Kernel Normalized Cut: a Theoretical Revisit

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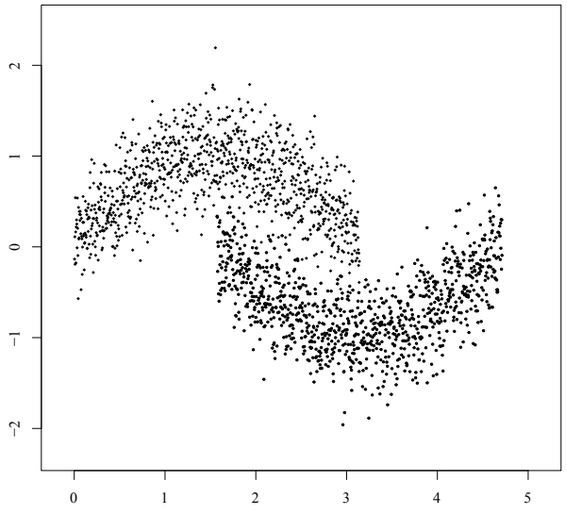

Unsupervised Learning (Room 103)

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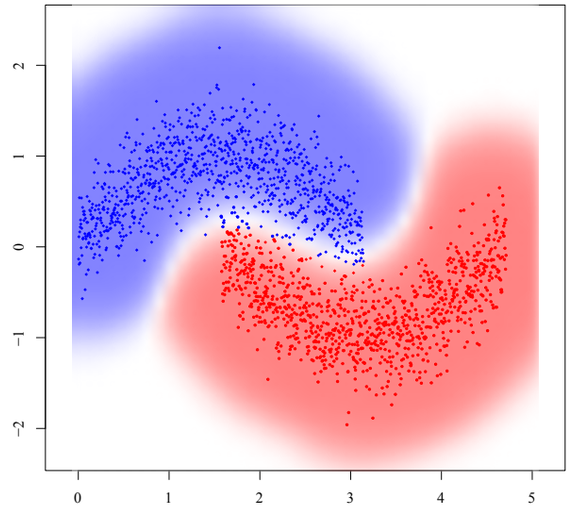
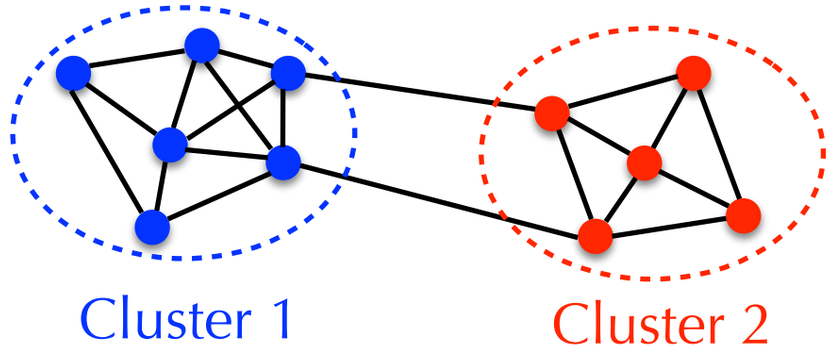
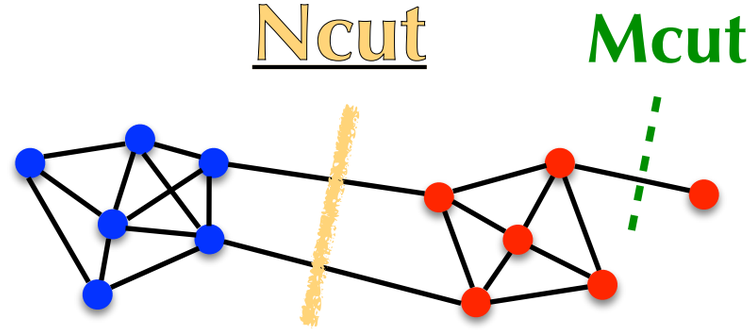


What is Normalized cut?

