

Conditioning by adaptive sampling for robust design

David Brookes

Biophysics Graduate Group

University California, Berkeley

Jennifer Listgarten

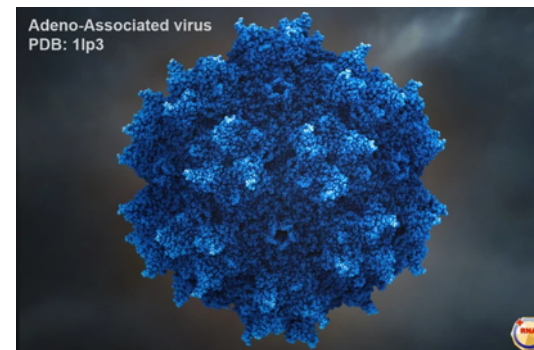
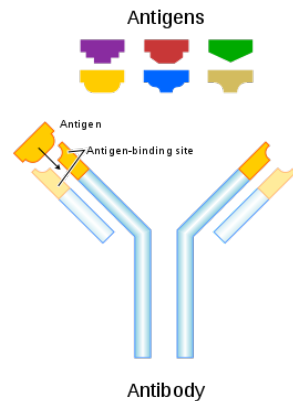
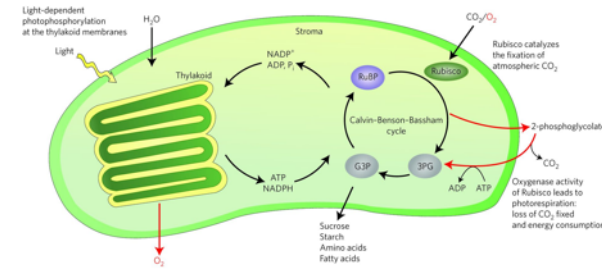
EECS and Center for Computational Biology

University California, Berkeley



Motivating problem: design protein sequences

- Proteins are made up of sequences of amino acids (20 possibilities)
- Huge variety of proteins whose function we would like to improve

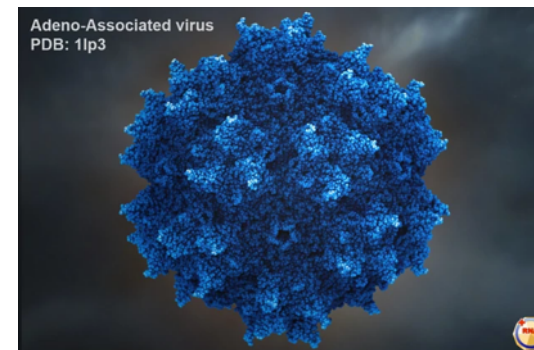
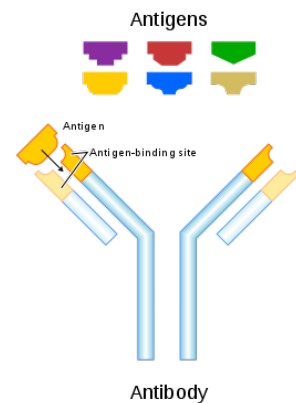
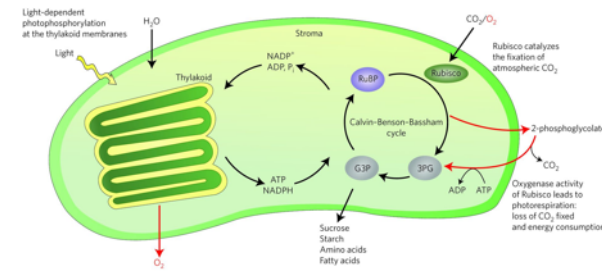


