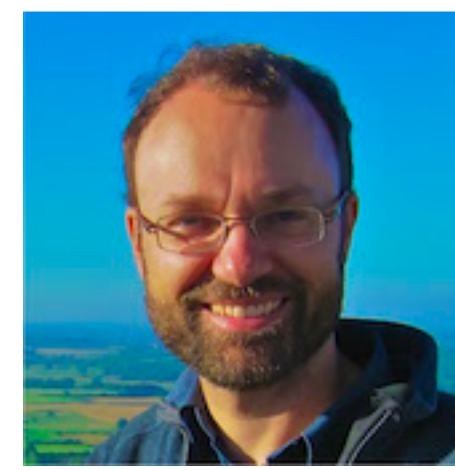
Supervised Hierarchical Clustering with Exponential Linkage

Nishant Yadav Ari Kobren Nicholas Monath Andrew McCallum

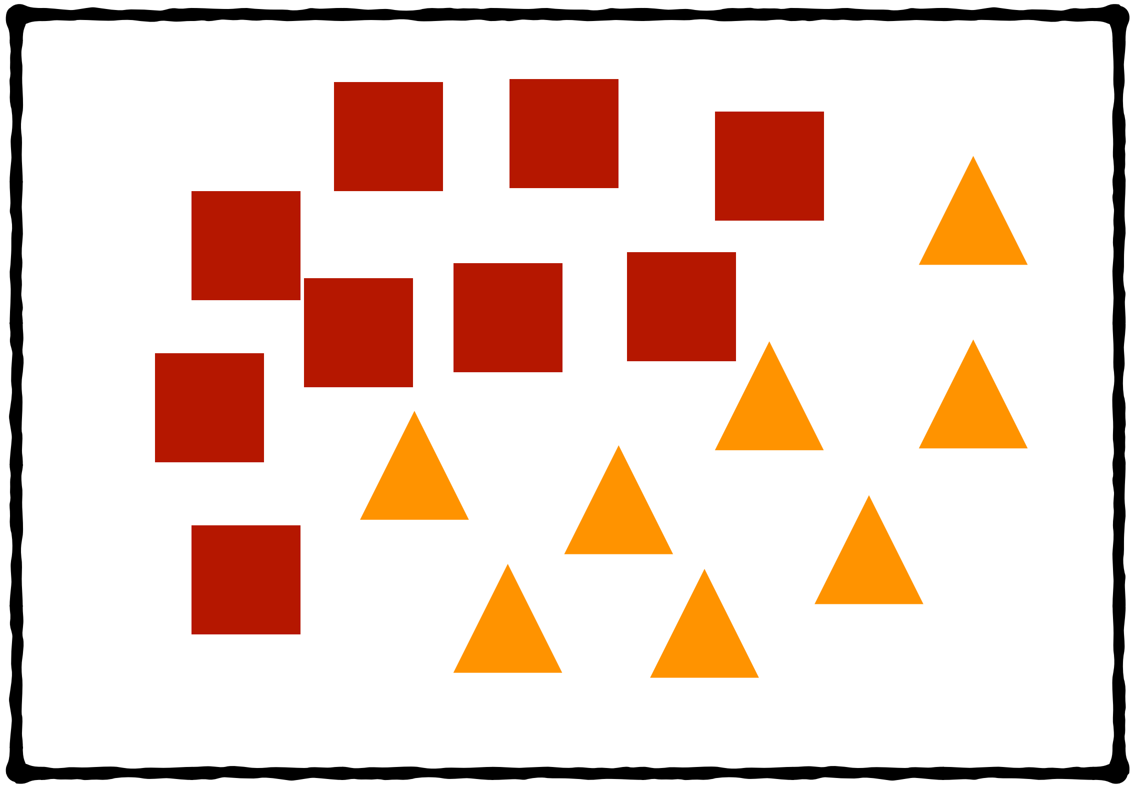


UMass Amherst

College *of* Information and Computer Sciences

Supervised Clustering

At train time, learn $\mathcal{A} : 2^{\mathcal{X}} \rightarrow \mathcal{Y}$



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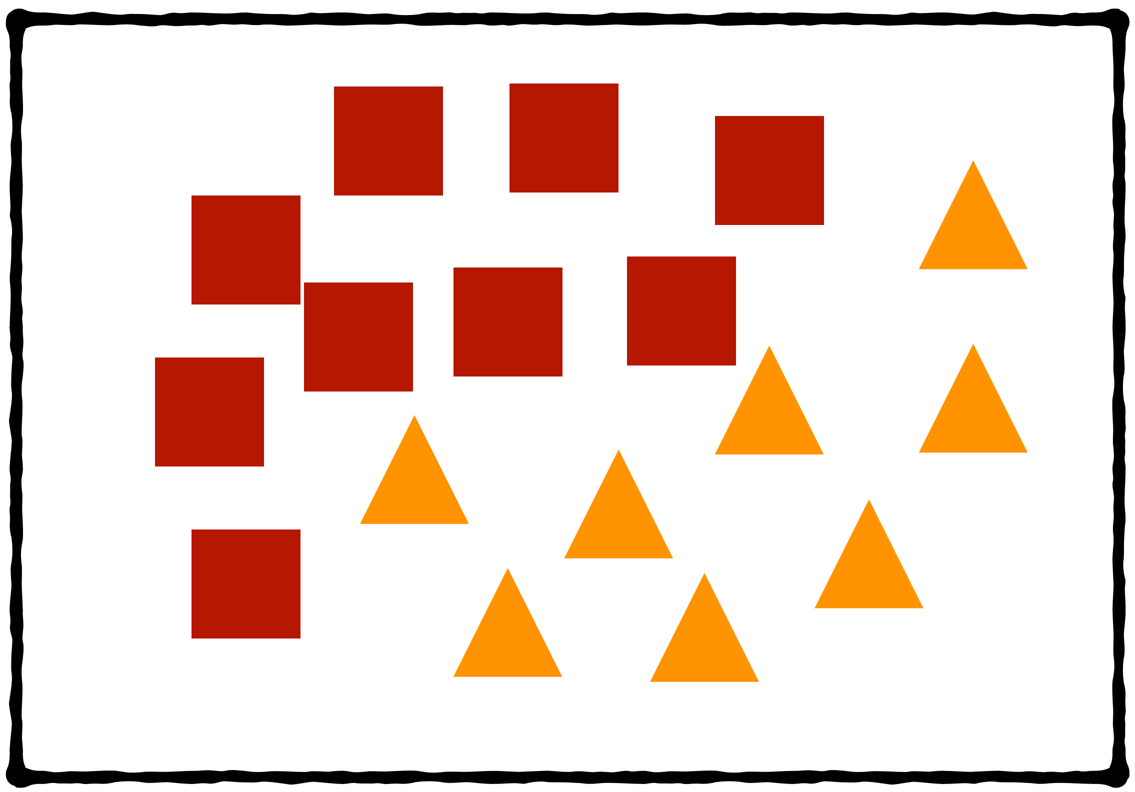
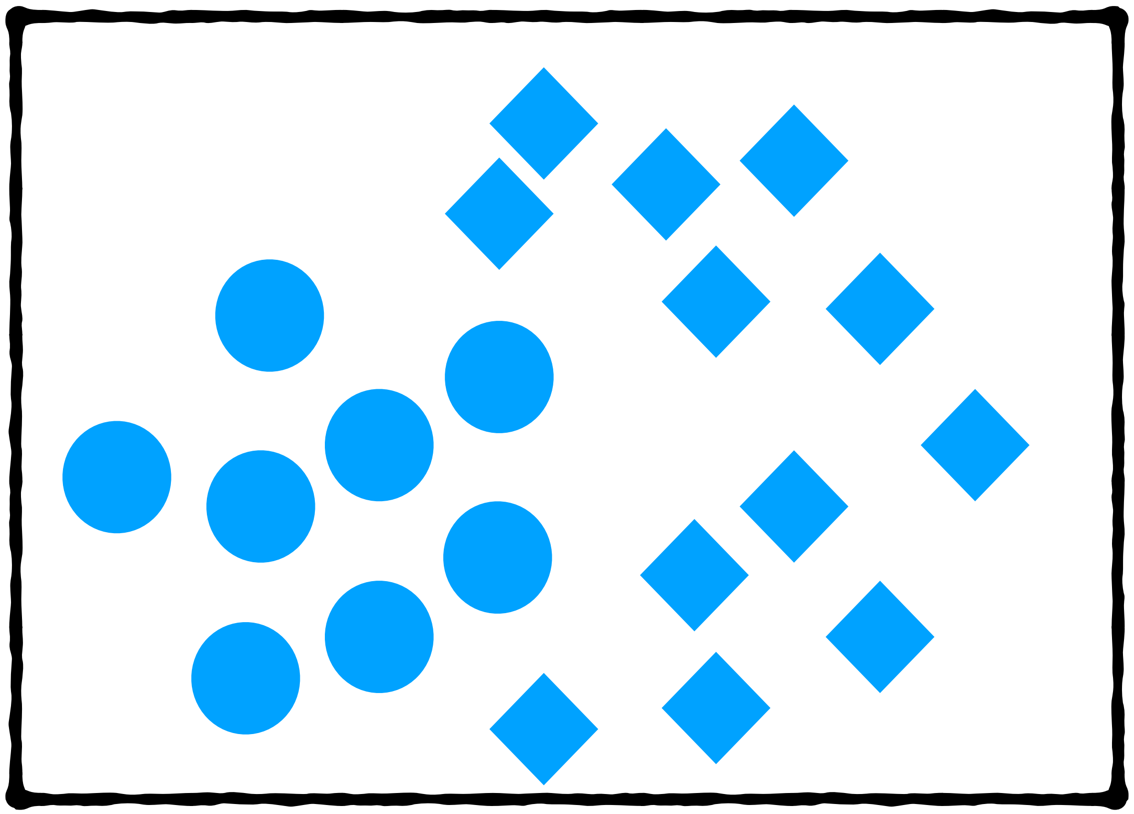


At test time, use $\mathcal{A}$ on new set of points



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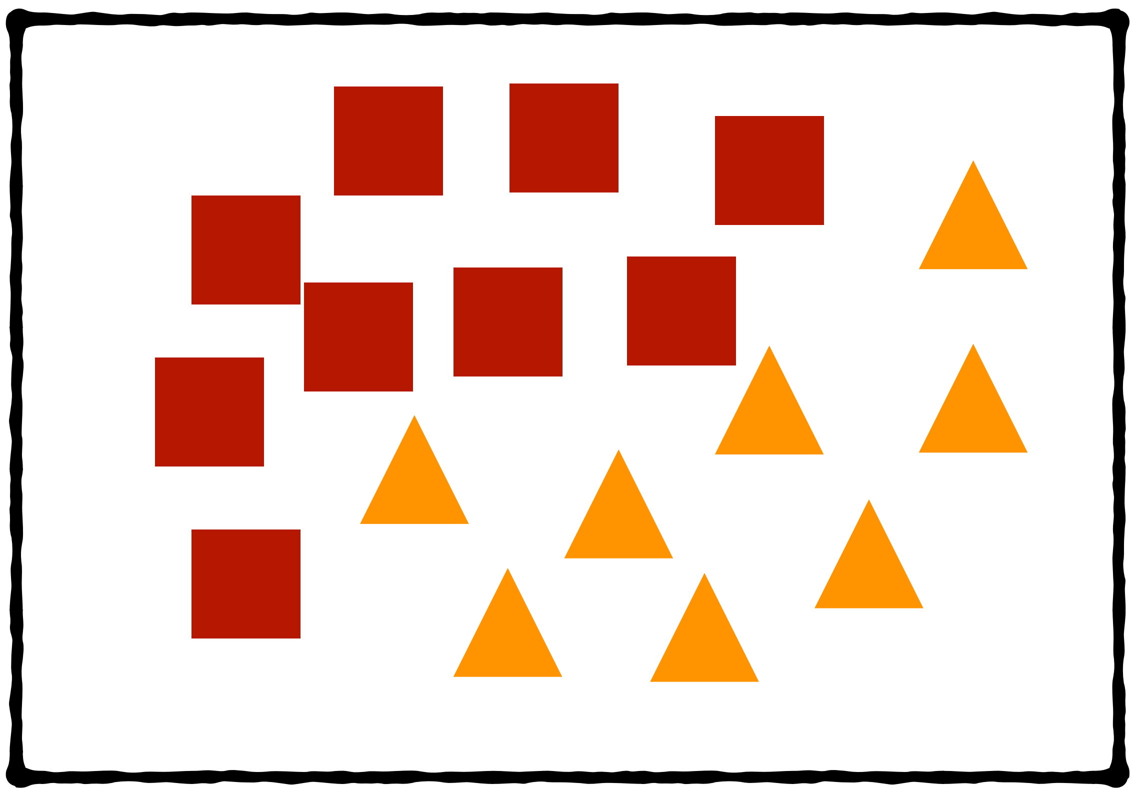

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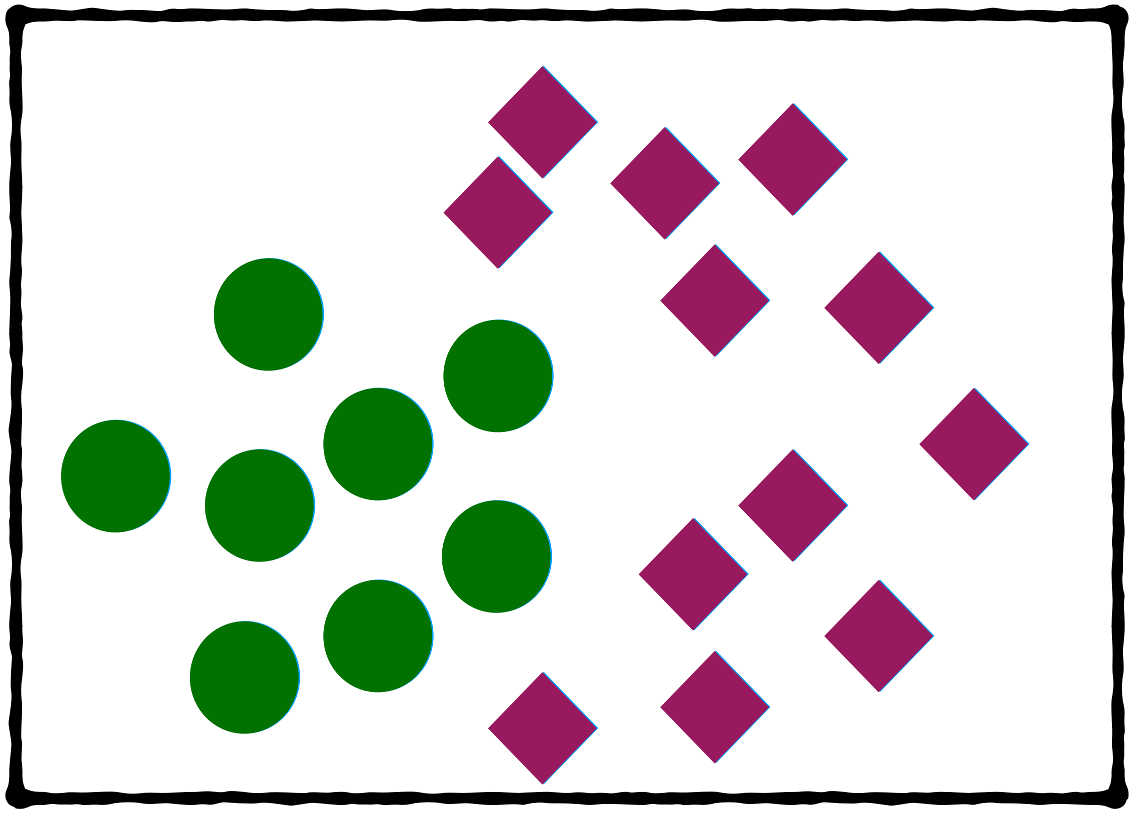

