LAPRAS

ICML 2019 Jun 13th 5:00-5:05 PM @ Hall A

Hyperbolic Disk Embeddings for Directed Acyclic Graphs

Ryota Suzuki, Ryusuke Takahama, Shun Onoda

Poster Session
Today @ Pacific Ballroom #27

Contents

Introduction
Our Contributions
Disk Embedding Models

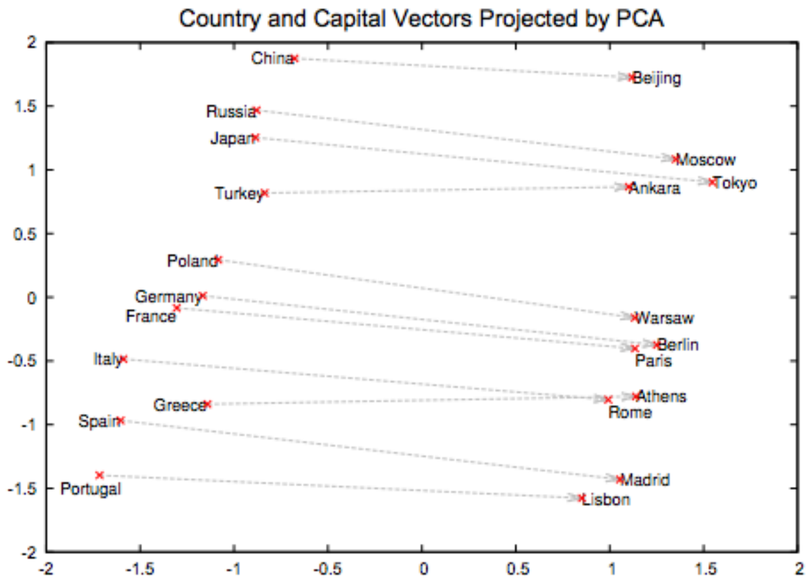
Introduction

Embedding Methods

Vector representation of discrete entities (natural languages, graphs, …)

- Data structure is encoded as geometrical properties (e.g. metric)

Used as first layer of neural networks



[Mikolov, NIPS 2013]

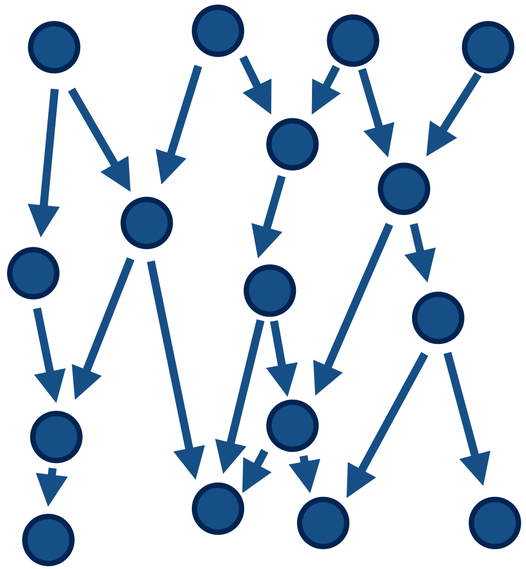
DAG Embedding Models

Embedding Directed Acyclic Graphs (DAGs)

- Implication of words, citation networks, genealogical networks, etc…
- Asymmetric / transitive relation of nodes → Partially ordered set (poset)

DAG Embedding

- Embedding nodes into continuous poset so that transitive relation is preserved.