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Proteins that fluoresce

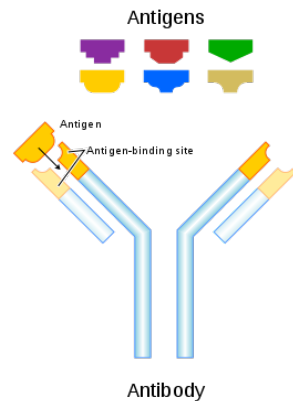
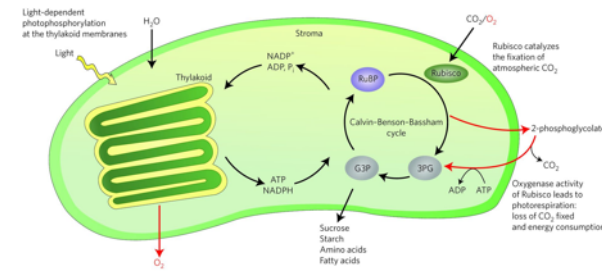


Motivating problem: design protein sequences

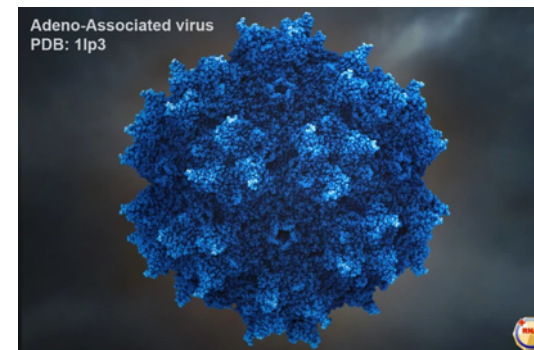
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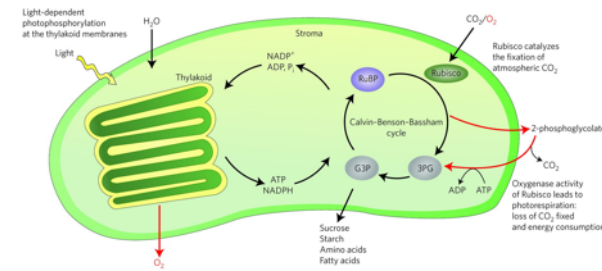


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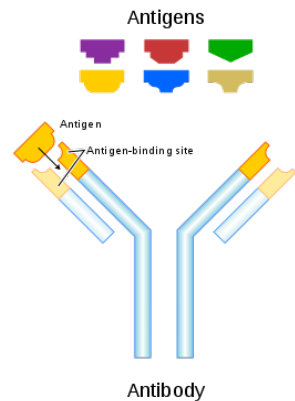
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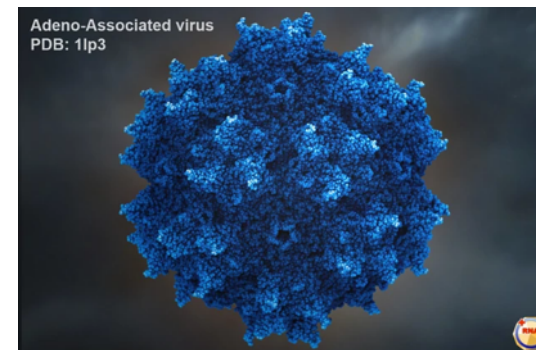
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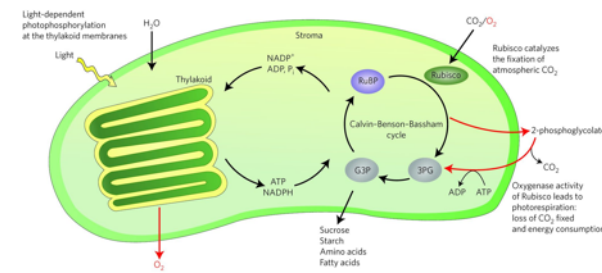


