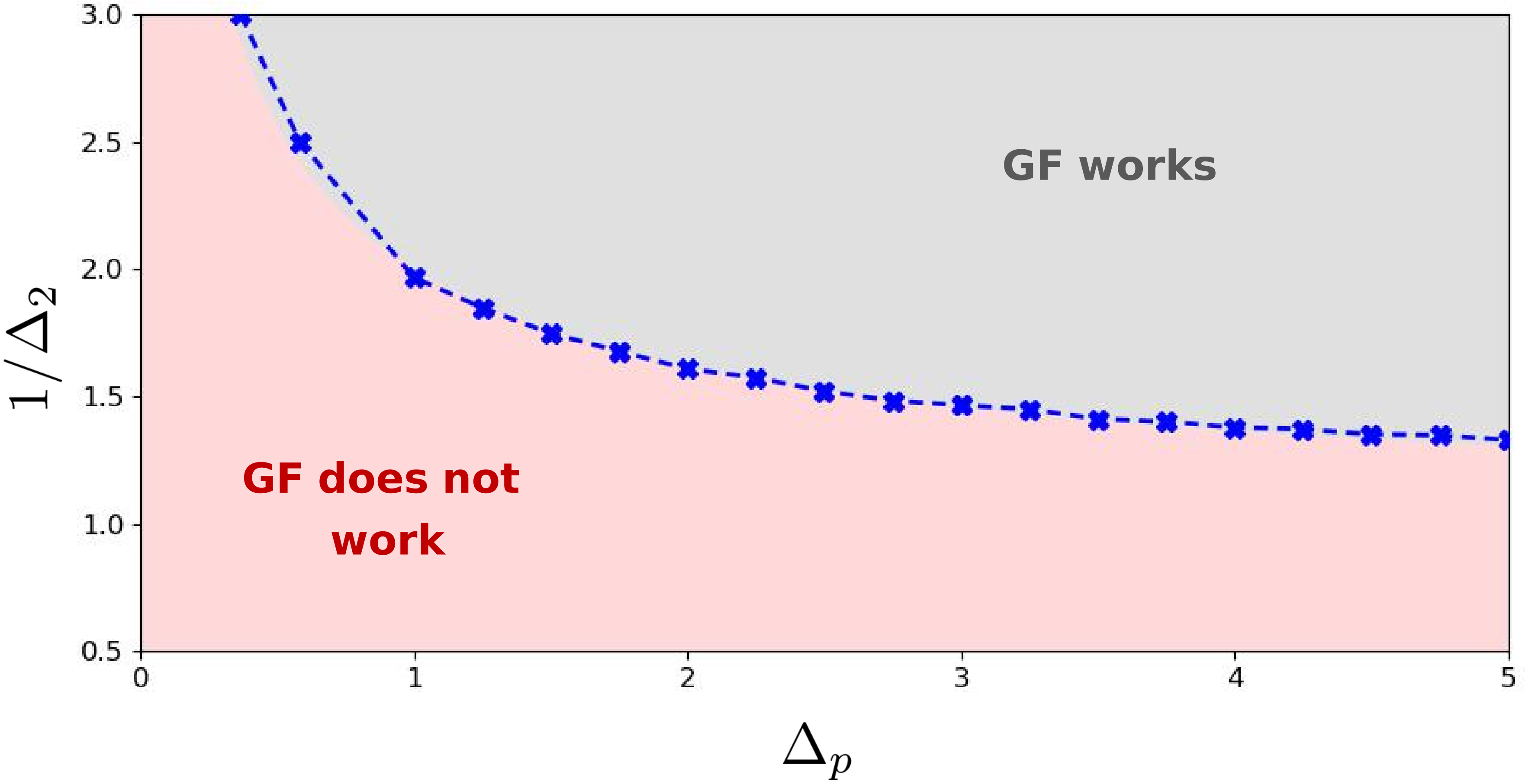
$$\frac{\partial}{\partial t}R(t,t') = \delta(t-t') - \mu(t)R(t,t') + \int_{t'}^t dt'' R(t,t'')Q''(C(t,t''))R(t'',t')\,,$$

$$\frac{\partial}{\partial t}m(t) = -\mu(t)m(t) + Q'(m(t)) + \int_0^t dt'' R(t,t'')m(t'')Q(C(t,t''))\,,$$