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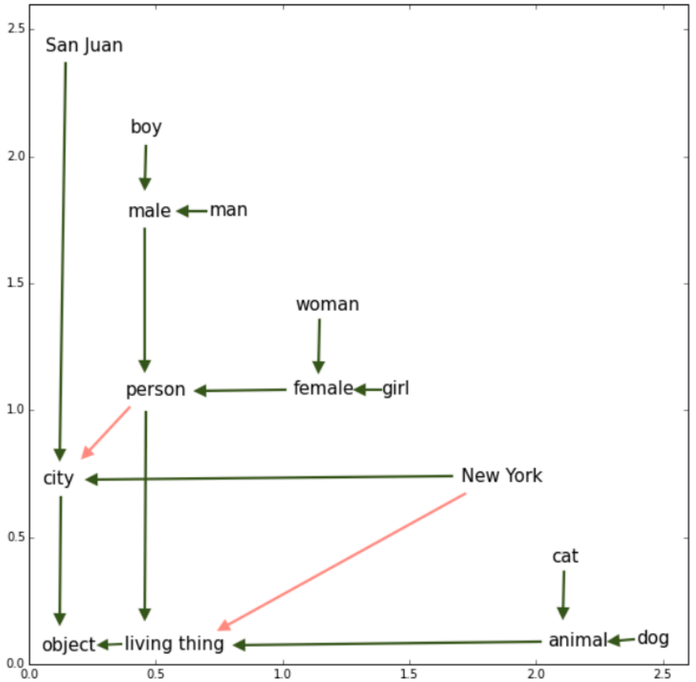
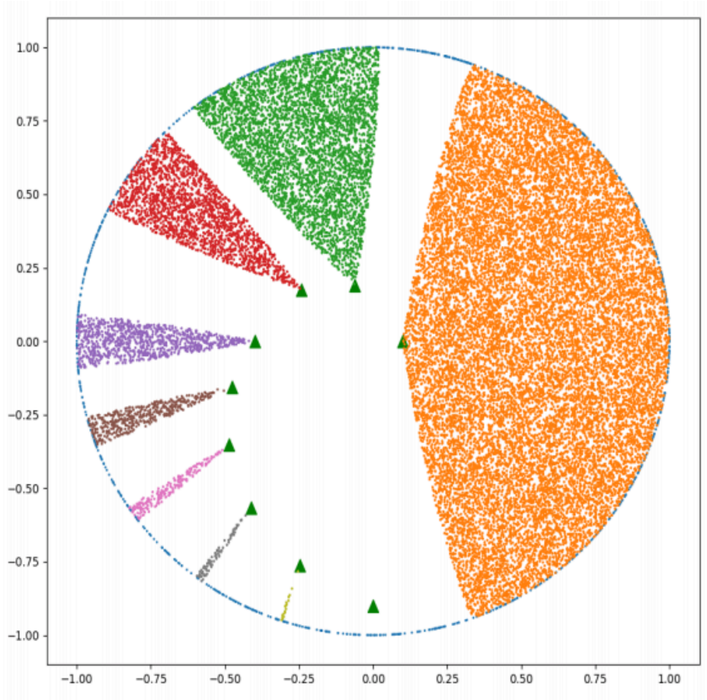
Existing Methods



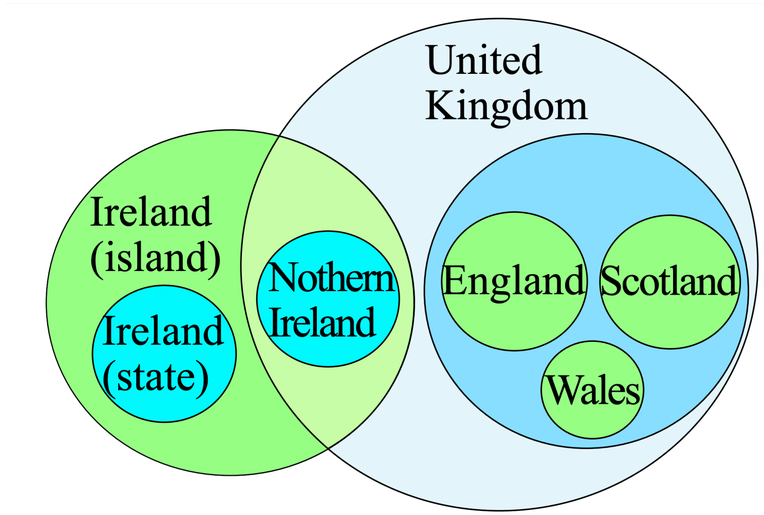
Order Embeddings
[Vendrov, ICLR 2016]



Hyperbolic Entailment Cones
[Ganea, ICML 2018]