- **Normalized cut (Ncut;** Shi and Malik, 2000)
  - **Ncut** = Graph partitioning method
  - Goal = To find "clusters" in the graph:
    - ★ Many edges inside the cluster
    - ★ Fewer edges between different clusters



  - **Ncut = Balanced cut**
    - ★ **Each cluster is "reasonably large"!**
    - ★ Cut between different clusters is small.



  - **Objective function of Ncut** (Number of clusters = 2)
    - $K := (k_{ij})_{n \times n}$: Similarity matrix, $d_i := \sum_{i=1}^{n} k_{ij}$, $\mathrm{vol}(A) := \sum_{i \in A} d_i$,
    - Min cut: $\mathrm{Mcut}(A, B) := \sum_{i \in A} \sum_{j \in B} k_{ij}$

$$\mathrm{Ncut}(A, B) = \mathrm{Mcut}(A, B) \left\{ \frac{1}{\mathrm{vol}(A)} + \frac{1}{\mathrm{vol}(B)} \right\} \leftarrow \textbf{Balancing term!}$$

Normalized cut and its related methods

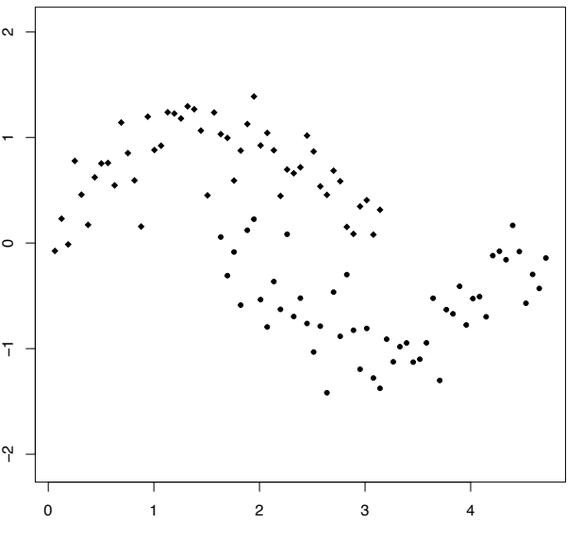
- **Normalized cut, Spectral clustering, Weighted kernel *k*-means**
  - Ncut is an **NP hard problem** $\Rightarrow$ **Normalized Spectral clustering (SC)** **= Continuous relaxation of Ncut**
  - **Ncut and Weighted Kernel *K*-Means (WKKM)** (Dhillon et al., 2007)
    - WKKM with kernel h and weight w_i : $H = (h_{ij})_{n\times n},\ W = \mathrm{diag}(w_1, \ldots, w_n)$

$$\sum_{i=1}^{n} w_i \min_m \|\psi_h(X_i) - \mu_m\|^2_{\mathcal{H}_h} = Const. - \mathrm{tr}(\tilde{U}^T W^{1/2} H W^{1/2} \tilde{U})$$

    - **Ncut = WKKM** with $H = D^{-1}KD^{-1}$ and $W = D$ $(D = \mathrm{diag}(d_1, \ldots, d_n))$