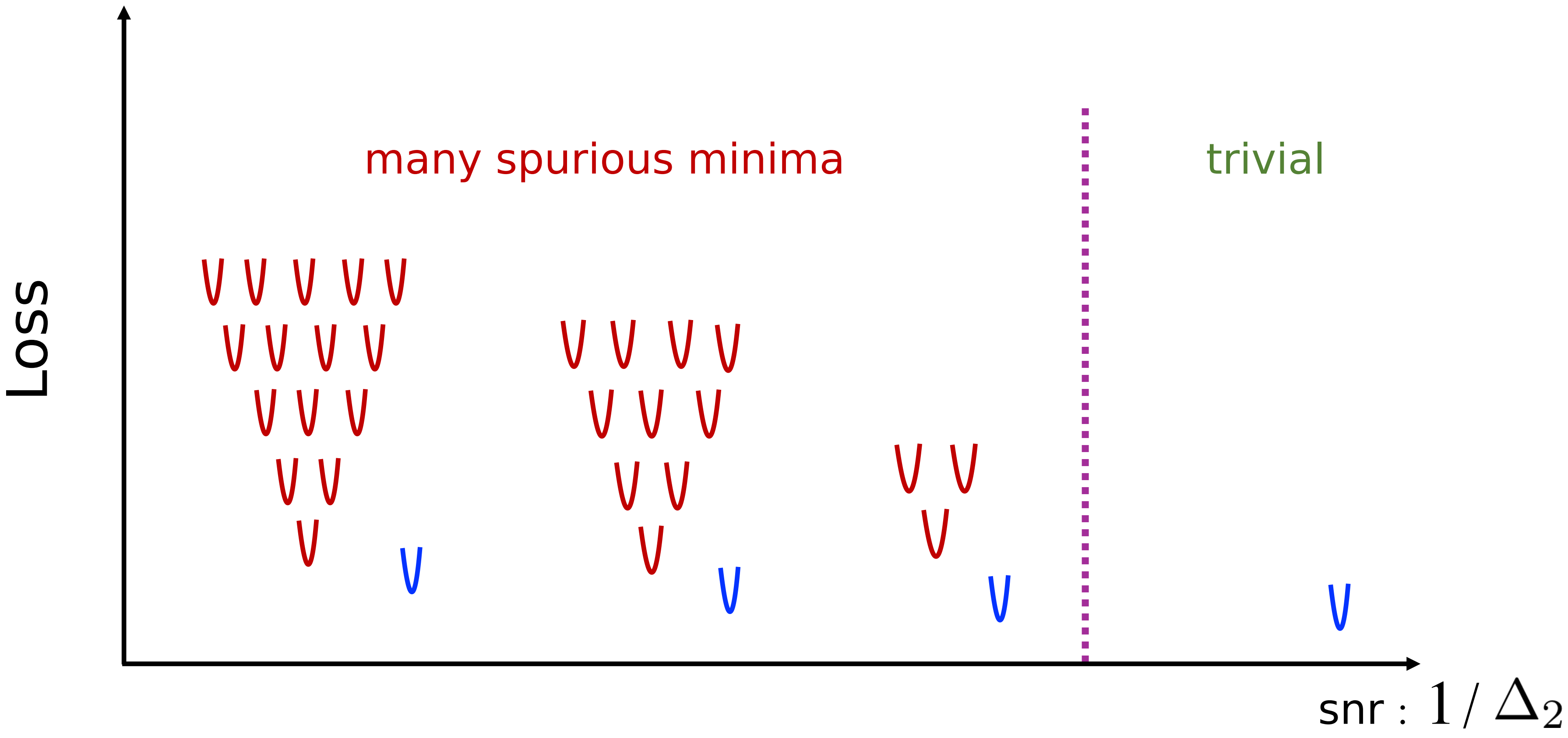
with $Q(x) = x^2/(2\Delta_2) + x^p/(p\Delta_p)$.