Our Contributions

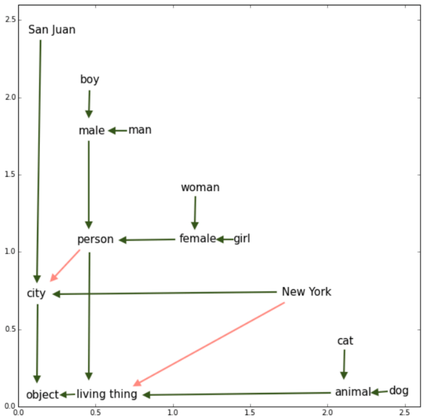
- **Disk Embedding** (DE): General framework for embedding DAGs



2D Euclidean Disk Embedding = Eular Diagram

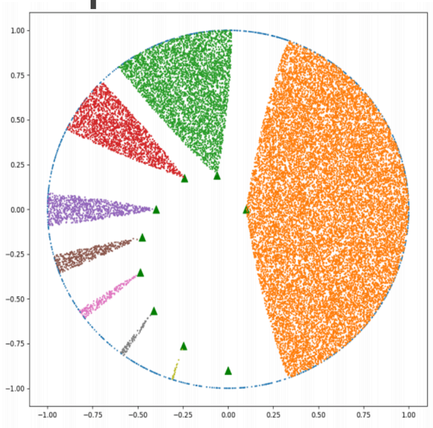
- **Disk Embedding** (DE): General framework for embedding DAGs
- Theorems: **Existing methods are special cases** of DE (+ extra restrictions)

Euclidean DE



Order Embeddings
[Vendrov, ICLR 2016]

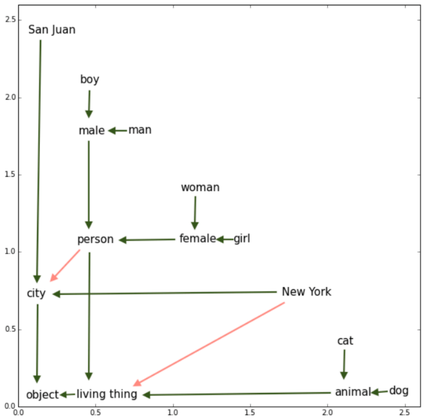
Spherical DE



Hyperbolic Entailment Cones
[Ganea, ICML 2018]

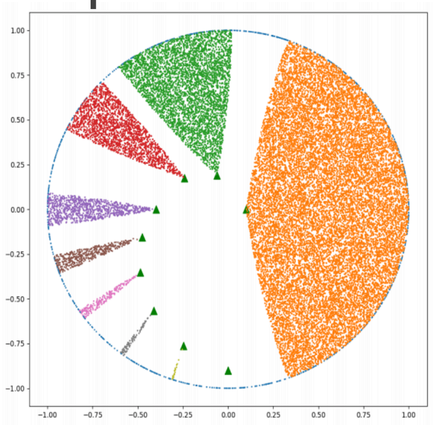
- **Disk Embedding** (DE): General framework for embedding DAGs
- Theorems: **Existing methods are special cases** of DE (+ extra restrictions)
- Novel **Hyperbolic Disk Embedding**

Euclidean DE



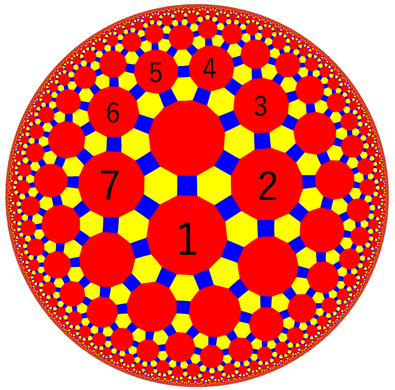
Order Embeddings
[Vendrov, ICLR 2016]

Spherical DE



Hyperbolic Entailment Cones
[Ganea, ICML 2018]