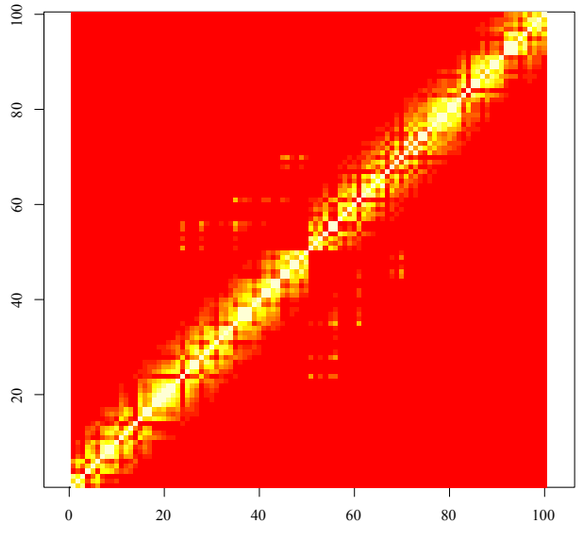
$\to 0$ as $n \to \infty$

- **Setting**



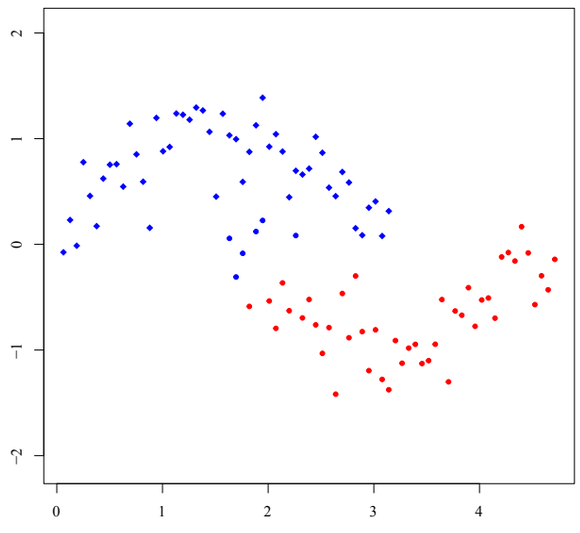
Data points

Kernel function



Similarity matrix

Graph cut

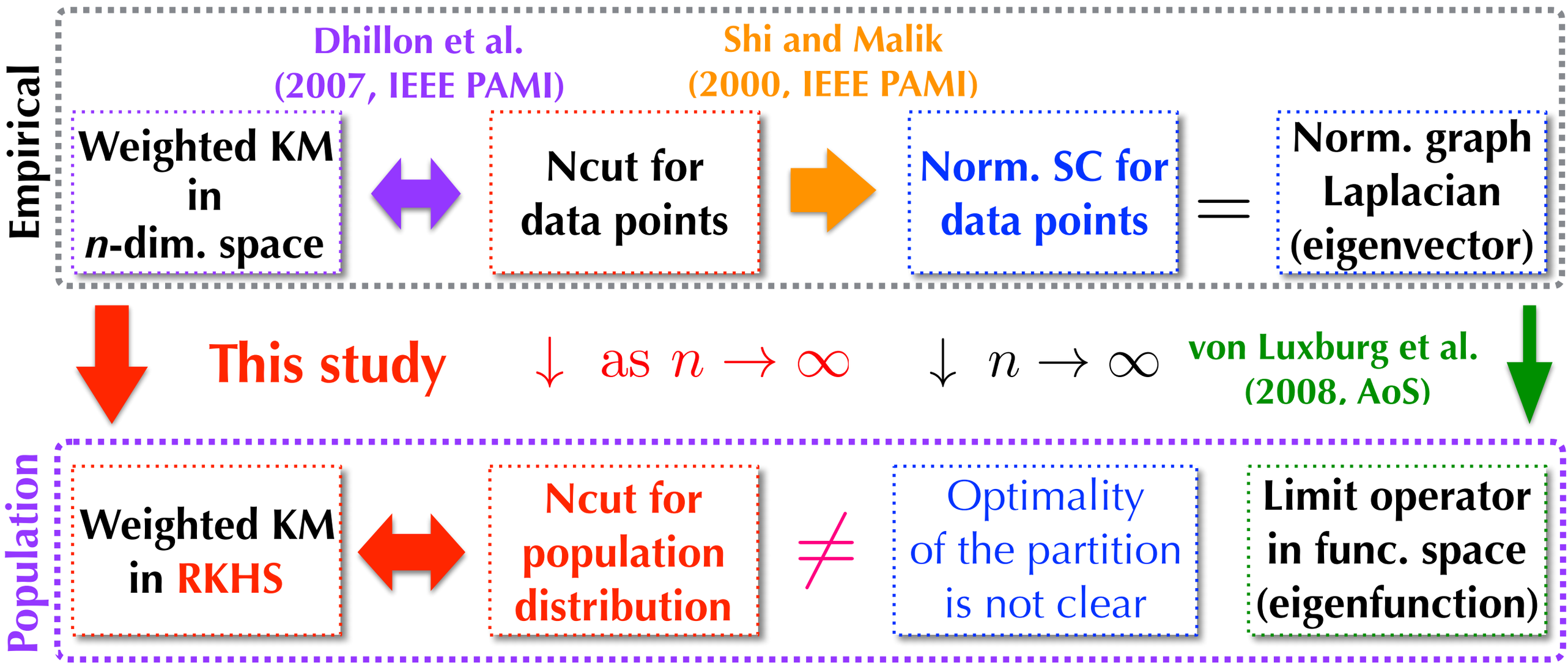


Clustering result!