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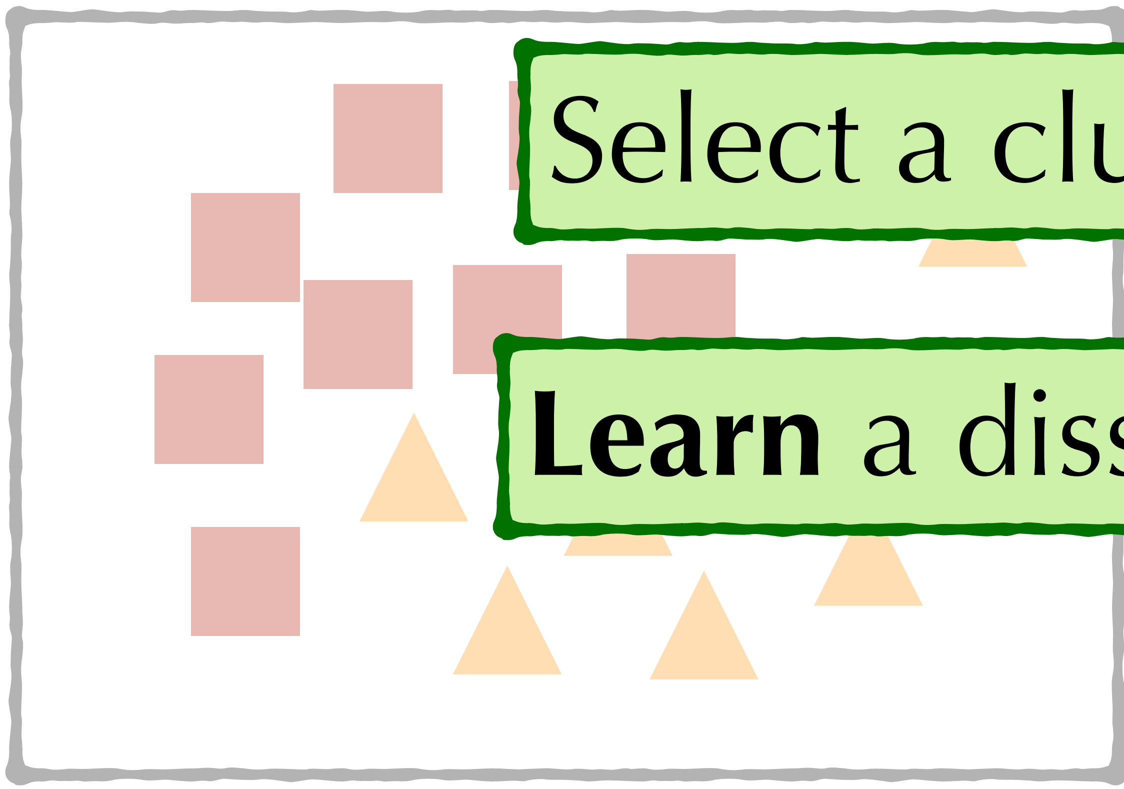

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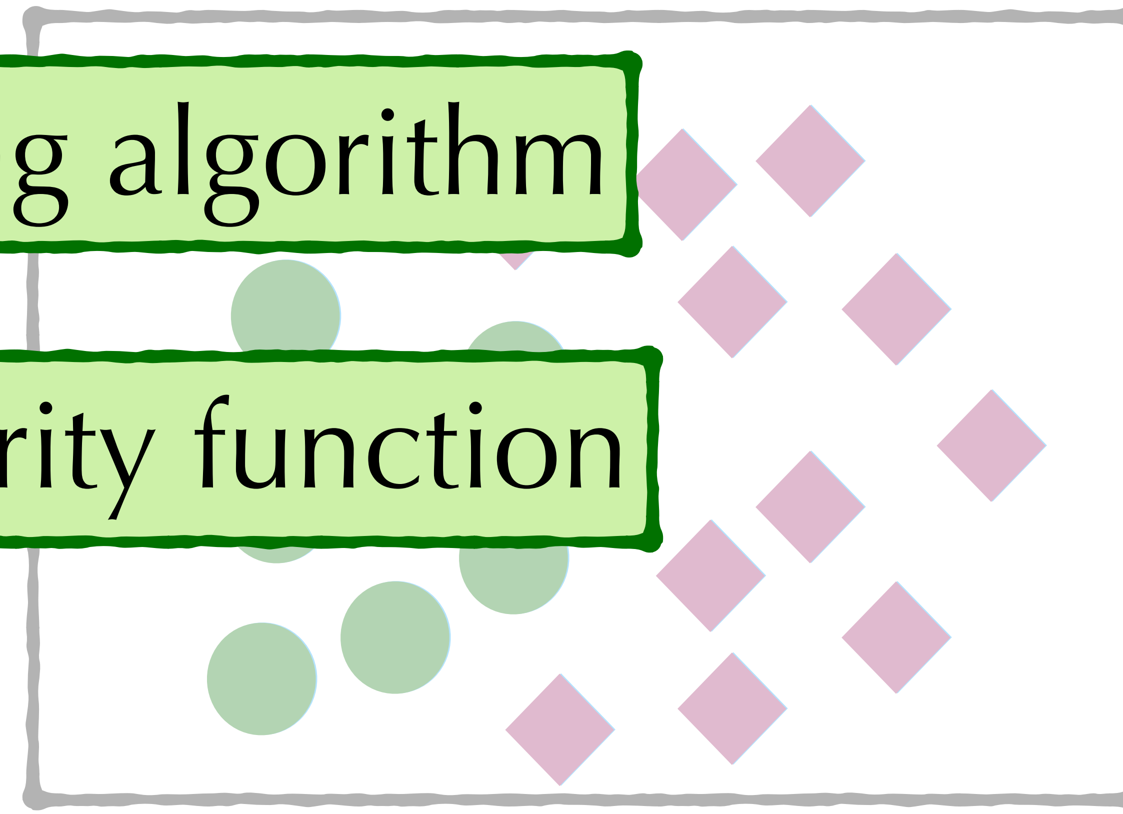

Select a clustering algorithm

Learn a dissimilarity function

Supervised Clustering

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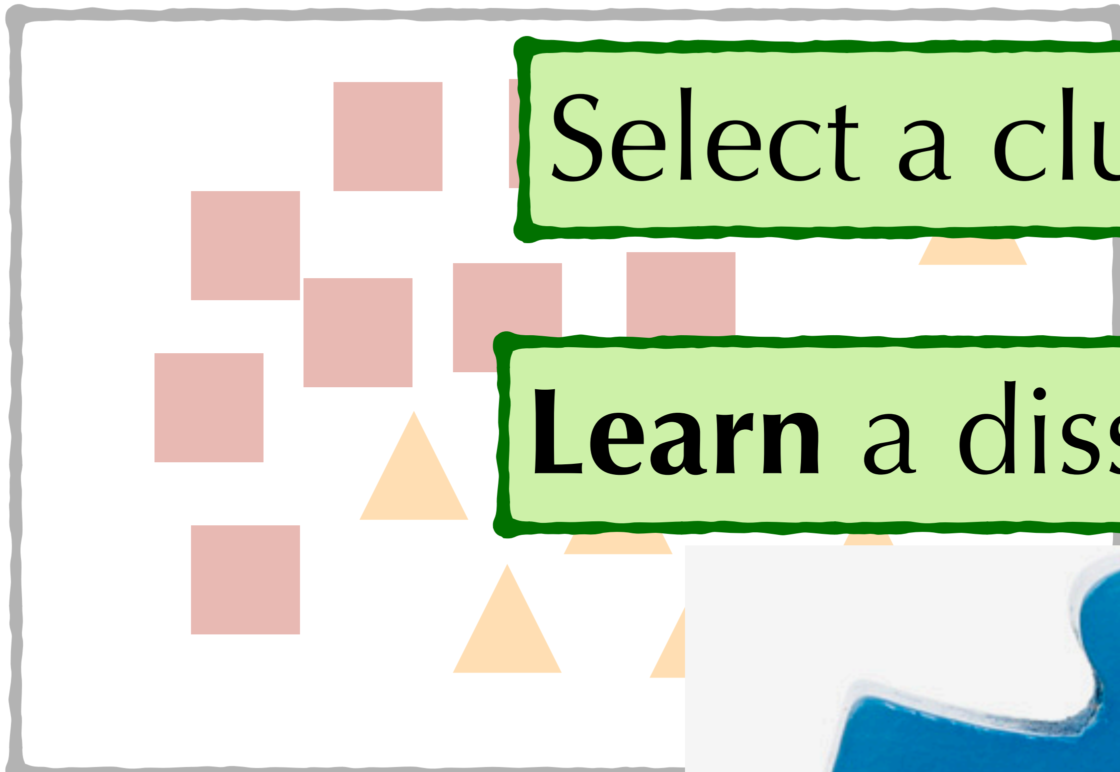

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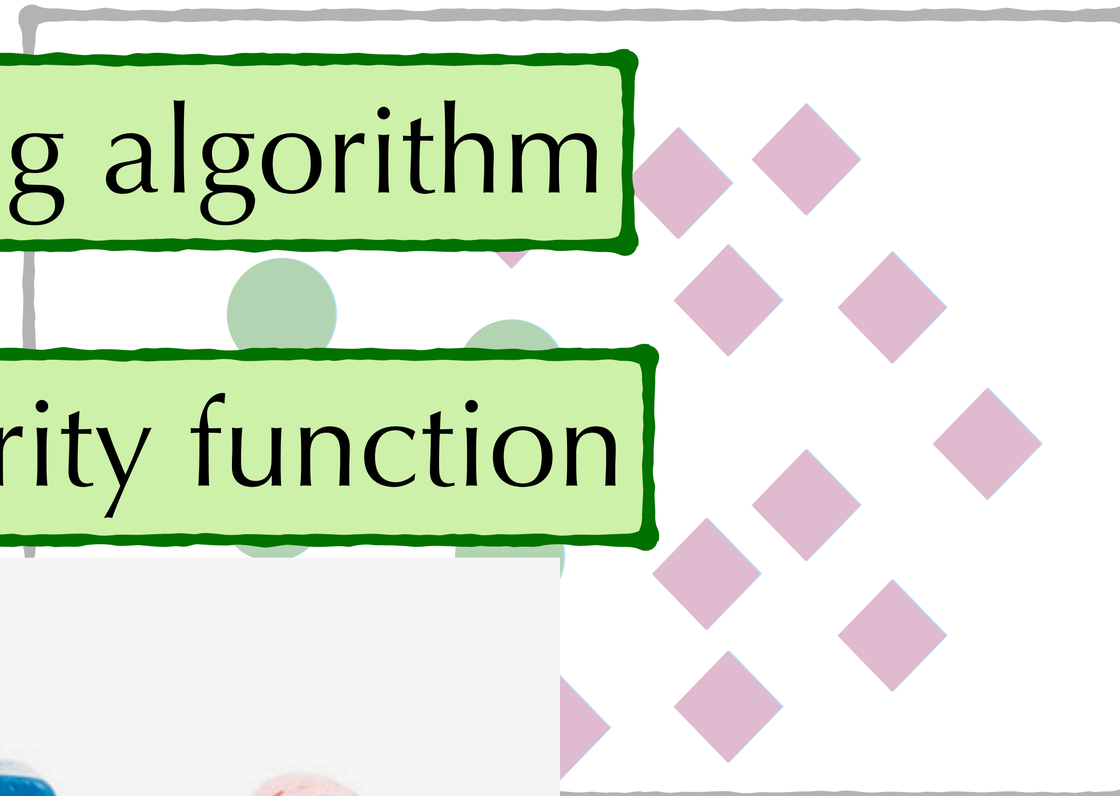