$$C(t,t') = \frac{1}{N}\sum_{i=1}^{N}\langle x_i(t)x_i(t')\rangle\,,\;\; R(t,t') = \frac{1}{N}\sum_{i=1}^{N}\left\langle \frac{\delta x_i(t)}{\delta \eta_i(t')}\right\rangle,\;\; m(t) = \frac{1}{N}\sum_{i=1}^{N}\langle x_i(t)x_i^*\rangle\,.$$

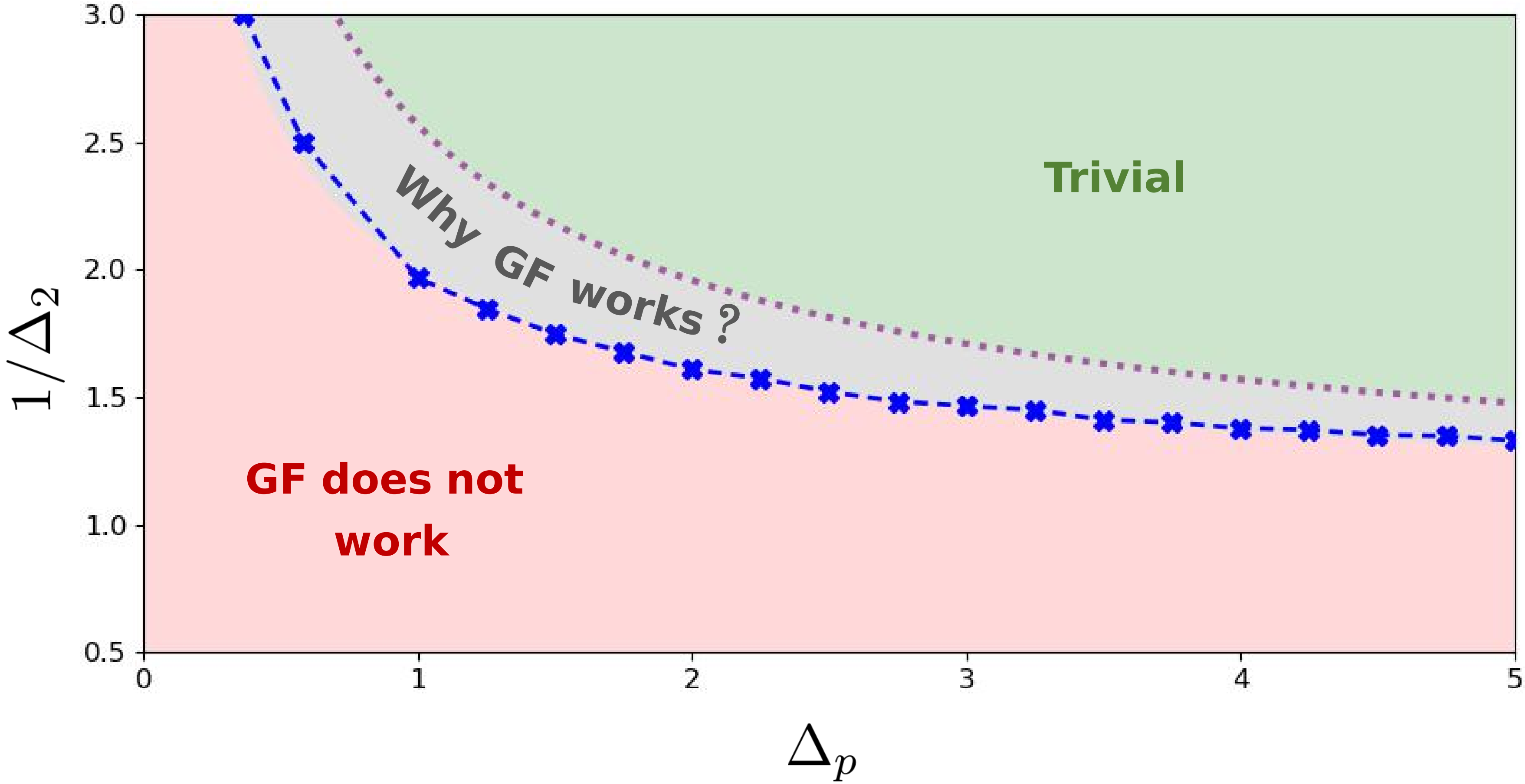