Motivating problem: design protein sequences

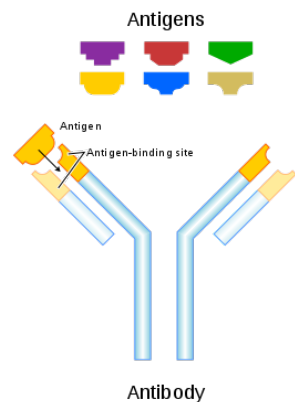
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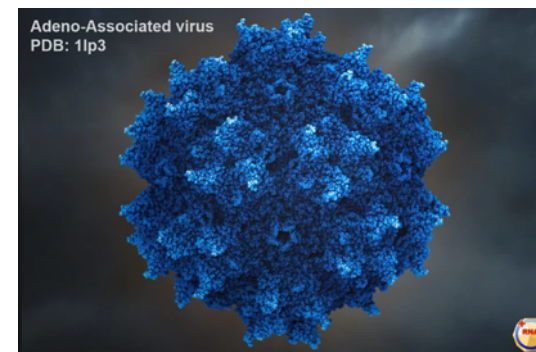
Proteins that fluoresce



... that fixate carbon in the atmosphere



.... that act as drugs



... that deliver gene-editing tools to tissues

How to map sequence to function?

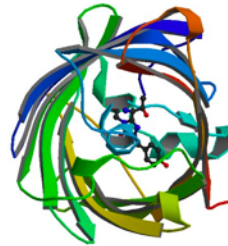
A law of molecular biology:

Sequence

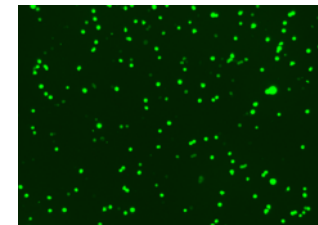
```
VTLDLQNSTKFGGFLRSALDV
VTLDLQNSTKFGGFLRSALDV
VTLDLQNSTKFGGFLRSALDV
VTLDLQNSTKFGGFLRSALDV
VTLDLQNSTKFGGFLRSALDV
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```



Structure



Function



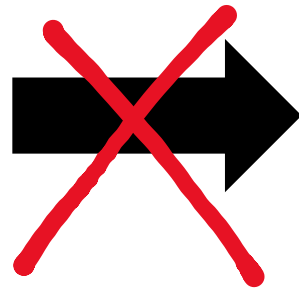
ex: fluorescence

Bypassing the structure relationships

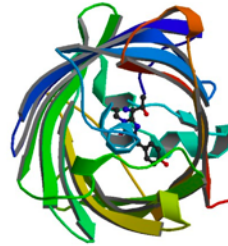
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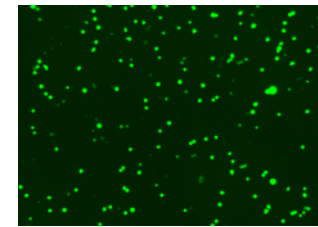
VTLDLQNSTEKFGGFLRSALDV
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Function



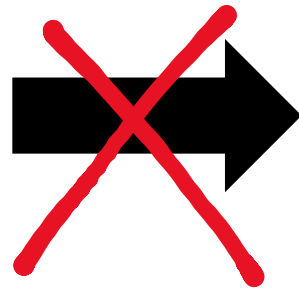
High throughput experiments (& ML)

Can we solve the inverse problem?

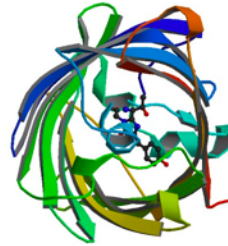
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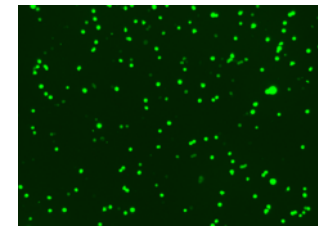
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```



Structure



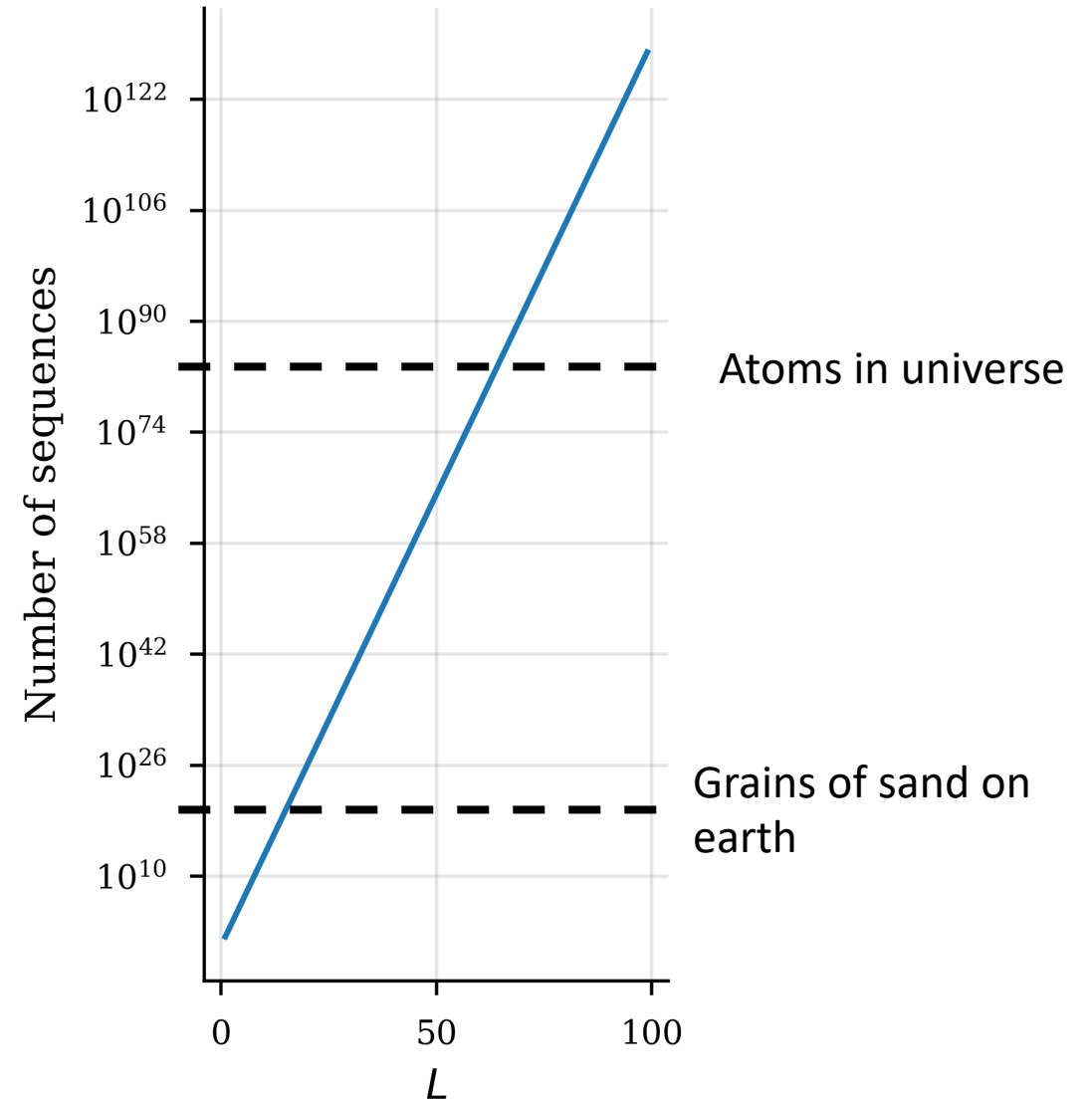