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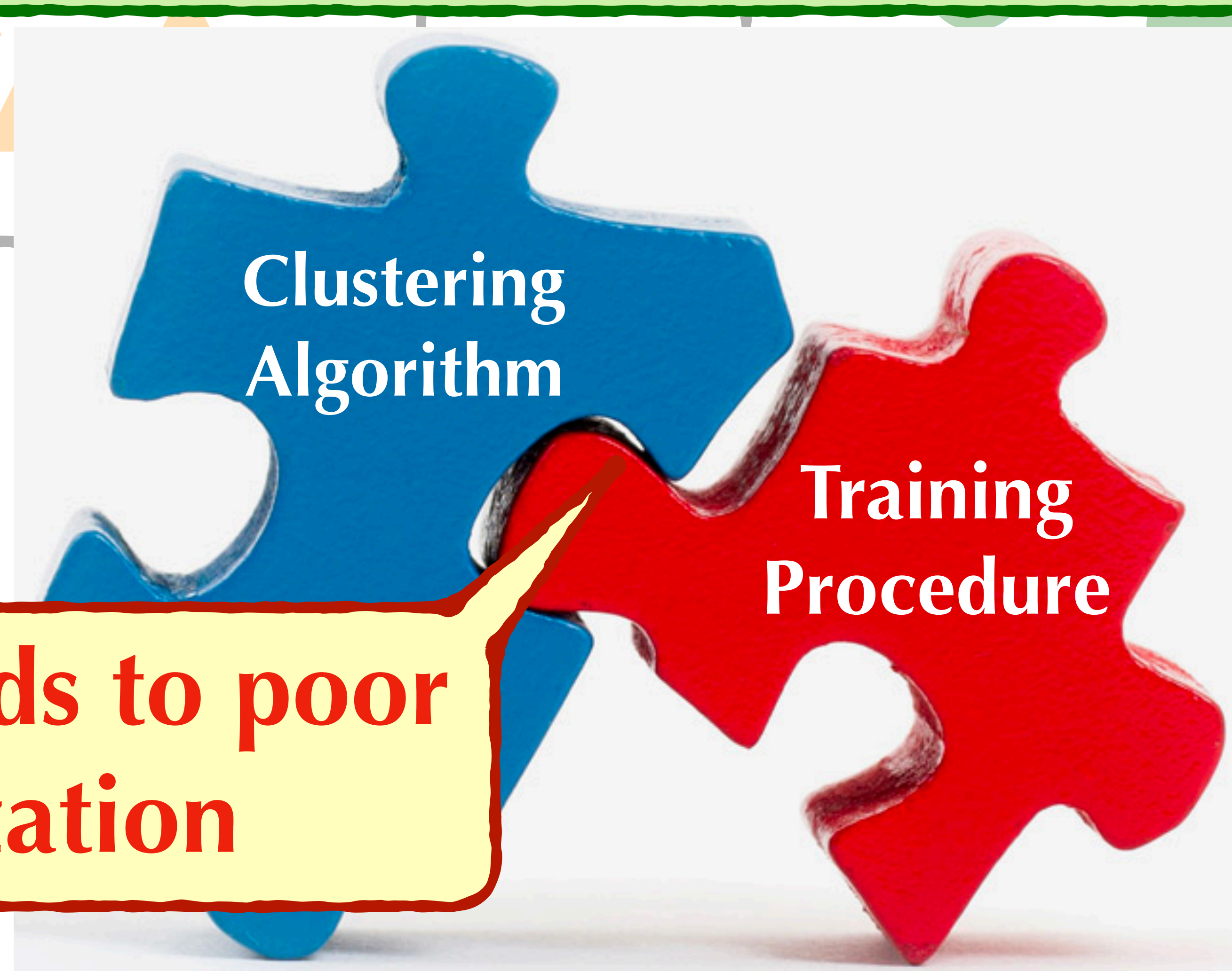

Mismatch leads to poor generalization

Hierarchical Agglomerative Clustering

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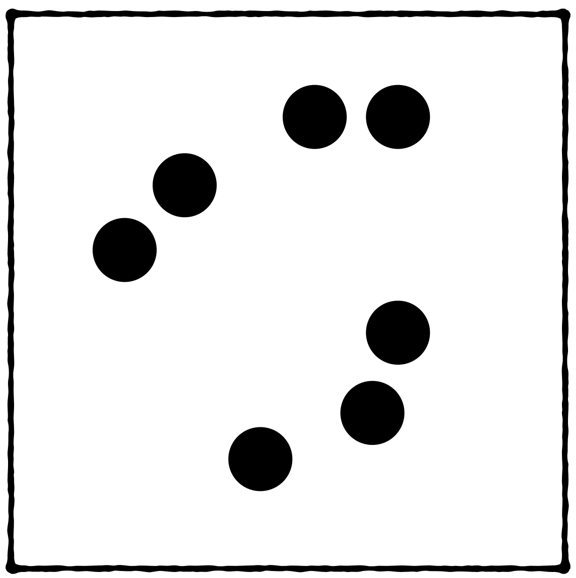

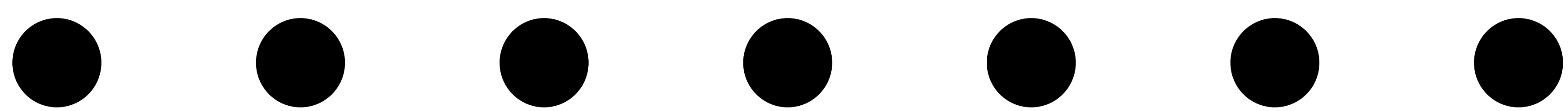

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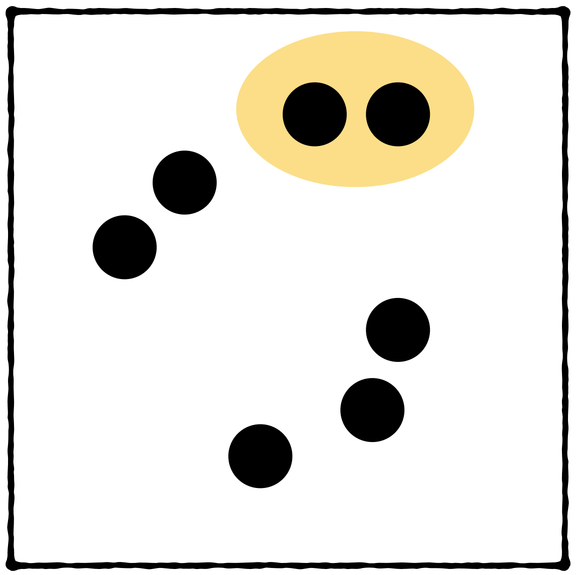
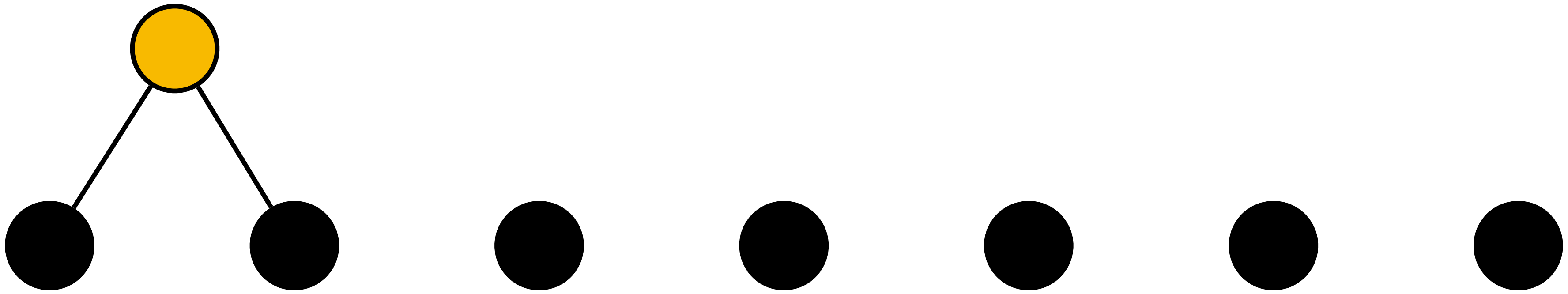
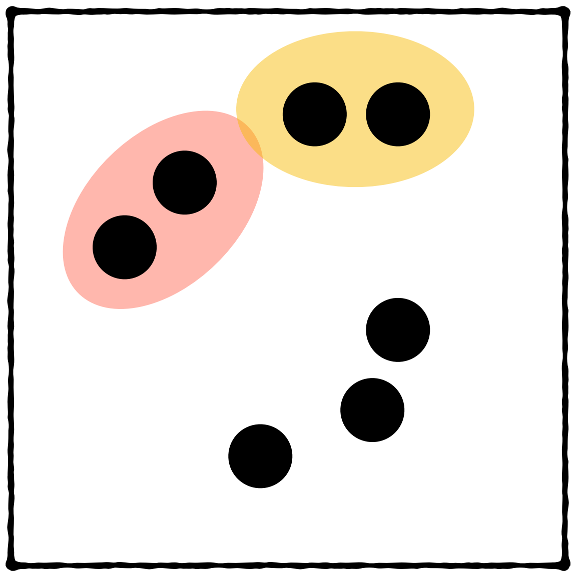
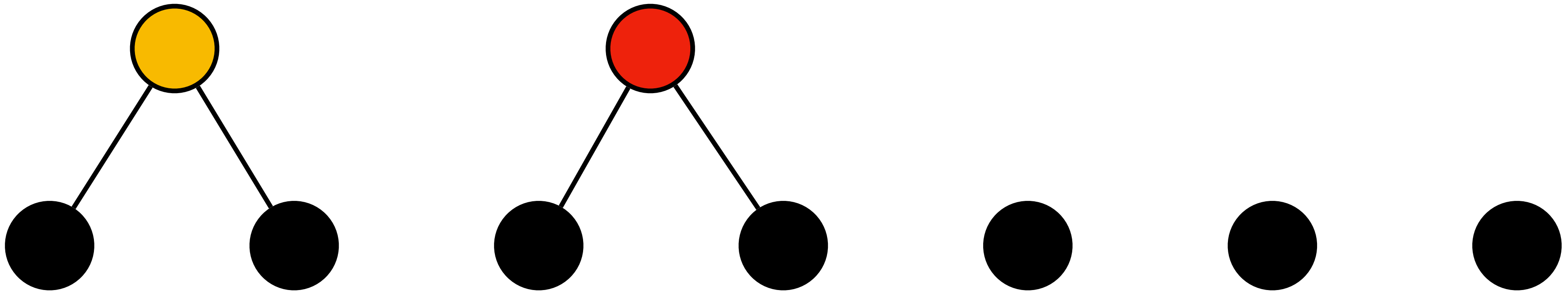




Iteratively merge two ***"closest"*** clusters

Hierarchical Agglomerative Clustering





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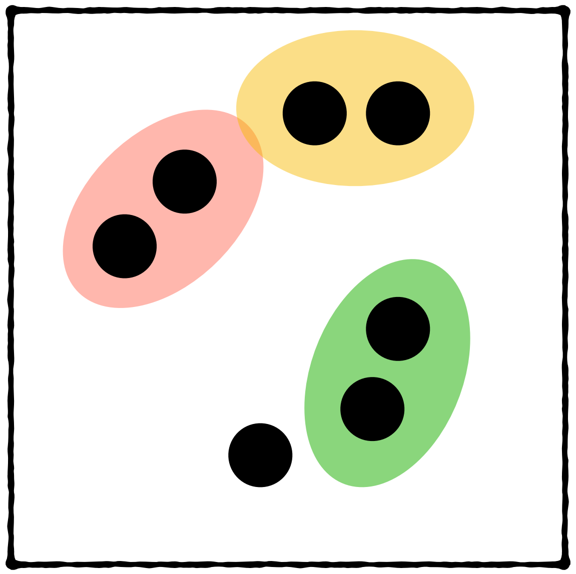

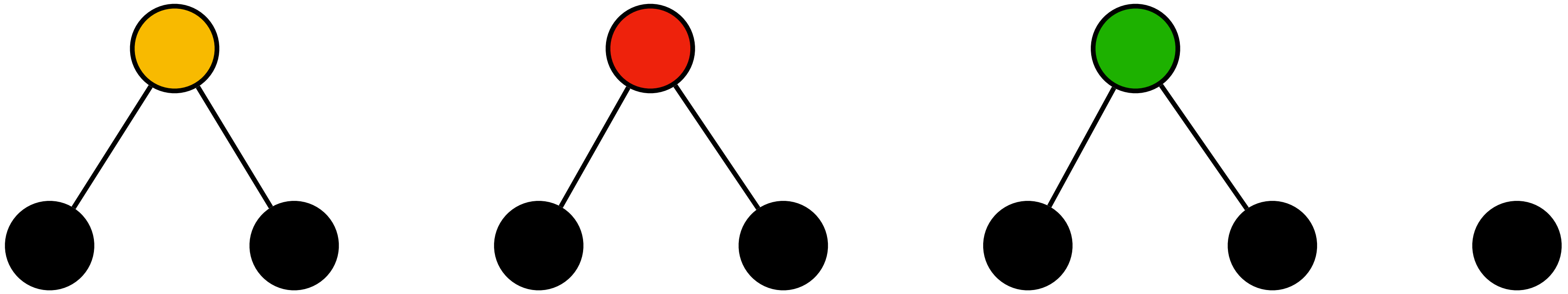

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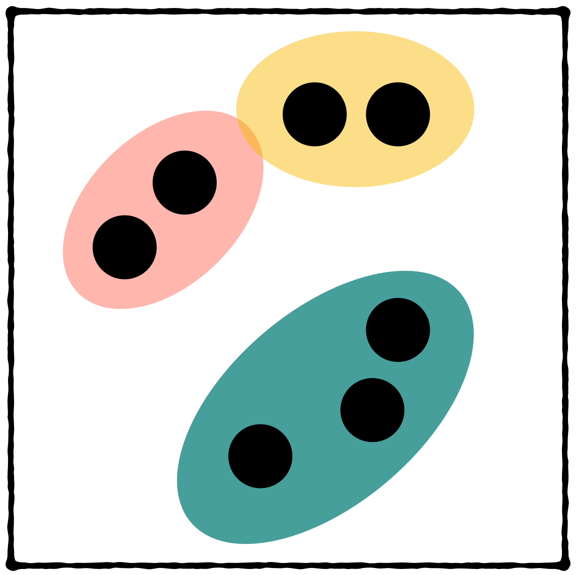