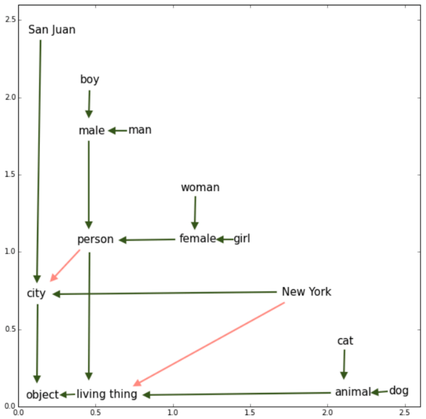
Hyperbolic DE



Disks tiled in the 2D hyperbolic space
(from Wikipedia)

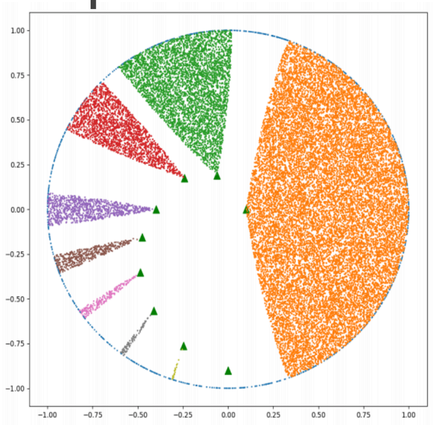
- **Disk Embedding** (DE): General framework for embedding DAGs
- Theorems: **Existing methods are special cases** of DE (+ extra restrictions)
- Novel **Hyperbolic Disk Embedding**
- Experiments

Euclidean DE



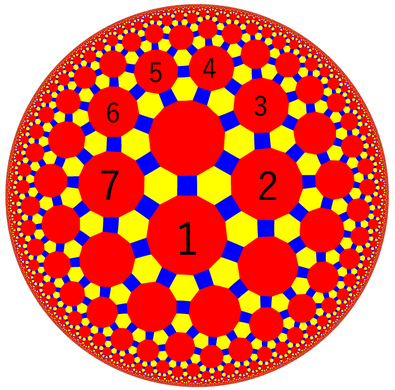
Order Embeddings
[Vendrov, ICLR 2016]

Spherical DE



Hyperbolic Entailment Cones
[Ganea, ICML 2018]

Hyperbolic DE

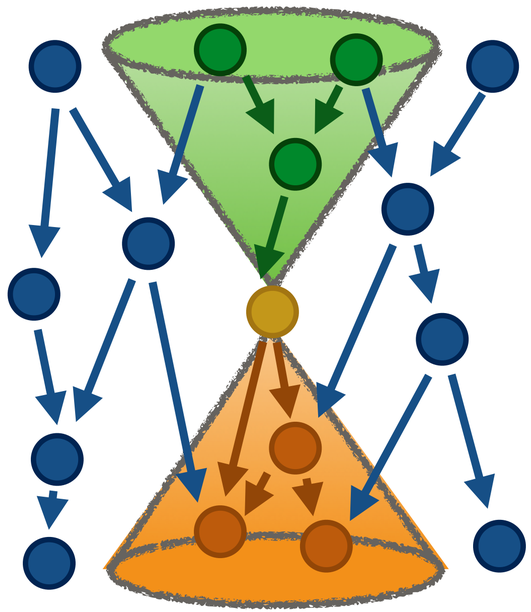


Disks tiled in the 2D hyperbolic space
(from Wikipedia)

Disk Embedding Models

Embedding DAGs

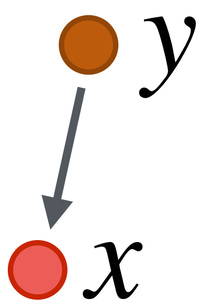
Transitive relation of DAG induces “cones” in the embedding space.