Function



Design problem: Given a model, find sequences with desired function

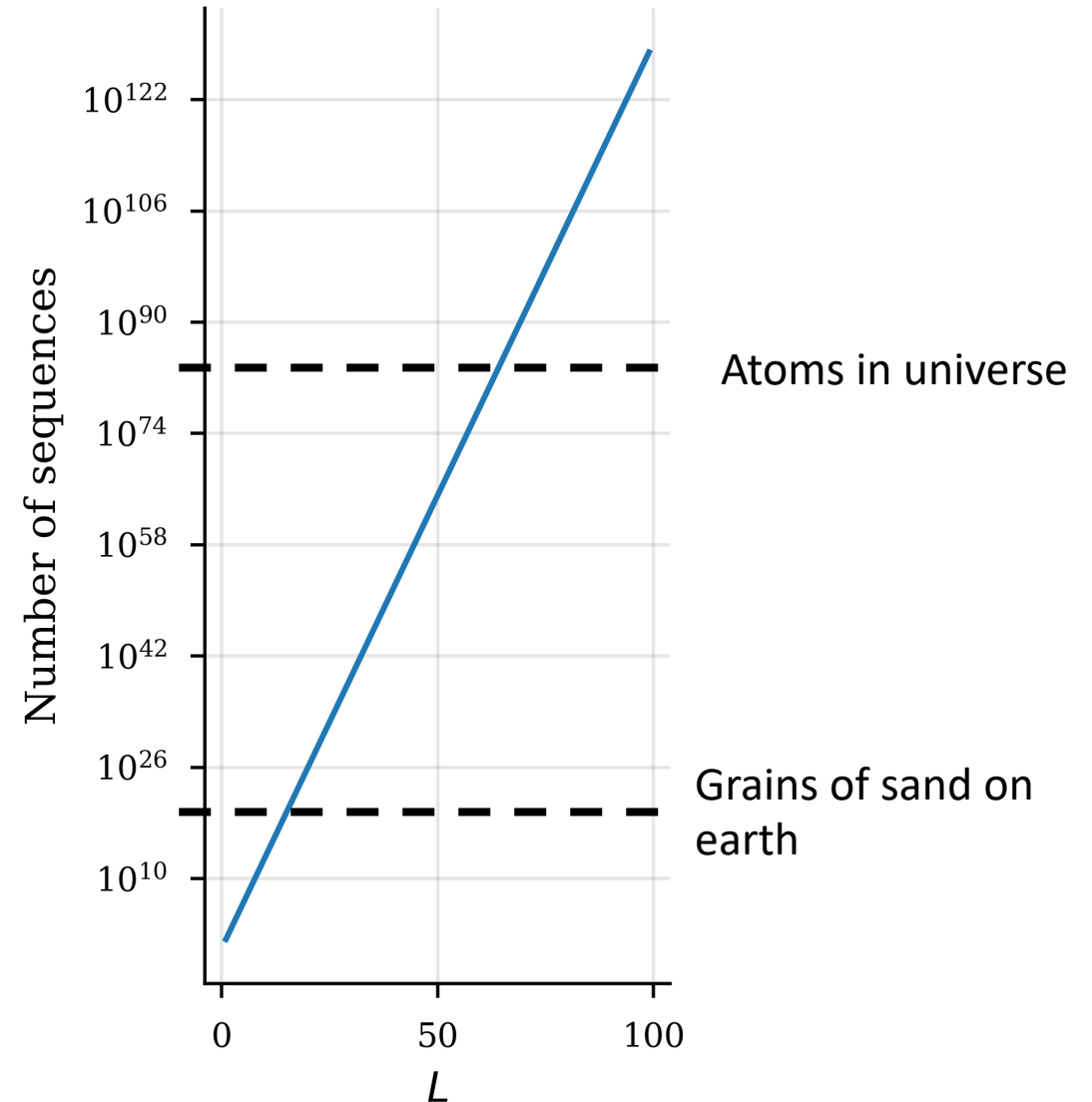
Why is protein design difficult?

- Huge, rugged search space
⇒ size scales as 20^L



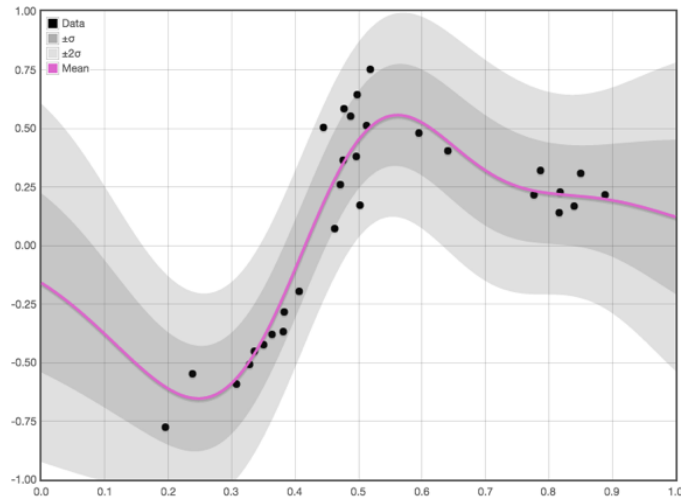
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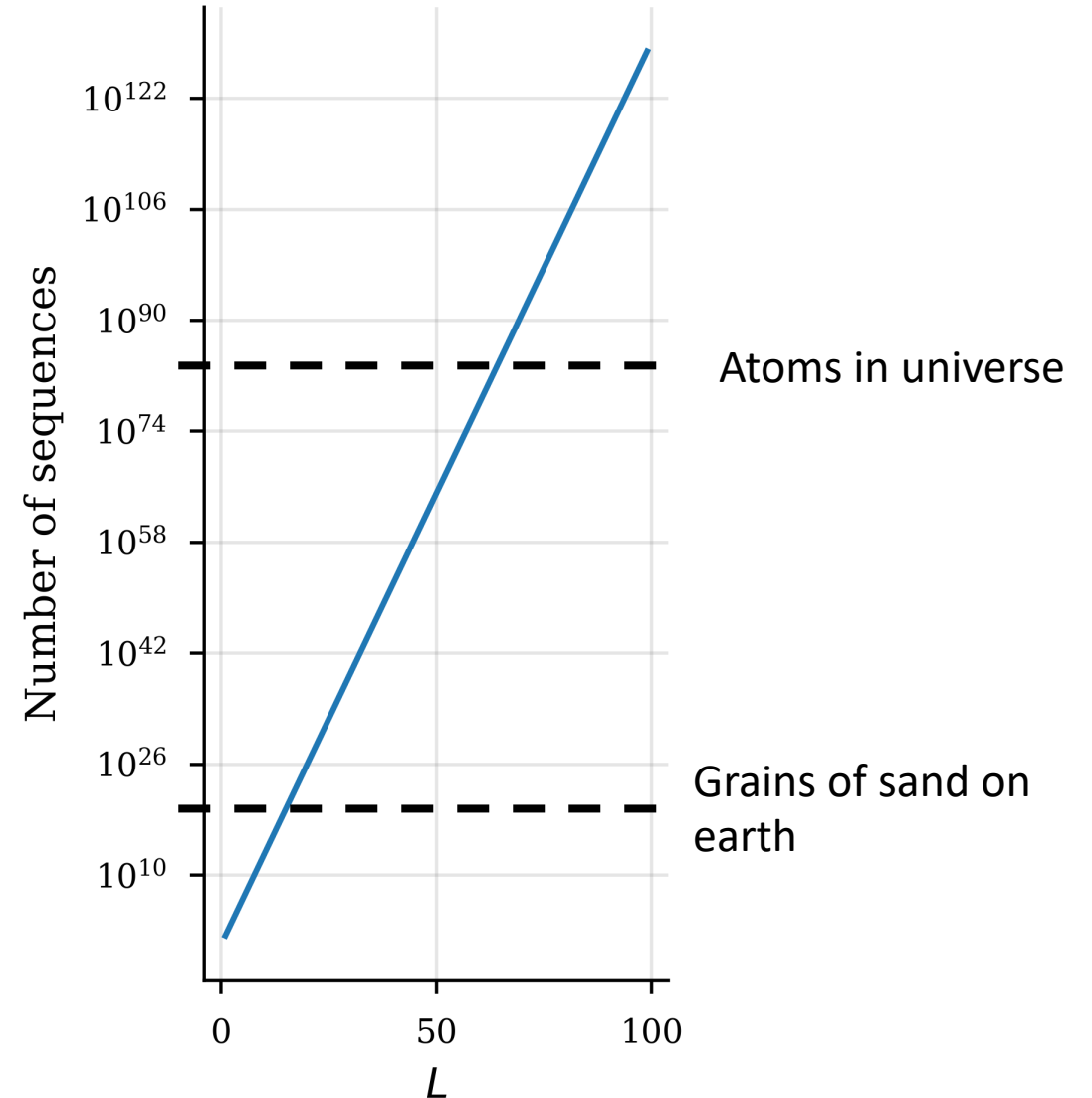


Why is protein design difficult?

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⇒ size scales as 20^L
- Discrete search space (no gradients)
- Uncertainty in predictor



https://livingthing.danmackinlay.name/gaussian_processes.html#69



Possible solution: model-based optimization (MBO)

Idea: replace the standard (hard) objective

$$\max_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x})$$

e.g. the space of sequences



Possible solution: model-based optimization (MBO)

Idea: replace the standard (hard) objective with a potentially easier one