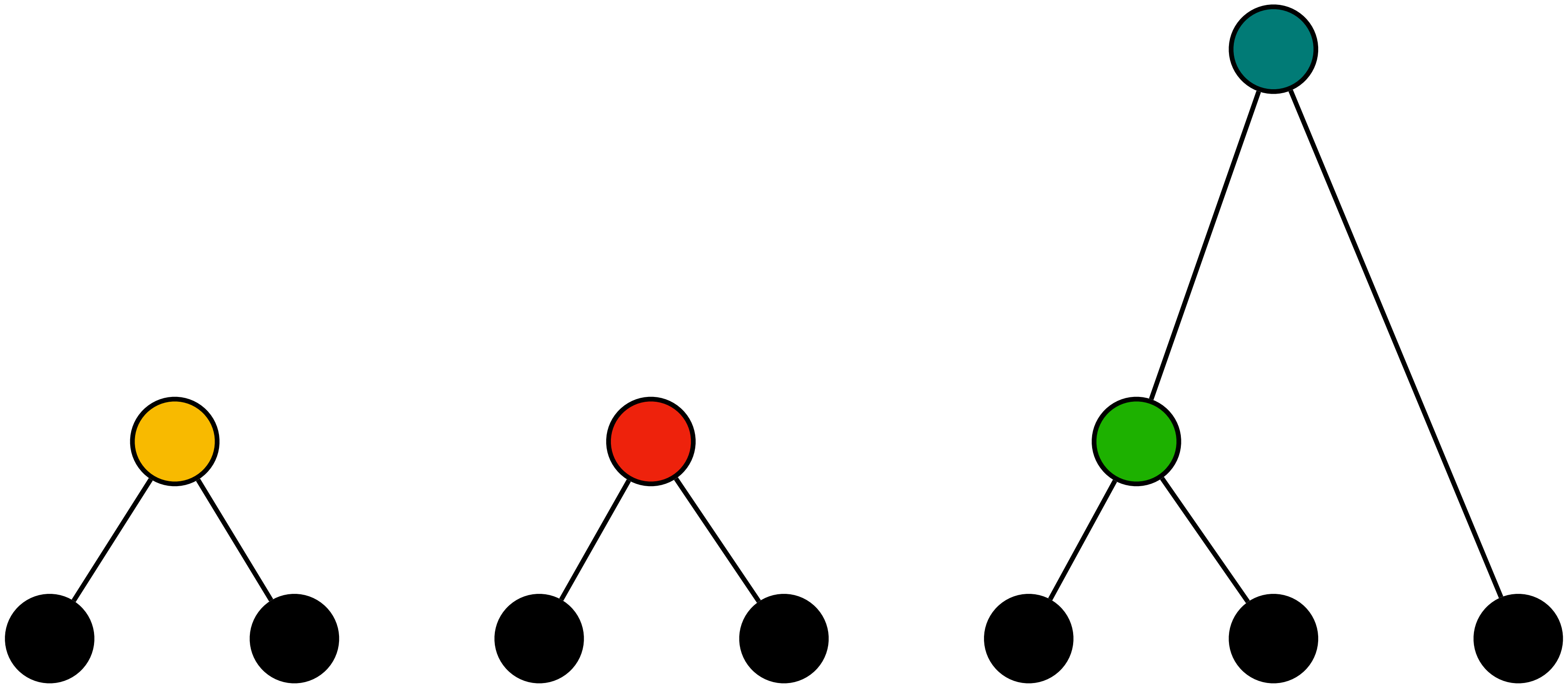

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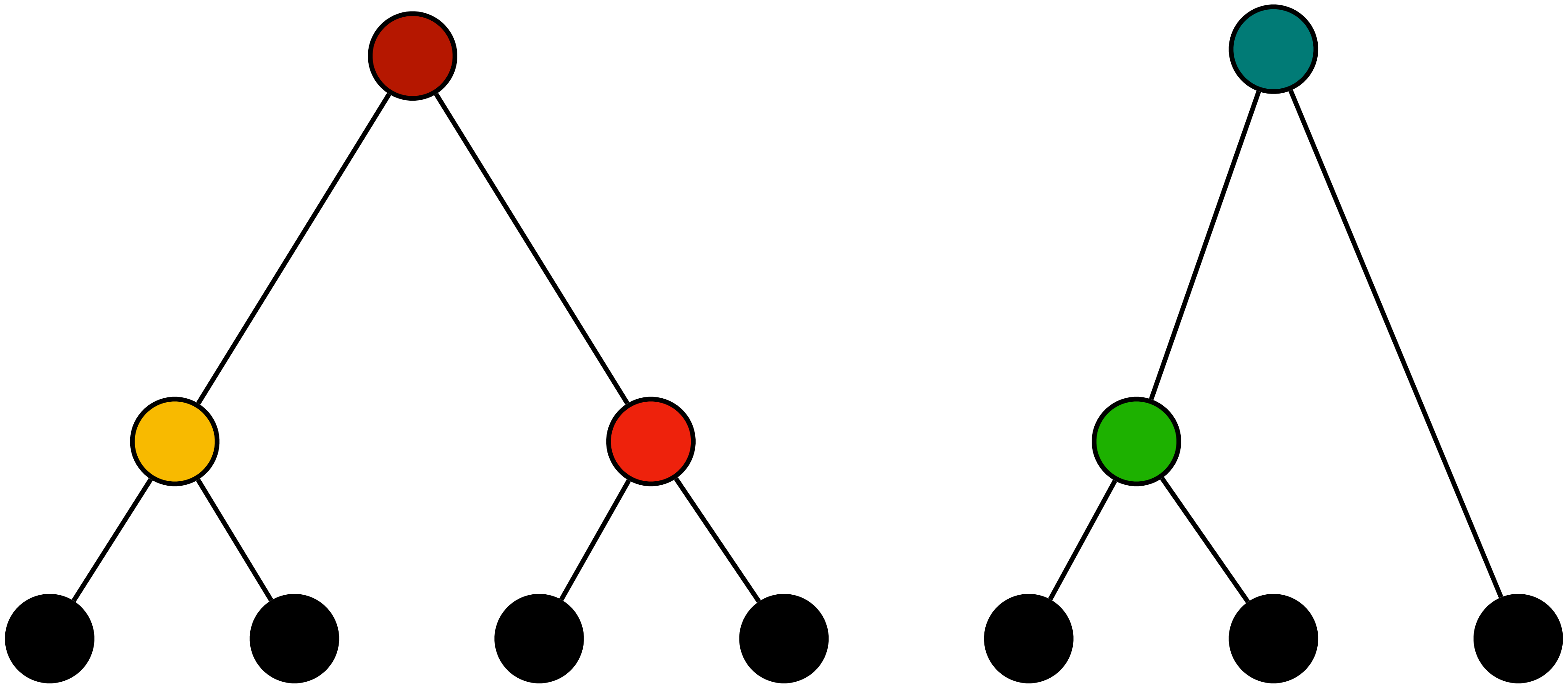

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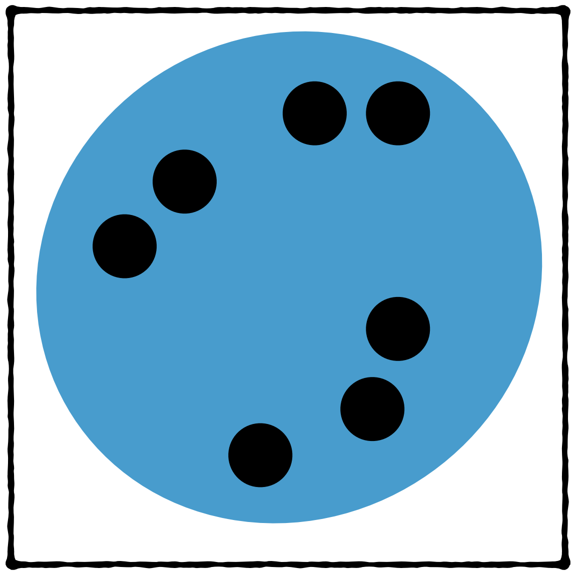

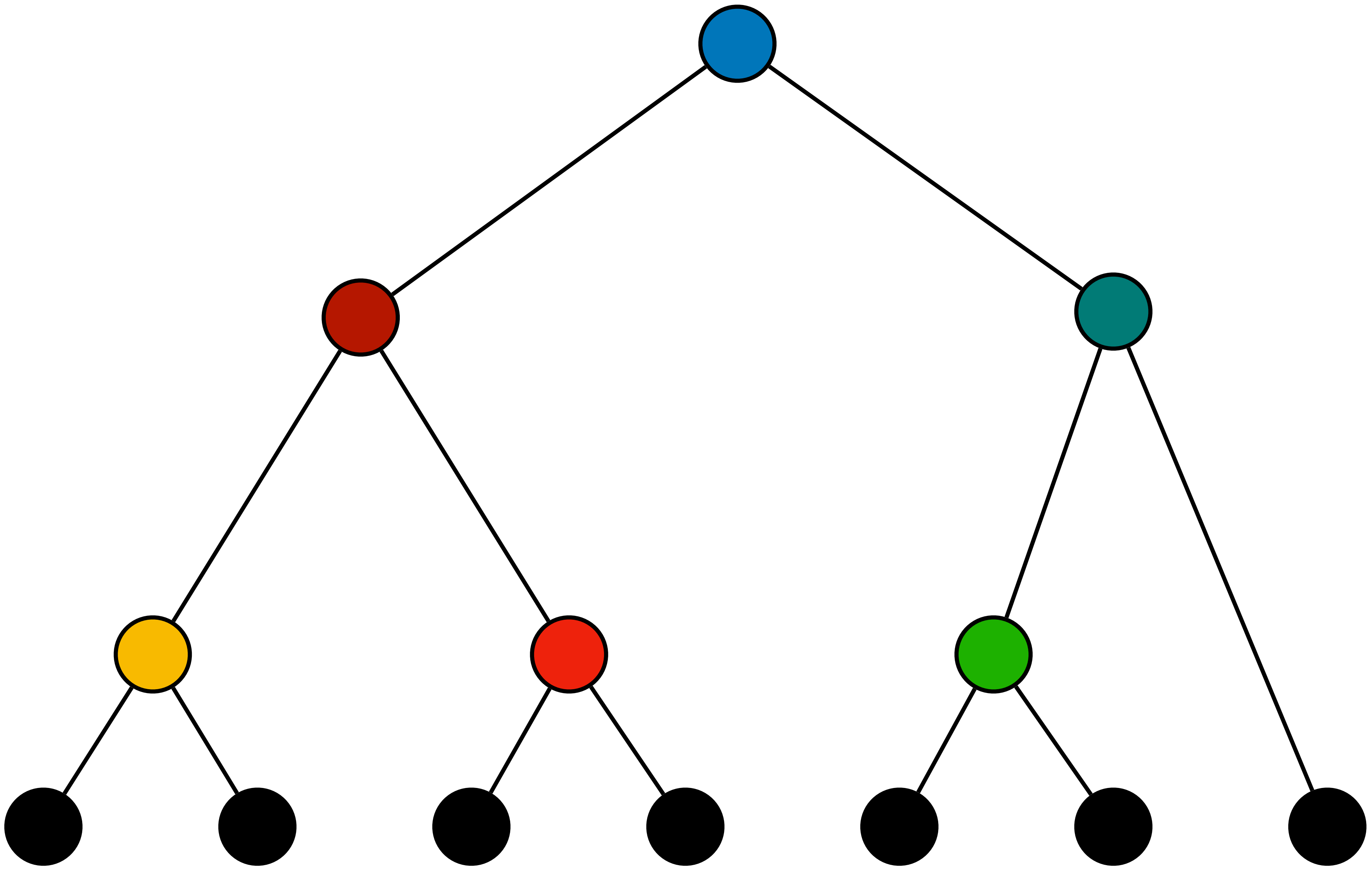

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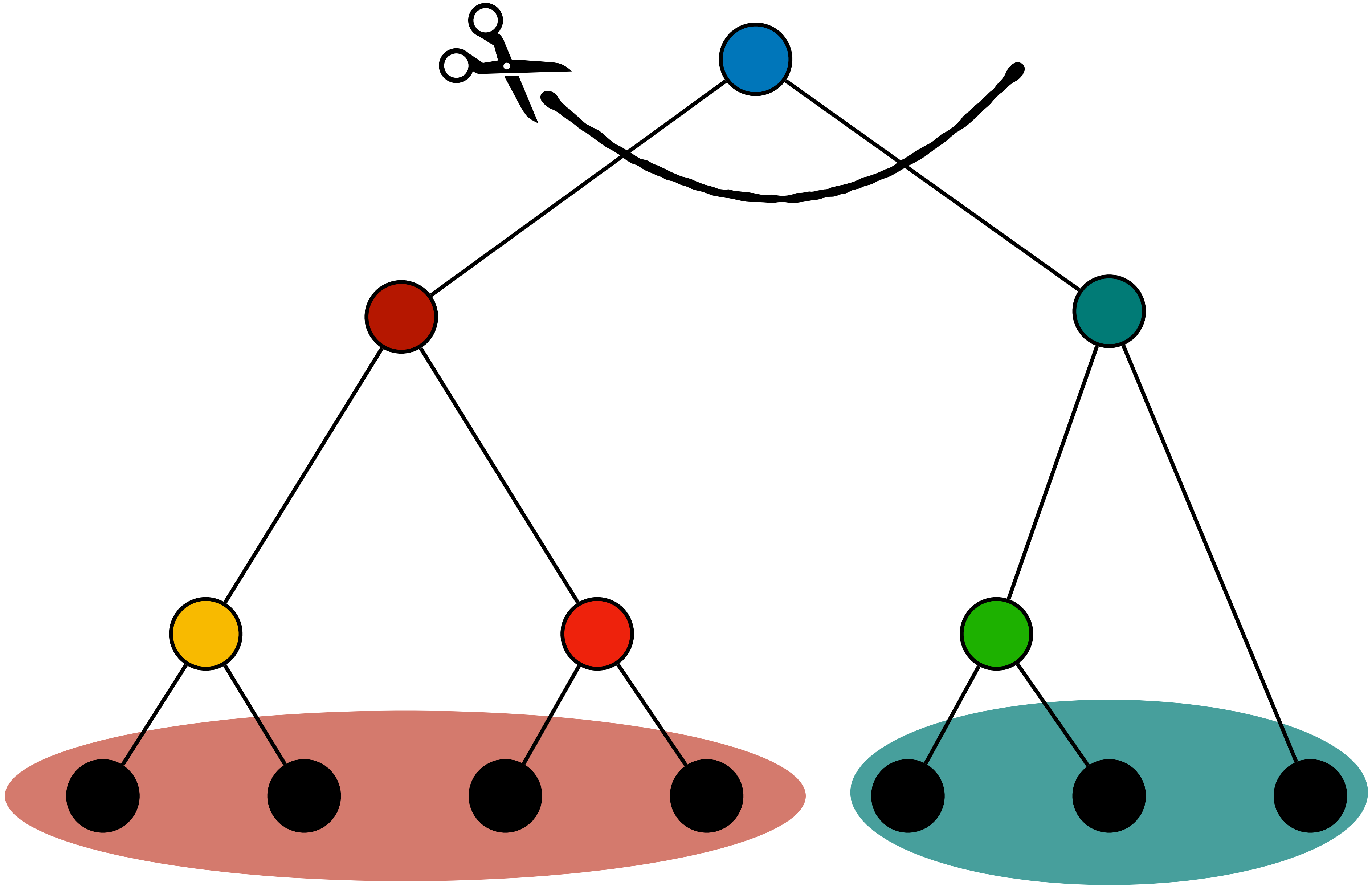