Ancestors = Upper cone

Descendants = Lower cone

Key Idea



Relation of DAG nodes

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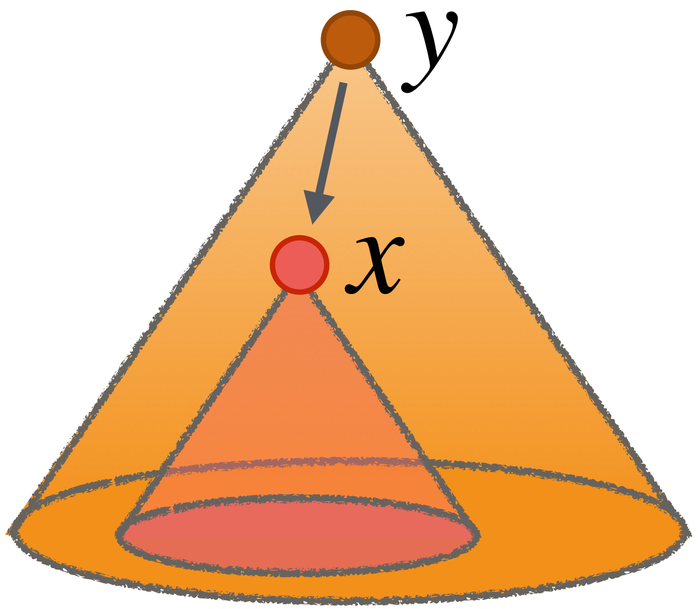
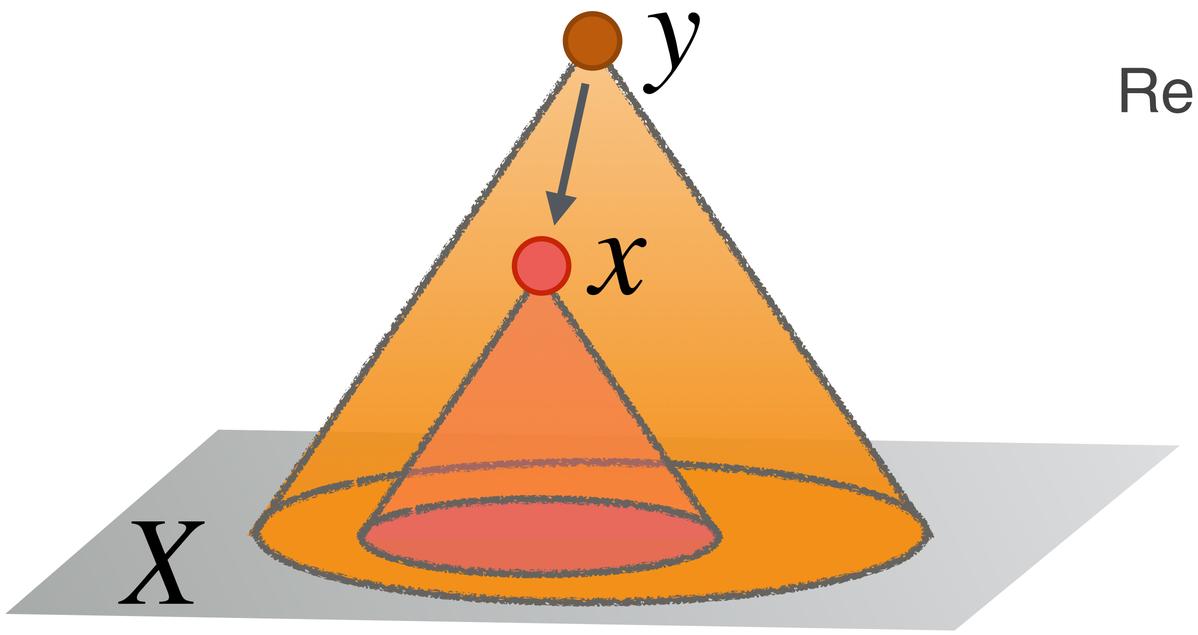
Key Idea