Phase Diagram



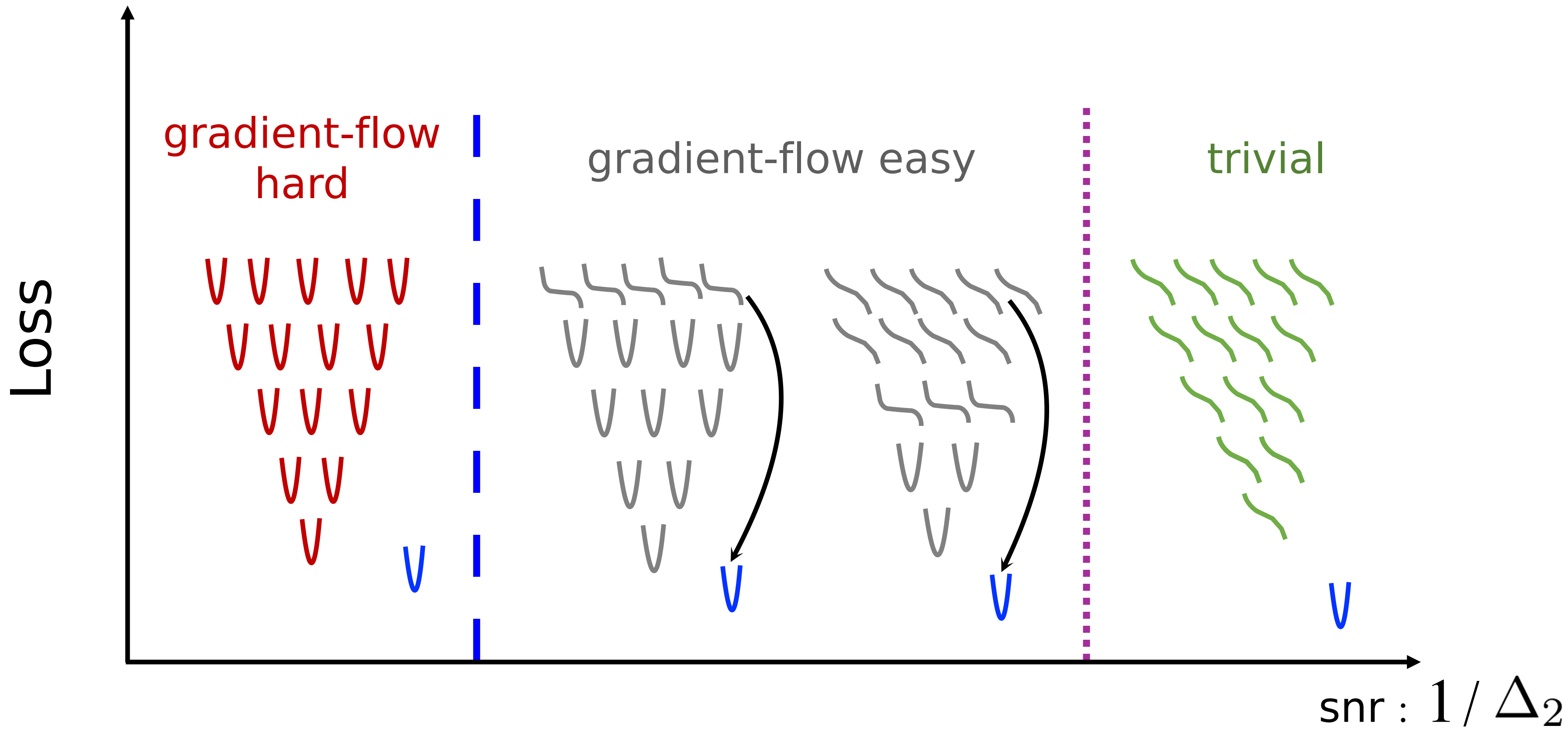
Phase Diagram



Phase Diagram



Phase Diagram



*joint work with Giulio Biroli and Chiara Cammarota


PASSED&
SPURIOUS

Thank you!
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