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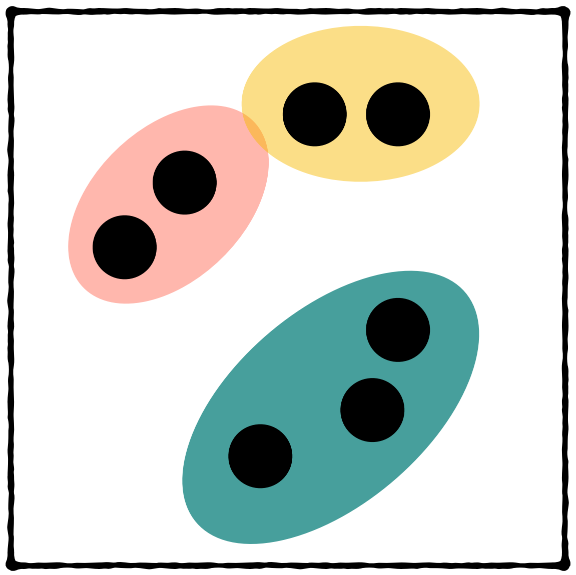
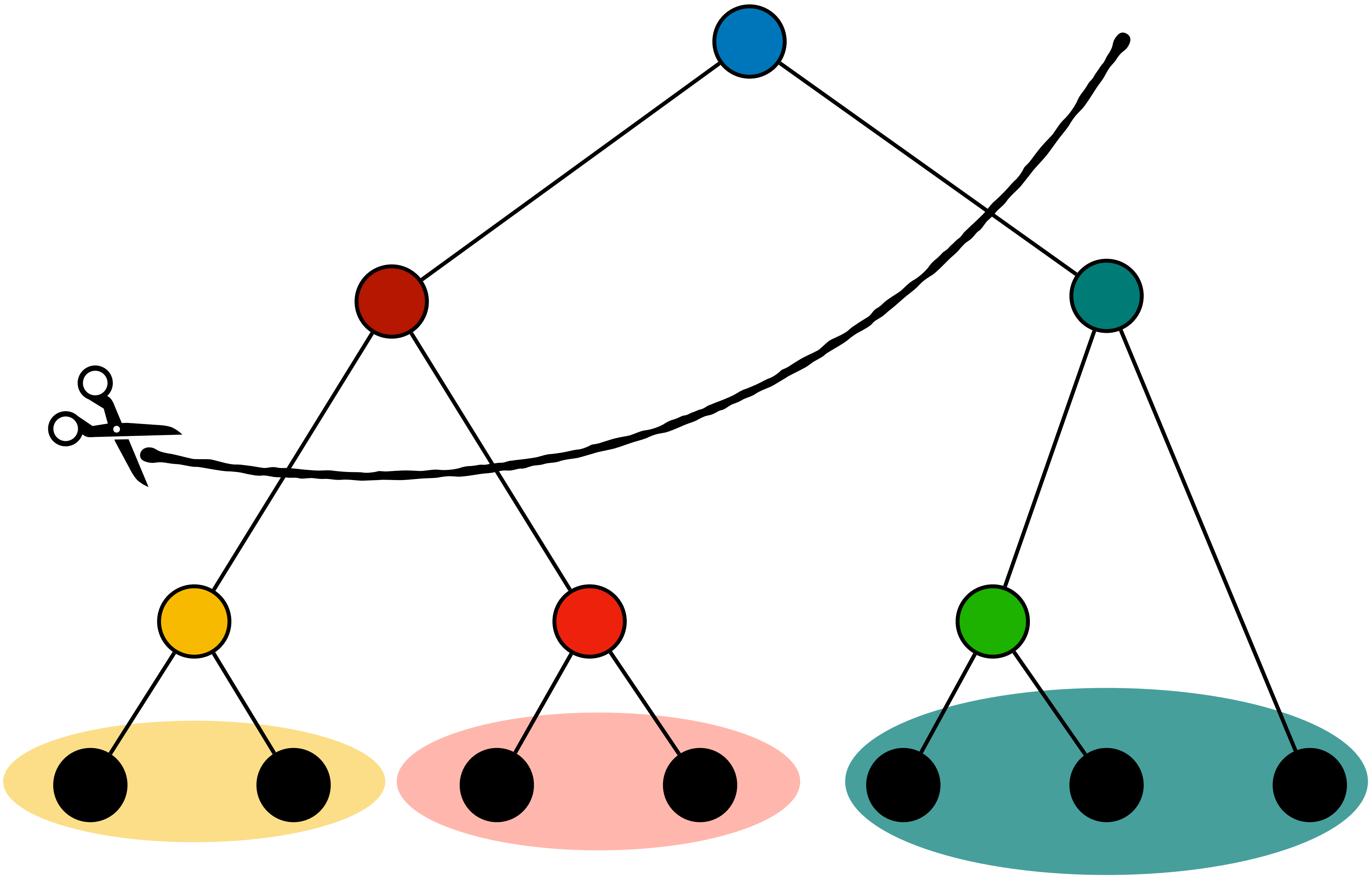
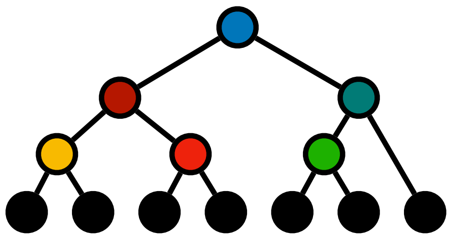
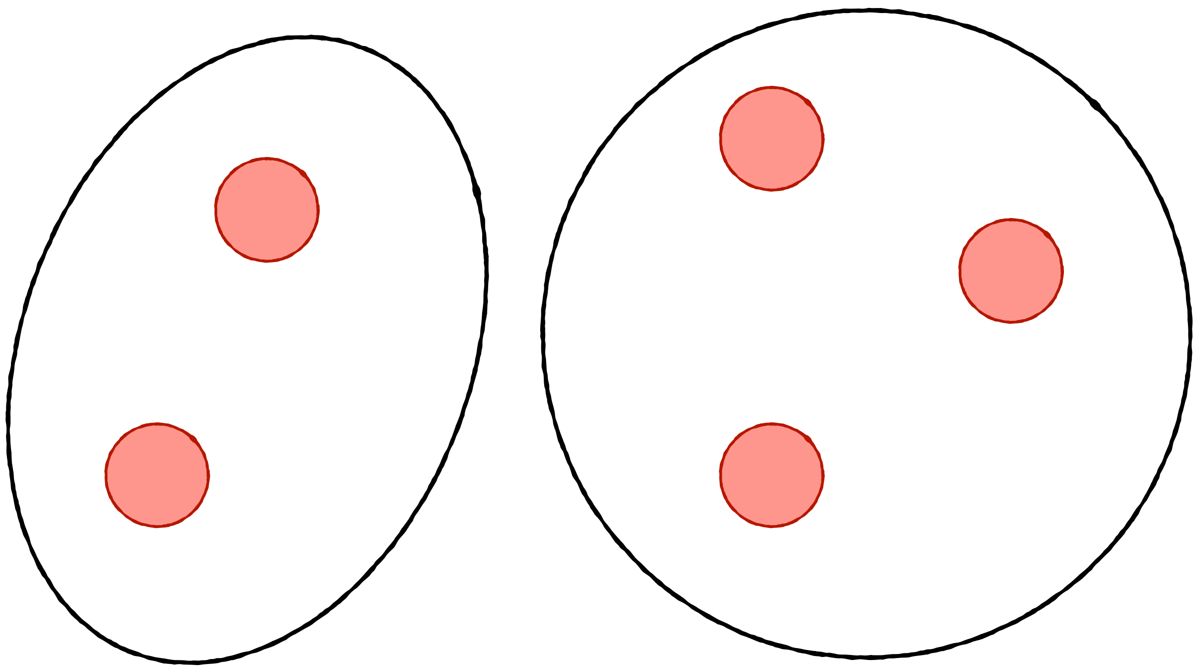




Iteratively merge two ***"closest"*** clusters

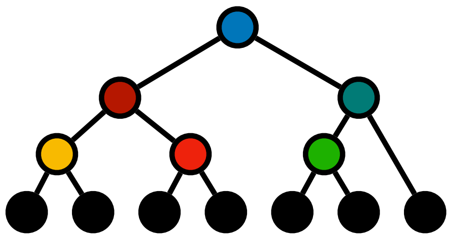
Linkage Function

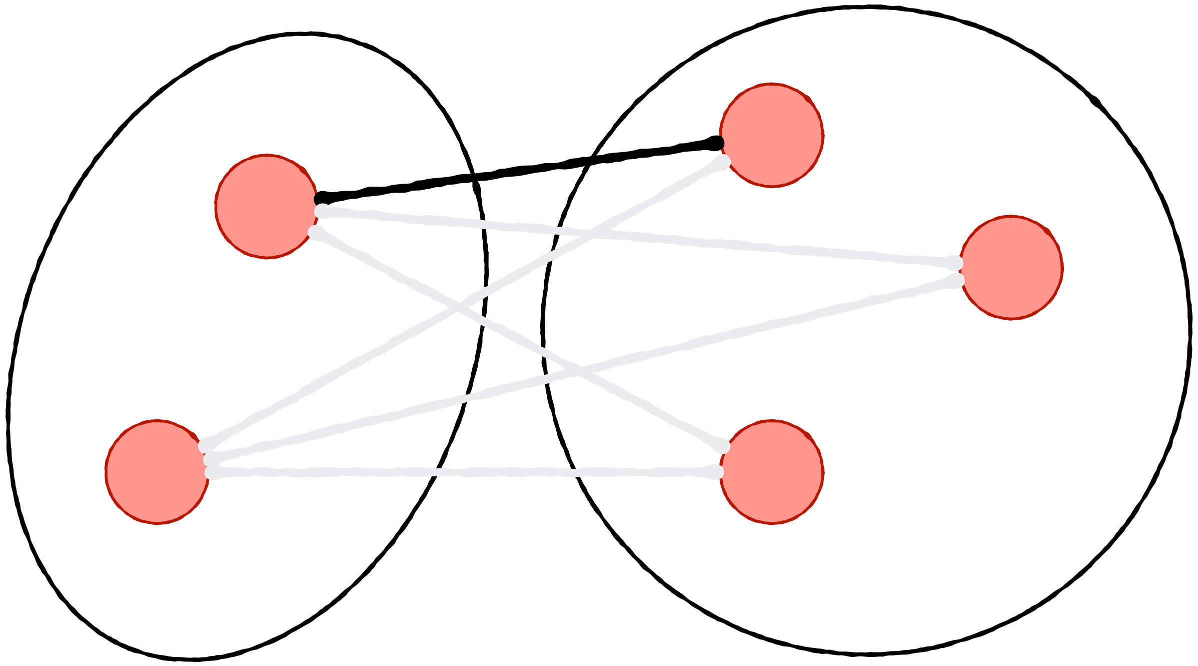




Inter-Cluster distance given by **Linkage Function**

Linkage Function





Single Linkage
(Minimum Pairwise Dissimilarity)

Inter-Cluster distance given by **Linkage Function**

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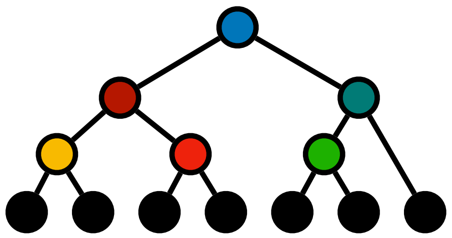
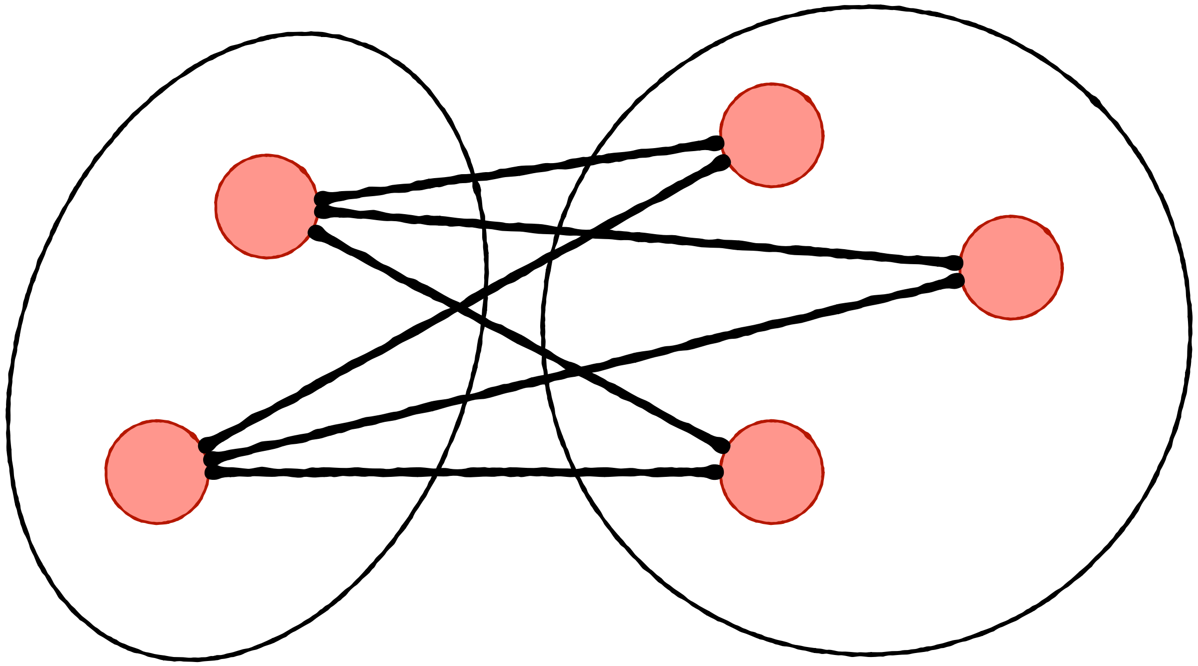
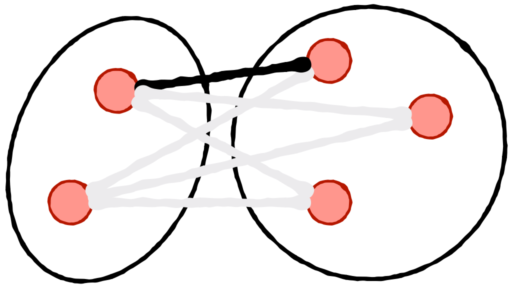