Relation of DAG nodes

= Inclusion of lower cones

Key Idea

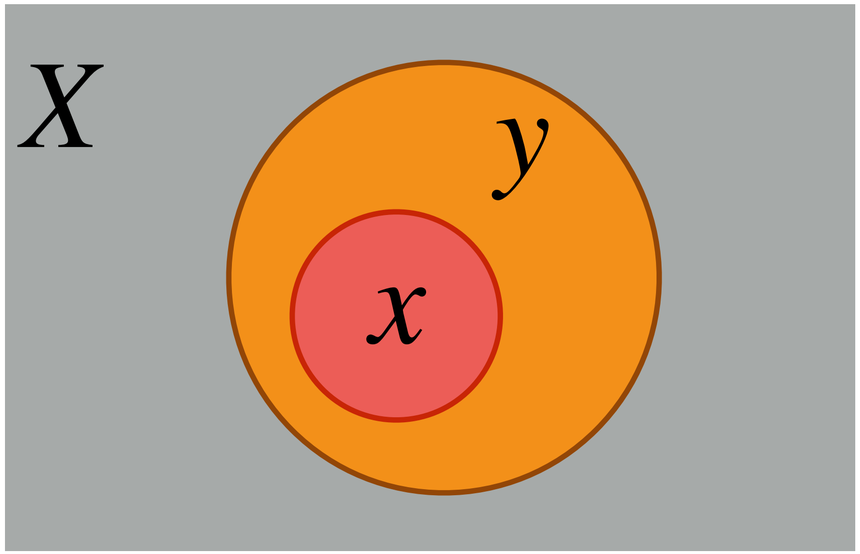


Relation of DAG nodes

= Inclusion of lower cones

= **Inclusion of projected disks**

Key Idea



$$d_X(x, y) + r_x - r_y \leq 0$$

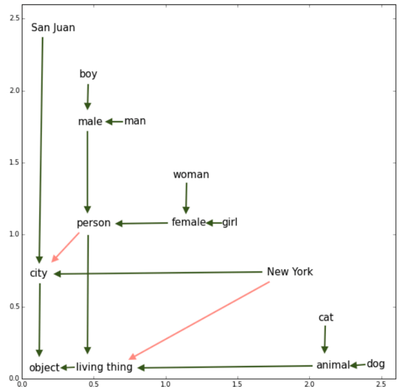
Relation of DAG nodes

= Inclusion of lower cones

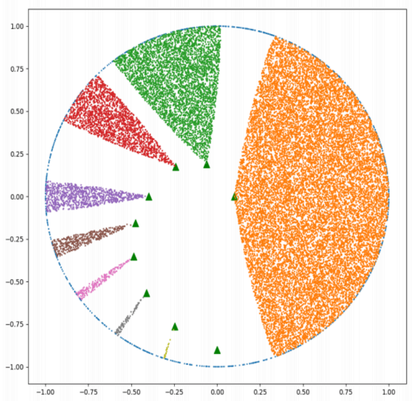
= **Inclusion of projected disks**

= **Disk Embedding**

Equivalence of models

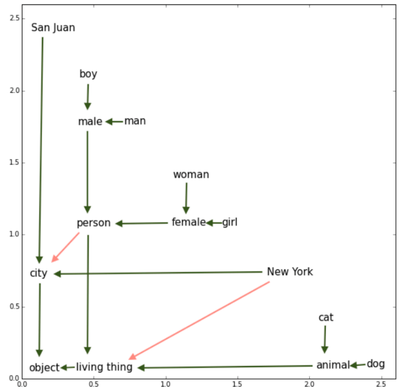


Order Embeddings
[Vendrov 2016]



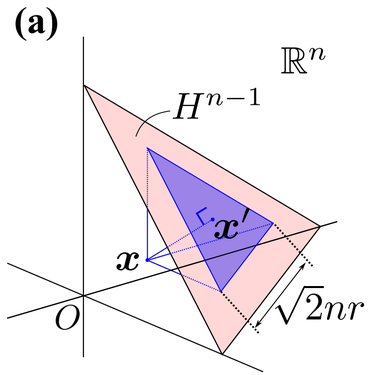
Hyperbolic Entailment Cones
[Ganea 2018]

Equivalence of models



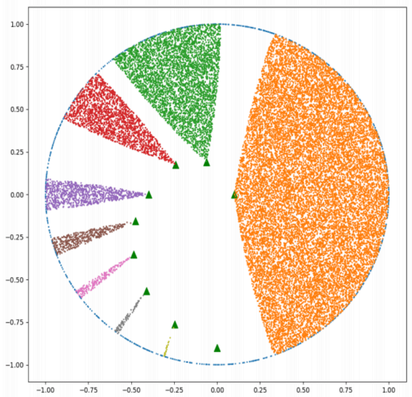
Order Embeddings
[Vendrov 2016]