Theoretical properties of Ncut

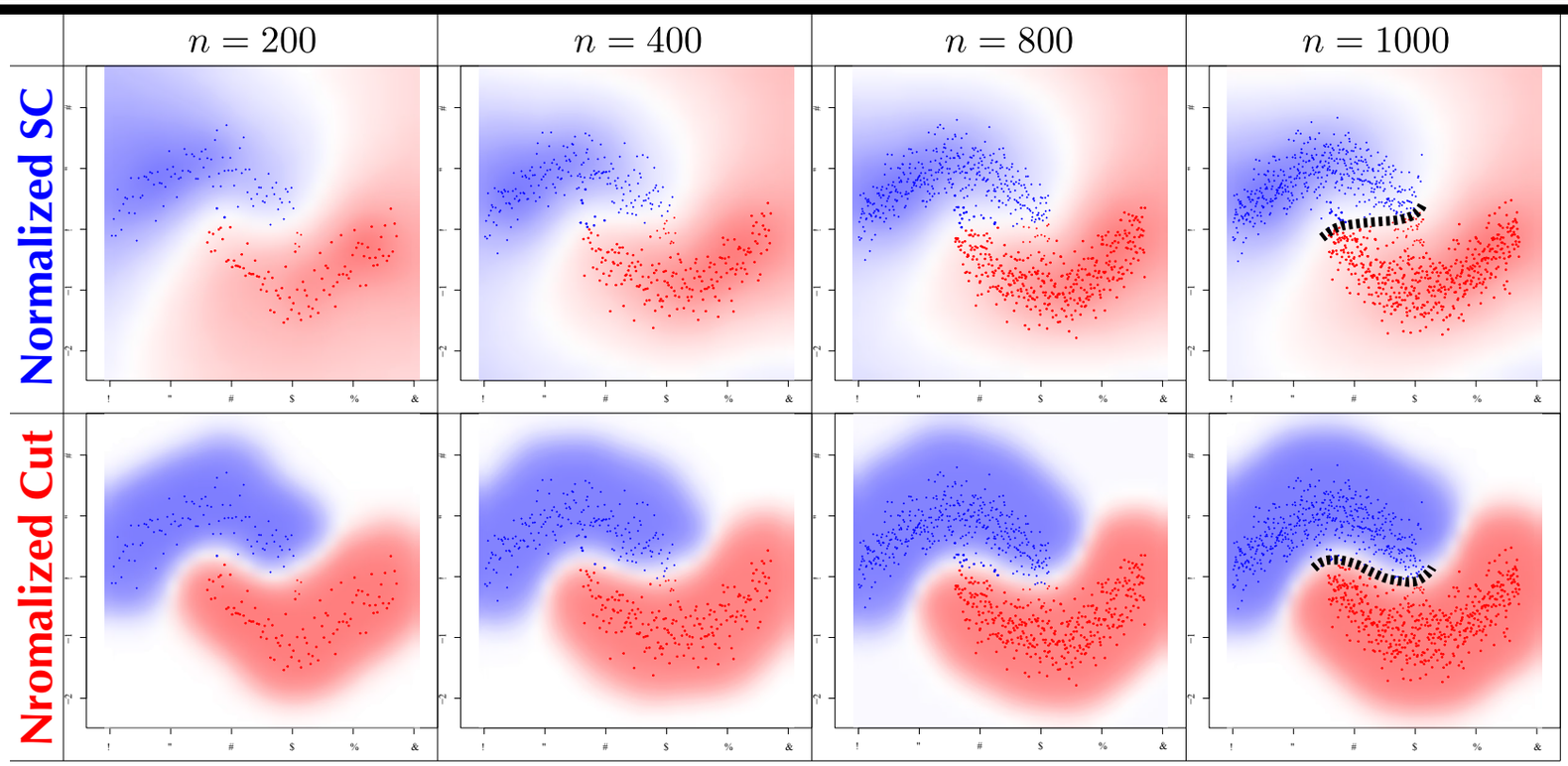
Overview of this study

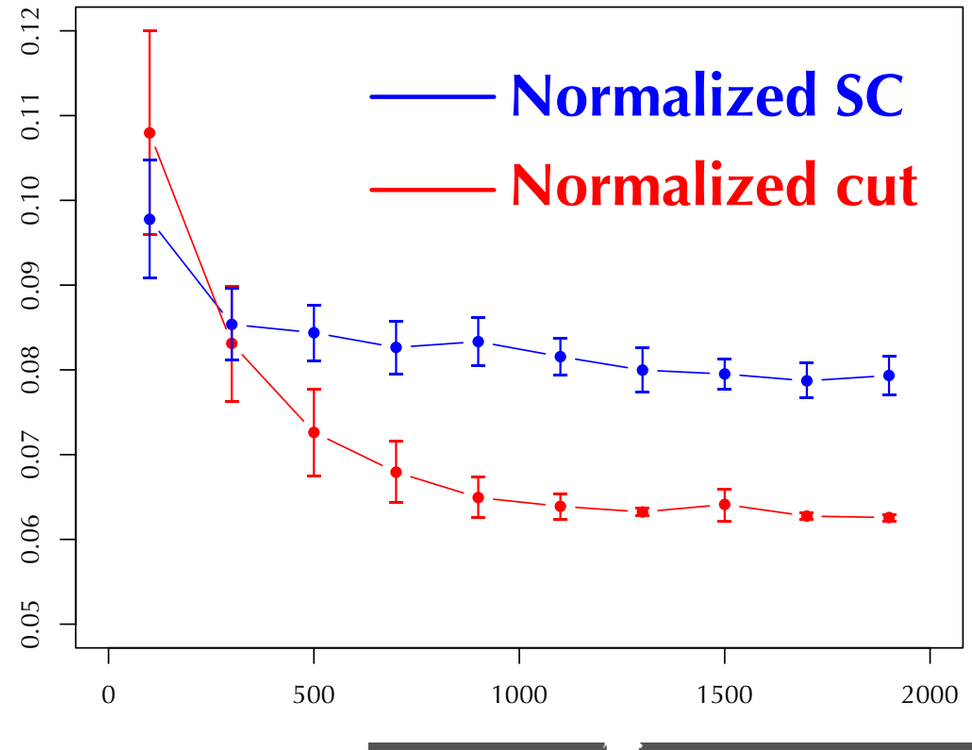
- We study theoretical properties of clustering based on Ncut!



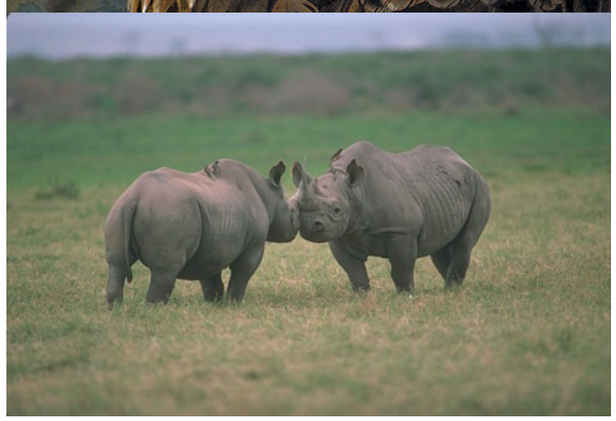
- We also derive the **fast rate of convergence of the normalized cut!**

Numerical experiments





Note that we used the same tuning parameter in both Ncut and SC!



Spectral clustering

Normalized cut