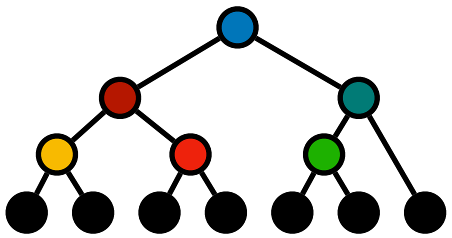
Average Linkage

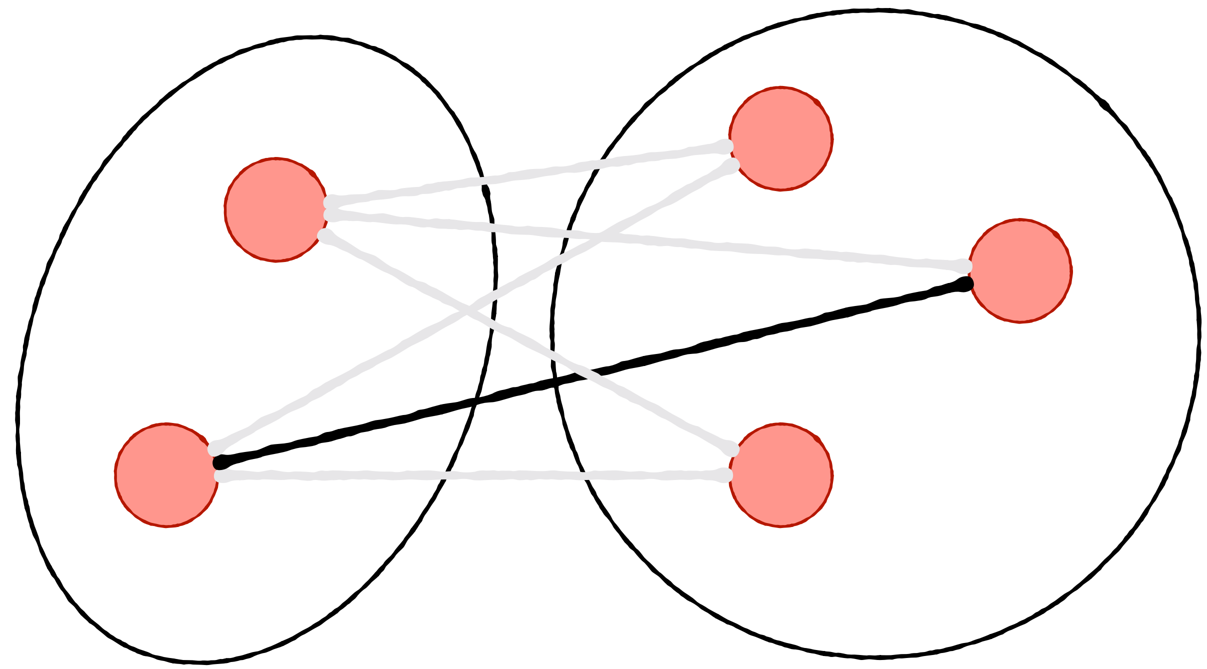
(Average Pairwise Dissimilarity)



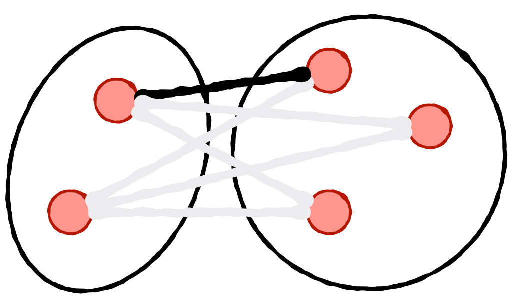
Single Linkage

Linkage Function





Complete Linkage
(Maximum Pairwise Dissimilarity)



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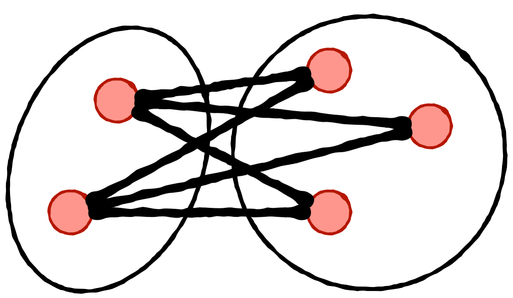
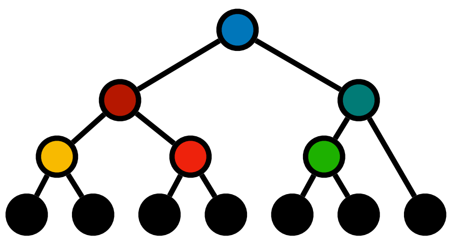
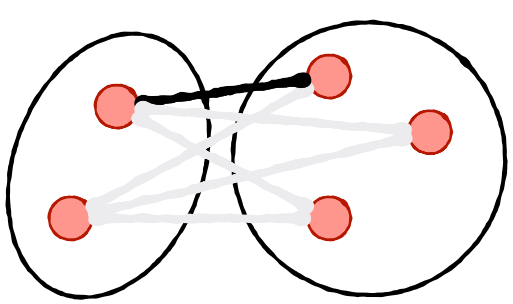


Average Linkage

Linkage Function



Best linkage function for a dataset is, *a priori*, unknown



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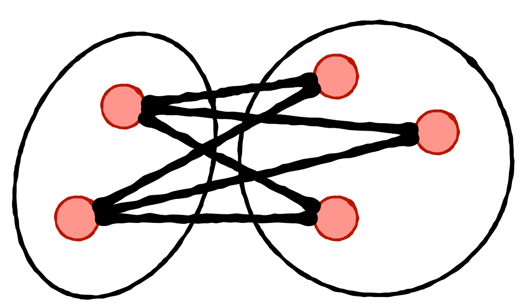
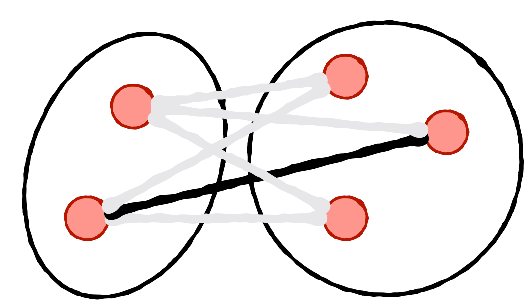


Average Linkage



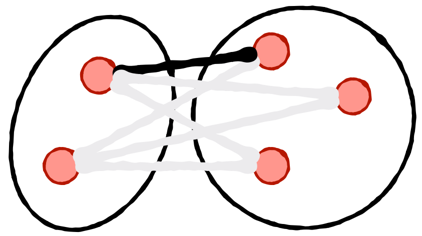
Complete Linkage

In this work:

1. **Exponential Linkage:** Learnable family of linkage functions

2. **Training objective** to jointly optimize linkage & dissimilarity function

Exponential Linkage



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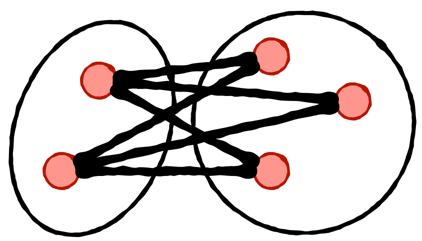

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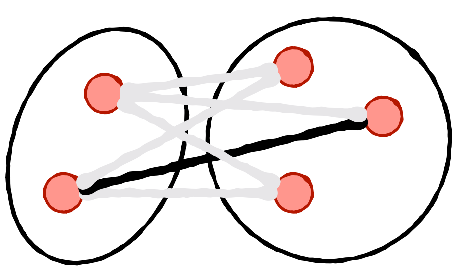

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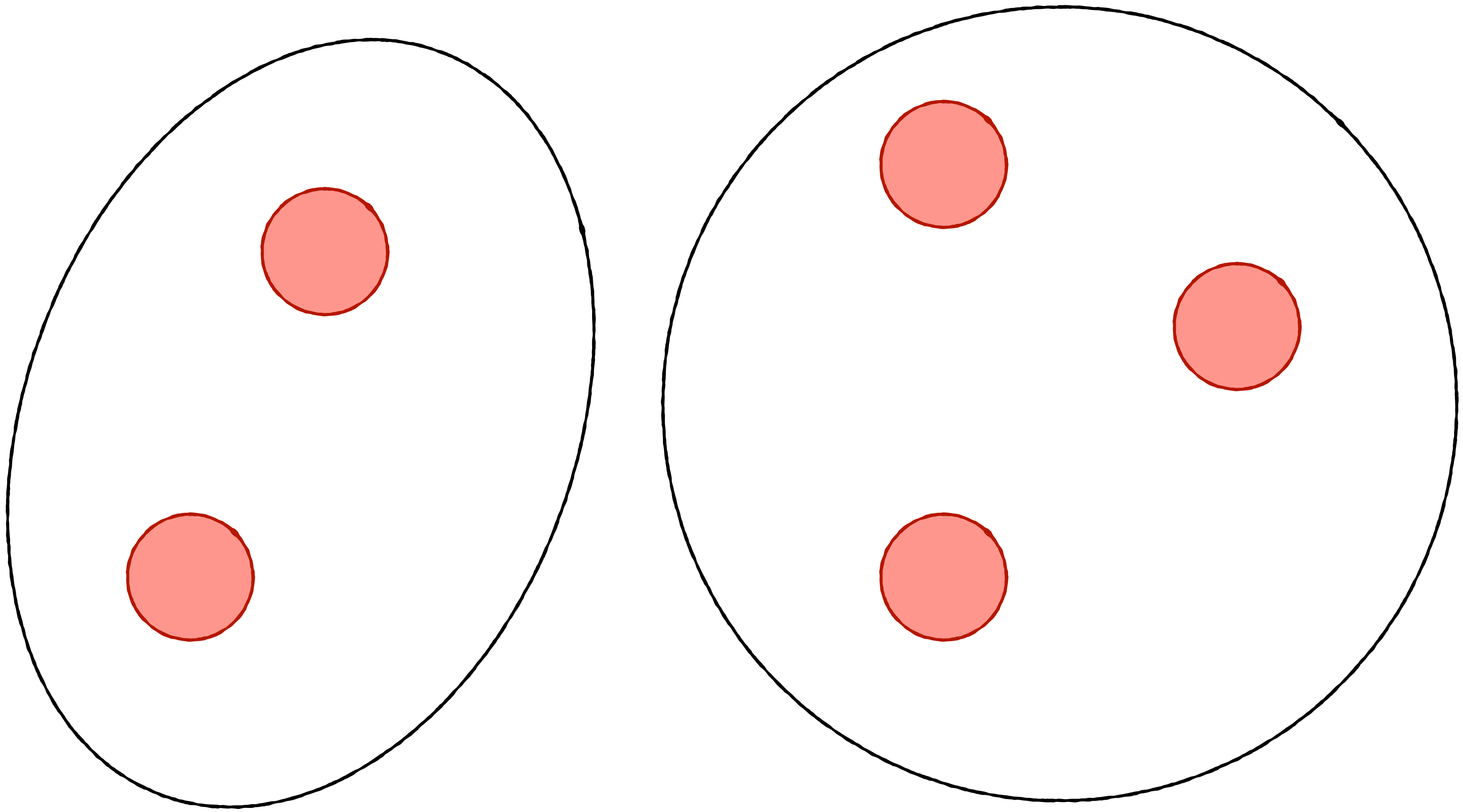

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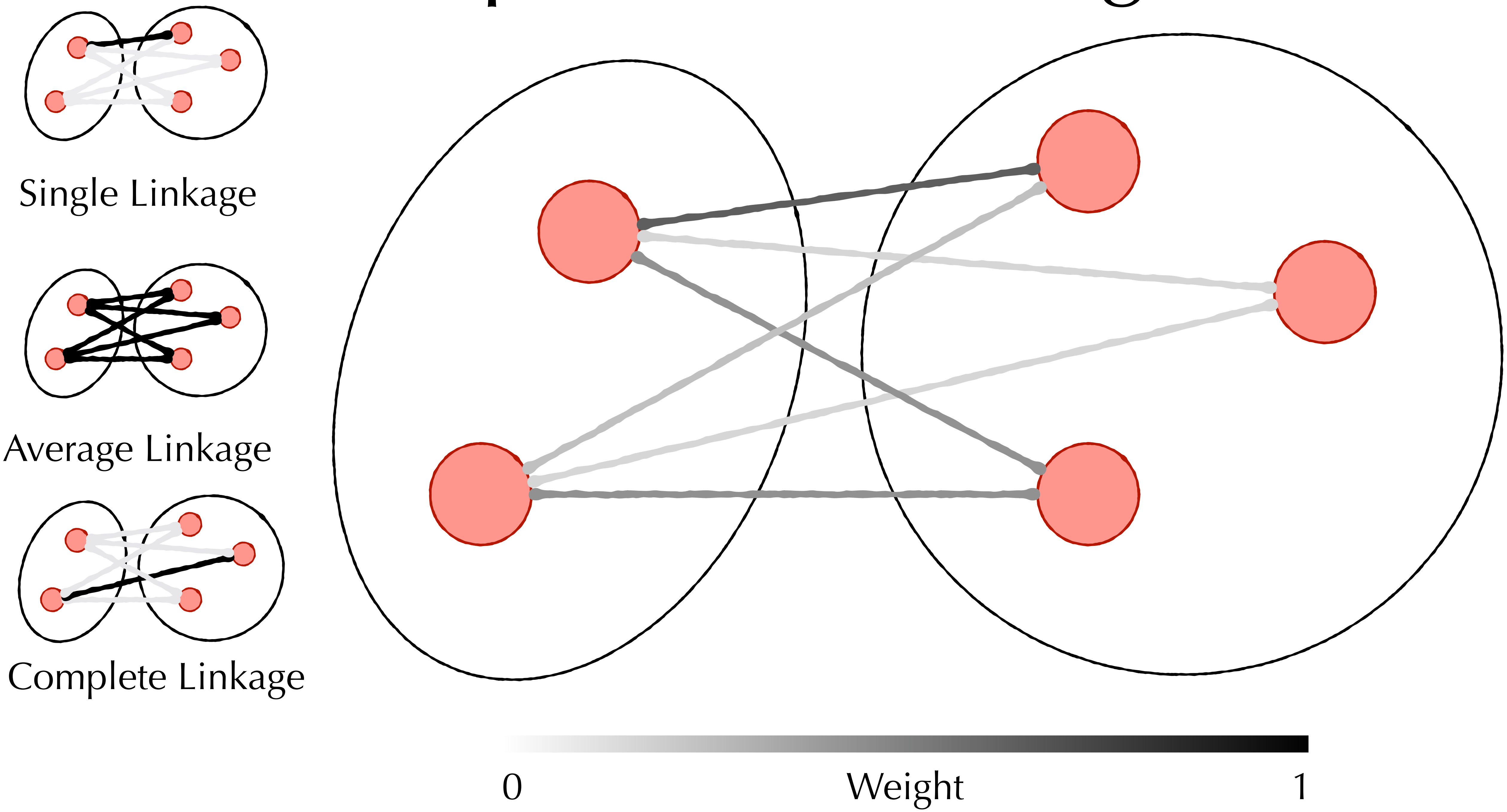
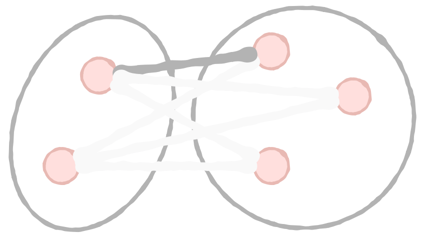