ϕ



Euclidean Disk Embedding
(w/ polyhedral quasimetric)

Order Isomorphism



Hyperbolic Entailment Cones
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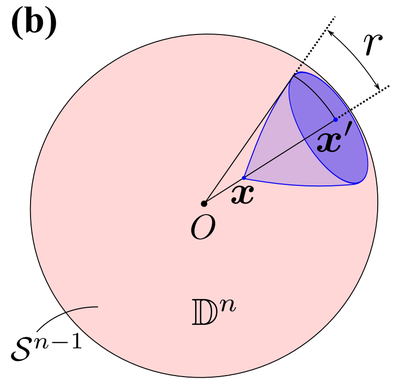
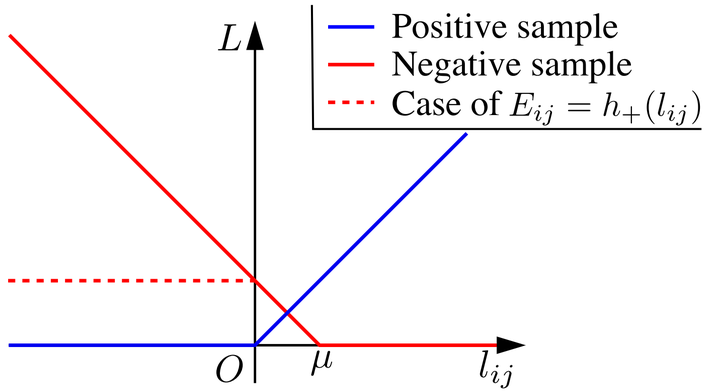


Spherical Disk Embedding

Advantage of Disk Embedding

Extends existing methods

- *Negative radius*: **reversibility & translational symmetry**
- **Avoid gradient vanishing** on loss functions



Applicable for various (quasi-)metric spaces

- Euclidean space, Sphere, …
- **Hyperbolic Disk Embedding**

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Experimental Results

Hierarchy

Non-Hierarchy

	Embedding Dimension = 5				Embedding Dimension = 10			
	Percentage of Transitive Closure (Non-basic) Edges in Training							
	0%	10%	25%	50%	0%	10%	25%	50%
WordNet nouns								
Our Euclidean Disk Embeddings	35.6%	38.9%	42.5%	45.1%	**45.6%**	54.0%	65.8%	72.0%
Our Hyperbolic Disk Embeddings	32.9%	69.1%	81.3%	83.1%	36.5%	79.7%	90.5%	**94.2%**
Our Spherical Disk Embeddings	**37.5%**	**84.8%**	**90.5%**	**93.4%**	42.0%	**86.4%**	**91.5%**	93.9%
Hyperblic Entailment Cones	29.2%	80.0%	87.1%	92.8%	32.4%	84.9%	90.8%	93.8%
Order Embeddings	34.4%	70.6%	75.9%	82.1%	43.0%	69.7%	79.4%	84.1%
Poincaré Embeddings	28.1%	69.4%	78.3%	83.9%	29.0%	71.5%	82.1%	85.4%
WordNet nouns reversed								
Our Euclidean Disk Embeddings	**35.4%**	38.7%	42.3%	44.6%	**46.6%**	55.9%	67.3%	70.6%
Our Hyperbolic Disk Embeddings	30.8%	49.0%	66.8%	78.5%	32.1%	53.7%	79.1%	88.2%
Our Spherical Disk Embeddings	34.8%	**59.0%**	**76.8%**	**84.9%**	38.0%	60.6%	**83.1%**	**90.1%**
Hyperblic Entailment Cones	17.3%	57.5%	71.8%	75.7%	20.5%	**61.9%**	73.1%	75.8%
Order Embeddings	32.9%	33.8%	34.8%	35.8%	34.7%	36.7%	38.8%	41.4%
Poincaré Embeddings	26.0%	48.4%	48.8%	51.4%	27.4%	49.7%	50.9%	51.9%

Our Disk Embedding models achieved significant improvement especially in non-hierarchical data

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Thank you for listening!

Poster Session

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