Weighted Average with **Learnable Parameter**

Exponential Linkage

$$J(\theta, \alpha) = \sum_{i=1}^{n'} \sum_{\mathbf{C}_{u,v} \in \mathcal{P}_-^{(i)}} \max\left\{0, \Psi\alpha(\mathbf{C}_{u',v'}) - \Psi\alpha(\mathbf{C}_{u,v})\right\}$$



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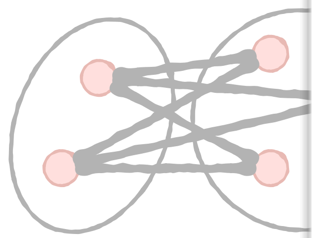
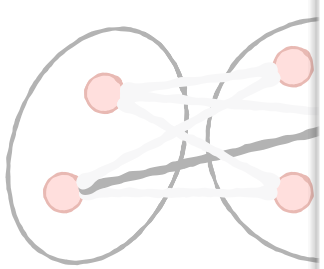


Average Link



Complete Li

Algorithm 1 `train_ExpLink`$(\mathcal{X}, \mathcal{C}^\star, T, \gamma_1, \gamma_2)$

Init: θ, α

for $t = 1, \ldots, T$ **do**

  $J \leftarrow 0$

  $\mathcal{T}_j^{(0)} \leftarrow \{x_j\} \quad \forall\, x_j \in \mathcal{X}$

  for round $i = 1, \ldots, n'$ **do**

    $\{\mathcal{T}_k^{(i)}\}_k^{l_i} \leftarrow \text{HAC-Round}(\{\mathcal{T}_k^{(i-1)}\}_k^{l_{i-1}})$

    $\{C^{(i)}\}_k^{l_i} \leftarrow \{\texttt{lvs}\mathcal{T}_k^{(i)}\}_k^{l_i}$

    $\mathcal{C}^{(i)} \leftarrow \{C^{(i)}\}_k^{l_i}$

    $\mathcal{P}^{(i)} \leftarrow \{\mathbf{C}_{u,v} \in \mathcal{C}^{(i)} \times \mathcal{C}^{(i)} : C_u \neq C_v\}$

    $\mathcal{P}_+^{(i)} \leftarrow \{\mathbf{C}_{u,v} \in \mathcal{P}^{(i)} : \exists C_j^\star \text{ s.t. } C_u, C_v \subset C_j^\star\}$

    $\mathcal{P}_-^{(i)} \leftarrow \mathcal{P}^{(i)} \setminus \mathcal{P}_+^{(i)}$

    $\mathbf{C}_{u',v'} \leftarrow \arg\min_{\mathbf{C}_{u,v} \in \mathcal{P}_+^{(i)}} \Psi^\alpha(\mathbf{C}_{u,v})$

    for $\mathbf{C}_{u,v} \in \mathcal{P}_-^{(i)}$ **do**

      $J \leftarrow J + \max\left\{0, \Psi^\alpha(\mathbf{C}_{u',v'}) - \Psi^\alpha(\mathbf{C}_{u,v})\right\}$

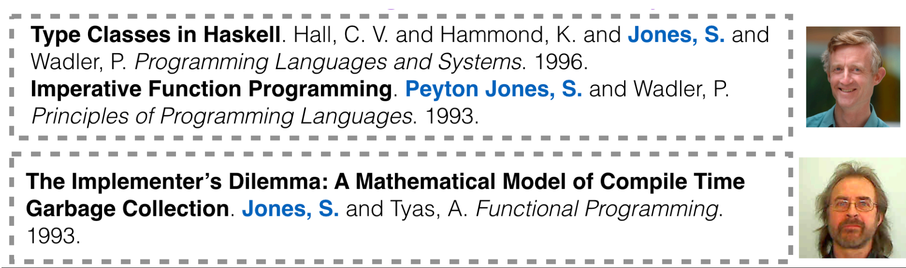
  $\theta \leftarrow \theta - \gamma_1 \frac{\partial J}{\partial \theta}$

  $\alpha \leftarrow \alpha - \gamma_2 \frac{\partial J}{\partial \alpha}$

Experimental Setup

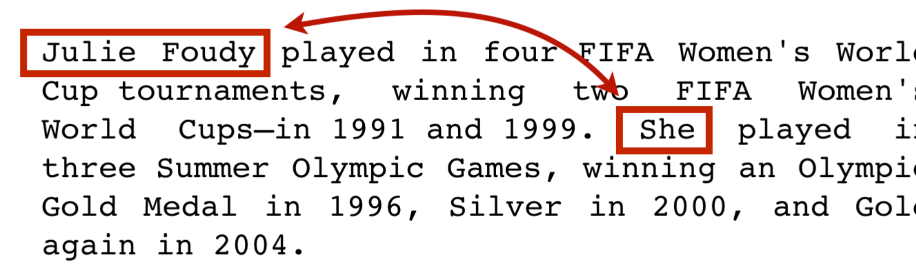
Entity Resolution

REXA **AMINER**



UMIST Faces

Noun Phrase Coreference



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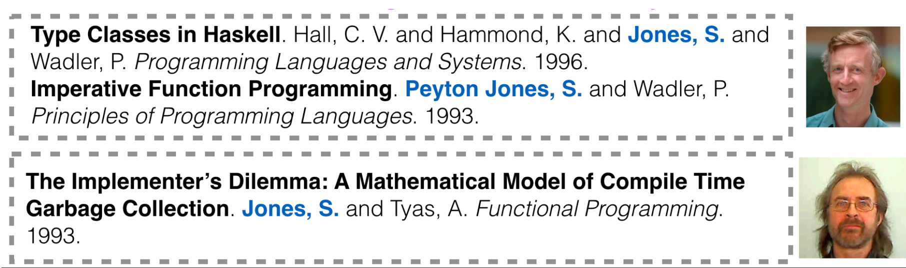
Type Classes in Haskell. Hall, C. V. and Hammond, K. and **Jones, S.** and Wadler, P. *Programming Languages and Systems.* 1996.
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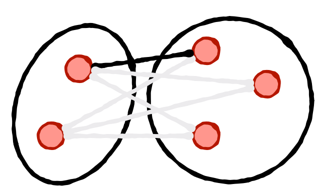
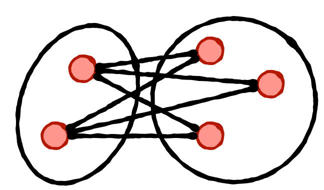
Noun Phrase Coreference

Julie Foudy played in four FIFA Women's World Cup tournaments, winning two FIFA Women's World Cups—in 1991 and 1999. She played in three Summer Olympic Games, winning an Olympic Gold Medal in 1996, Silver in 2000, and Gold again in 2004.

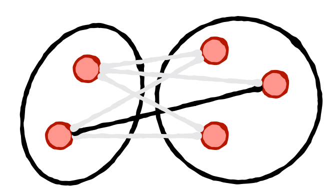
Four Linkage Functions



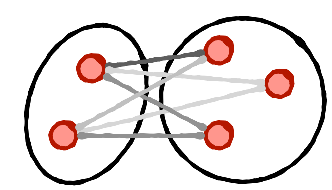
Single Linkage



Average Linkage



Complete Linkage

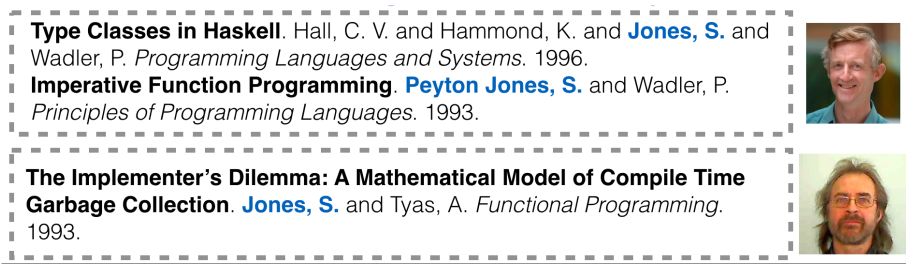


Exponential Linkage

Experimental Setup

Entity Resolution

REXA | AMINER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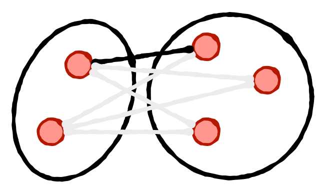
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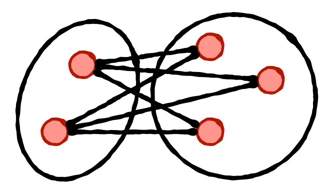
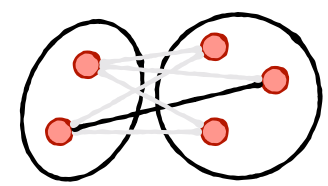
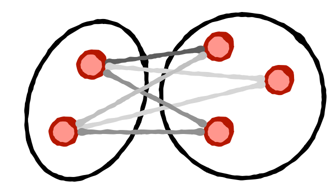

Average Linkage


Complete Linkage

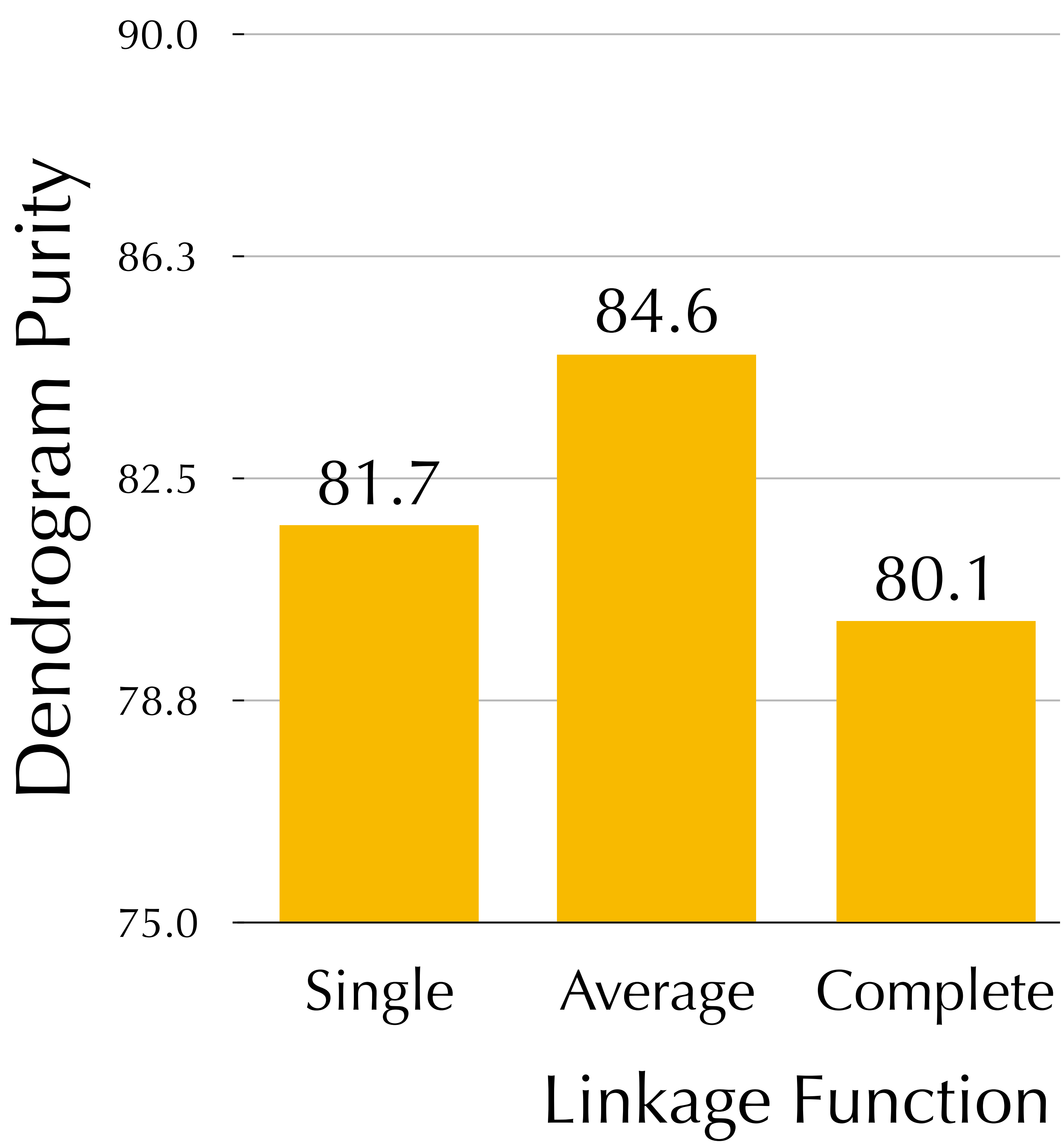

Exponential Linkage

Evaluated using **Dendrogram Purity**

Averaged across 50 different train/test/dev splits

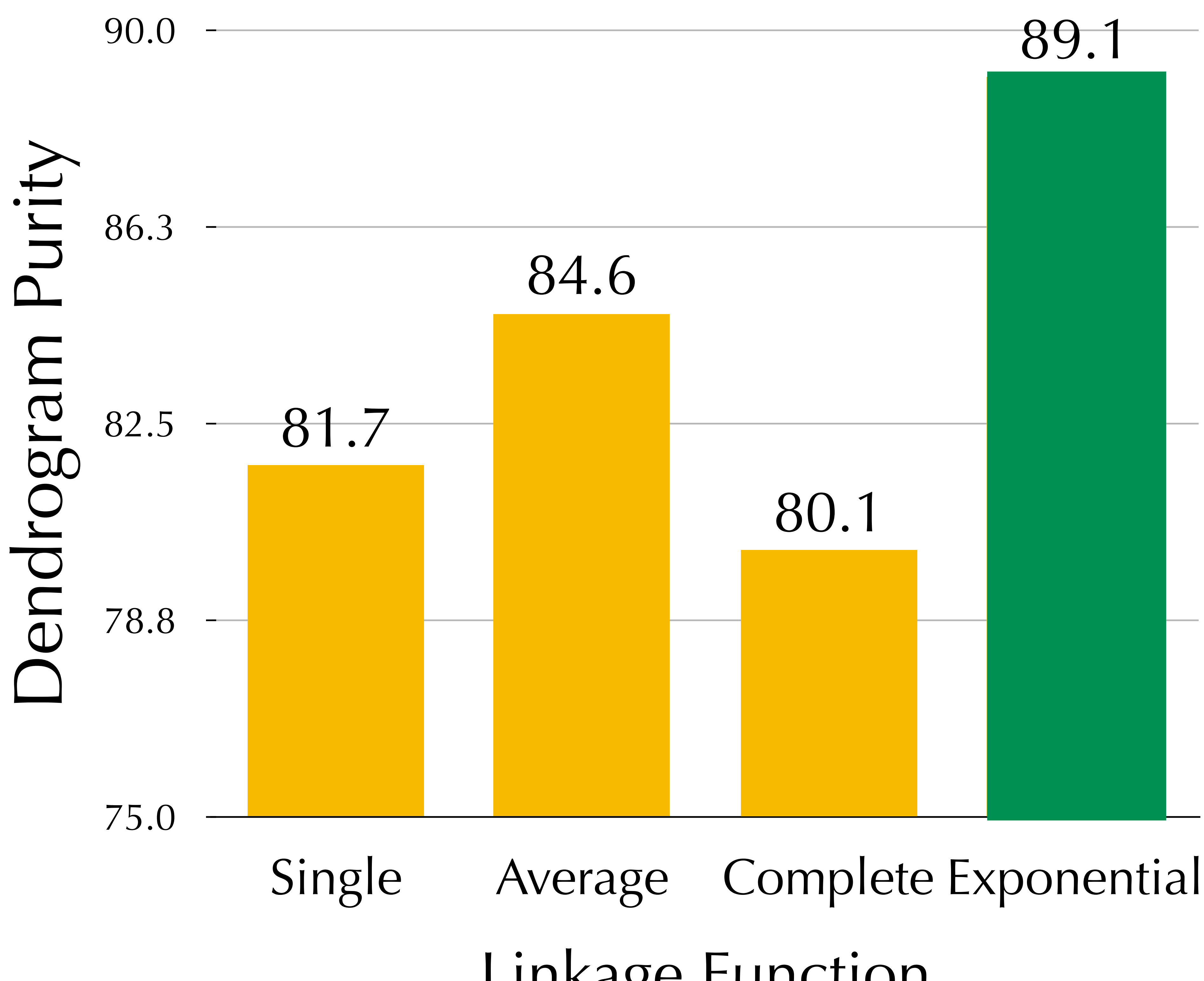
Results

Dataset: Rexa



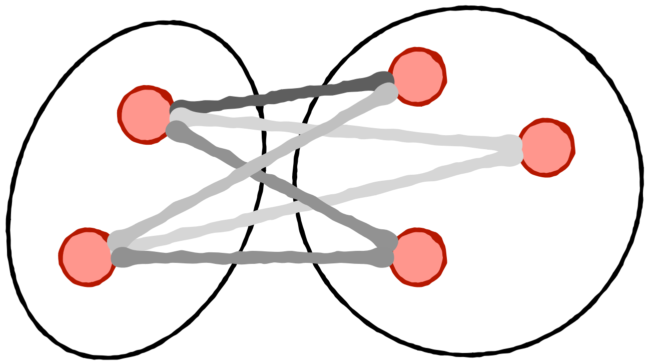
Results

Dataset: Rexa



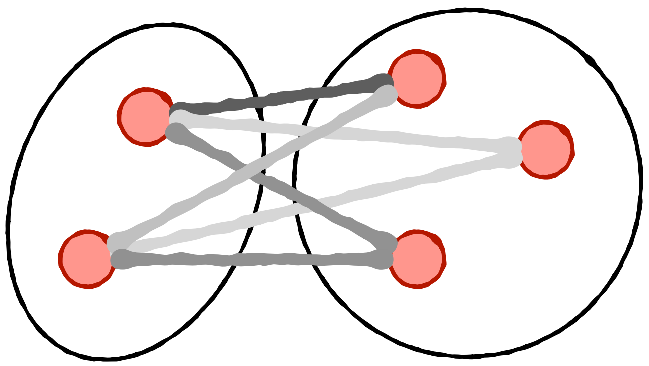
Summary

Summary



Exponential Linkage: Learnable family of linkage functions

Summary



Exponential Linkage: Learnable family of linkage functions

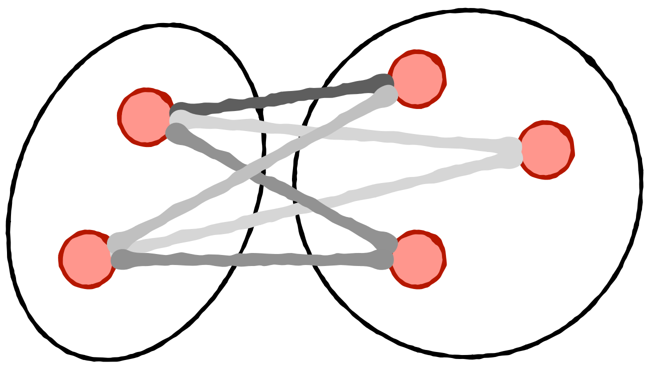
$$J(\theta, \alpha) = \sum_{i=1}^{n'} \sum_{\mathbf{C}_{u,v} \in \mathcal{P}_-^{(i)}} \max\left\{0, \Psi\alpha(\mathbf{C}_{u',v'}) - \Psi\alpha(\mathbf{C}_{u,v})\right\}$$

Algorithm 1 `train_ExpLink`$(\mathcal{X}, \mathcal{C}^\star, T, \gamma_1, \gamma_2)$

Init: θ, α
for $t = 1, \ldots, T$ **do**
  $J \leftarrow 0$
  $\mathcal{T}_j^{(0)} \leftarrow \{x_j\} \quad \forall\, x_j \in \mathcal{X}$
  for round $i = 1, \ldots, n'$ **do**
    $\{\mathcal{T}_k^{(i)}\}_k^{l_i} \leftarrow \text{HAC-Round}(\{\mathcal{T}_k^{(i-1)}\}_k^{l_{i-1}})$
    $\{C^{(i)}\}_k^{l_i} \leftarrow \{\texttt{lvs}\mathcal{T}_k^{(i)}\}_k^{l_i}$
    $\mathcal{C}^{(i)} \leftarrow \{C^{(i)}\}_k^{l_i}$
    $\mathcal{P}^{(i)} \leftarrow \{\mathbf{C}_{u,v} \in \mathcal{C}^{(i)} \times \mathcal{C}^{(i)} : C_u \neq C_v\}$
    $\mathcal{P}_+^{(i)} \leftarrow \{\mathbf{C}_{u,v} \in \mathcal{P}^{(i)} : \exists C_j^\star \text{ s.t. } C_u, C_v \subset C_j^\star\}$
    $\mathcal{P}_-^{(i)} \leftarrow \mathcal{P}^{(i)} \setminus \mathcal{P}_+^{(i)}$
    $\mathbf{C}_{u',v'} \leftarrow \arg\min_{\mathbf{C}_{u,v} \in \mathcal{P}_+^{(i)}} \Psi^\alpha(\mathbf{C}_{u,v})$
    for $\mathbf{C}_{u,v} \in \mathcal{P}_-^{(i)}$ **do**
      $J \leftarrow J + \max\left\{0, \Psi^\alpha(\mathbf{C}_{u',v'}) - \Psi^\alpha(\mathbf{C}_{u,v})\right\}$
  $\theta \leftarrow \theta - \gamma_1 \frac{\partial J}{\partial \theta}$
  $\alpha \leftarrow \alpha - \gamma_2 \frac{\partial J}{\partial \alpha}$

Training Objective & Algorithm: Jointly Optimizing Dissimilarity & Linkage Function

Summary



Exponential Linkage: Learnable family of linkage functions

$$J(\theta, \alpha) = \sum_{i=1}^{n'} \sum_{\mathbf{C}_{u,v} \in \mathcal{P}_{-}^{(i)}} \max\left\{0, \Psi\alpha(\mathbf{C}_{u',v'}) - \Psi\alpha(\mathbf{C}_{u,v})\right\}$$

Algorithm 1 `train_ExpLink`$(\mathcal{X}, \mathcal{C}^\star, T, \gamma_1, \gamma_2)$

Init: θ, α
for $t = 1, \ldots, T$ **do**
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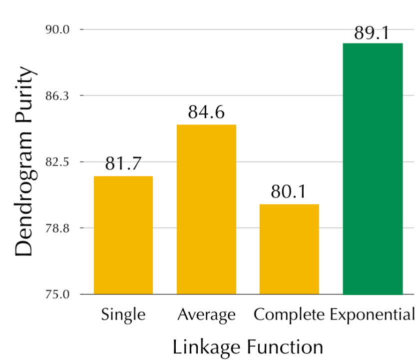
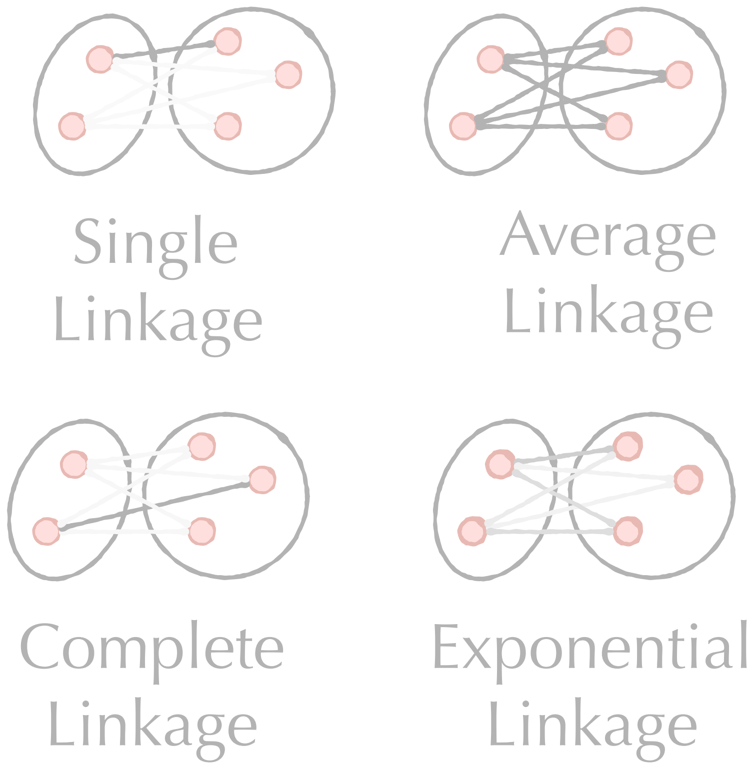
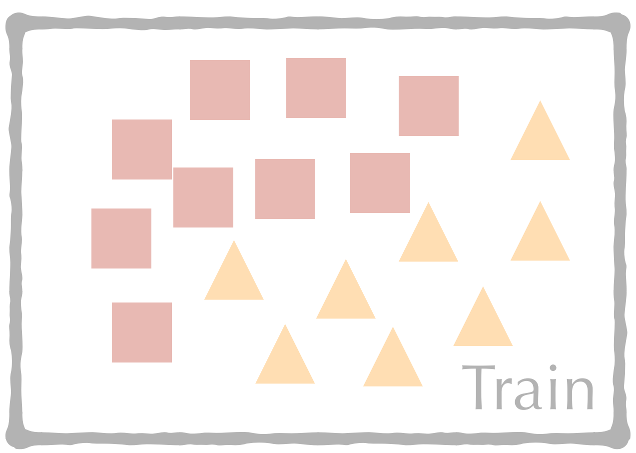
    $\mathcal{P}_{-}^{(i)} \leftarrow \mathcal{P}^{(i)} \setminus \mathcal{P}_{+}^{(i)}$
    $\mathbf{C}_{u',v'} \leftarrow \arg\min_{\mathbf{C}_{u,v} \in \mathcal{P}_{+}^{(i)}} \Psi^\alpha(\mathbf{C}_{u,v})$
    for $\mathbf{C}_{u,v} \in \mathcal{P}_{-}^{(i)}$ **do**
      $J \leftarrow J + \max\left\{0, \Psi^\alpha(\mathbf{C}_{u',v'}) - \Psi^\alpha(\mathbf{C}_{u,v})\right\}$
  $\theta \leftarrow \theta - \gamma_1 \frac{\partial J}{\partial \theta}$
  $\alpha \leftarrow \alpha - \gamma_2 \frac{\partial J}{\partial \alpha}$

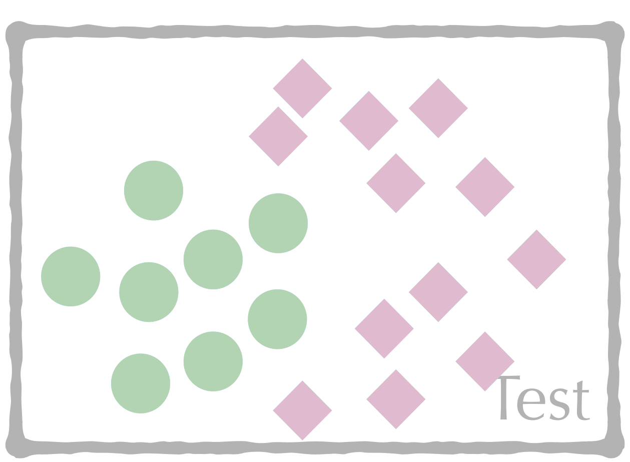
Training Objective & Algorithm: Jointly Optimizing Dissimilarity & Linkage Function



Effective Empirical Results







Thanks for listening!

Check out our poster **#196** today at 6:30pm in Pacific Ballroom!

Paper:

Scan me

Code:

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