$$\max_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x})$$

the space of sequences



$$\max_{\boldsymbol{\theta} \in \mathbb{R}^d} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} [f(\mathbf{x})]$$

model over sequence space

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$$\max_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}) \quad \longrightarrow \quad \max_{\boldsymbol{\theta} \in \mathbb{R}^d} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})}[f(\mathbf{x})]$$

Solution approach is to iterate:

1. Sample from “search model” $p(\mathbf{x}|\boldsymbol{\theta})$
2. Evaluate samples on $f(\mathbf{x})$
3. Adjust $\boldsymbol{\theta}$ so the model favors samples with large function evals

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Solution approach is to iterate:

1. Sample from “search model” $p(\mathbf{x}|\boldsymbol{\theta})$
 2. Evaluate samples on $f(\mathbf{x})$
 3. Adjust $\boldsymbol{\theta}$ so the model favors sequences with large function evals
- ✓ Model can sample broad areas of sequence space
 - ✓ Does not require gradients of f
 - ✓ Can incorporate uncertainty

First attempt at MBO for protein design: Design by Adaptive Sampling (DbAS)

Our aim is solve the MBO objective:

$$\operatorname{argmax}_{\boldsymbol{\theta}} \log \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} [P(S|\mathbf{x})]$$

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→ *e.g.* fluorescence $> \alpha$

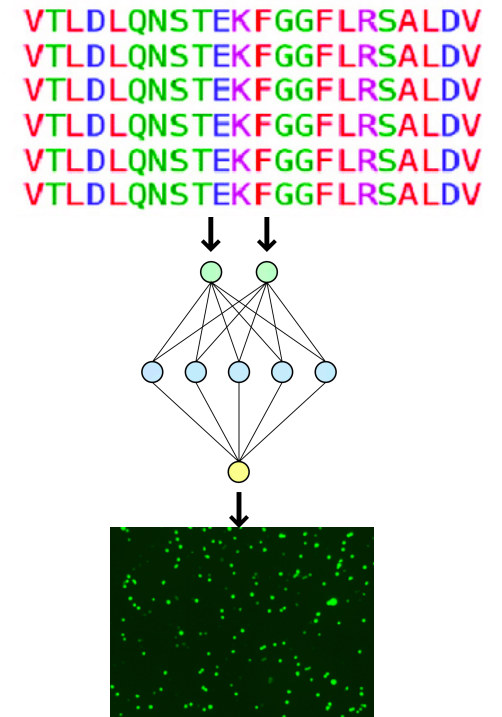
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- $p(x|\theta)$ is the search model (VAE, HMM...)
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→ *e.g.* fluorescence $> \alpha$
- $P(S|x)$ is a stochastic predictive model (“oracle”) that maps sequences to property



Design by Adaptive Sampling (cont.)

Two issues:

- 1. θ is in the expectation distribution.**

$$\operatorname{argmax}_{\theta} \log \mathbb{E}_{p(\mathbf{x}|\theta)} [P(S|\mathbf{x})]$$

Design by Adaptive Sampling (cont.)

maximize a lower bound

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↓ ≥

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Design by Adaptive Sampling (cont.)

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anneal a sequence of relaxations:

$S^t \rightarrow S$, where $S^t \supset S^{t+1}$

Design by Adaptive Sampling (cont.)

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↓

Anneal and MC

$$\theta^{(t+1)} = \operatorname{argmax}_{\theta} \sum_{i=1}^M P(S^{(t)}|\mathbf{x}_i^{(t)}) \log p(\mathbf{x}_i^{(t)}|\theta)$$

Design by Adaptive Sampling (cont.)

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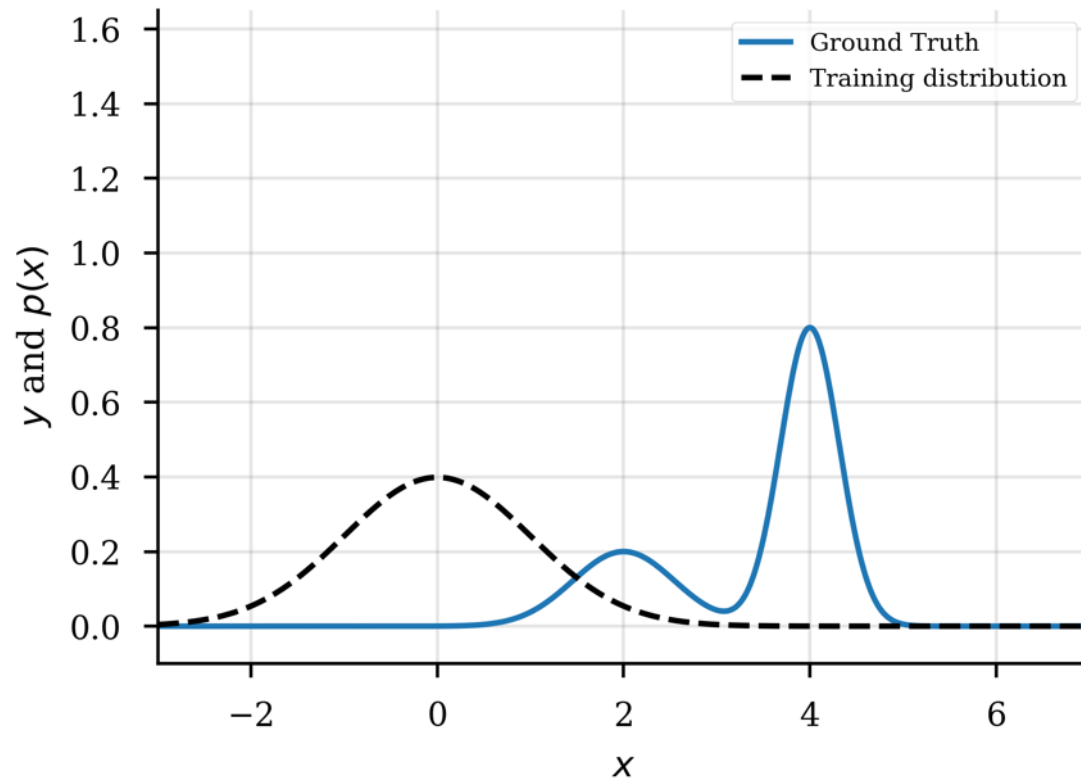
Assumes oracle is unbiased and has good uncertainty estimates



Anneal and MC

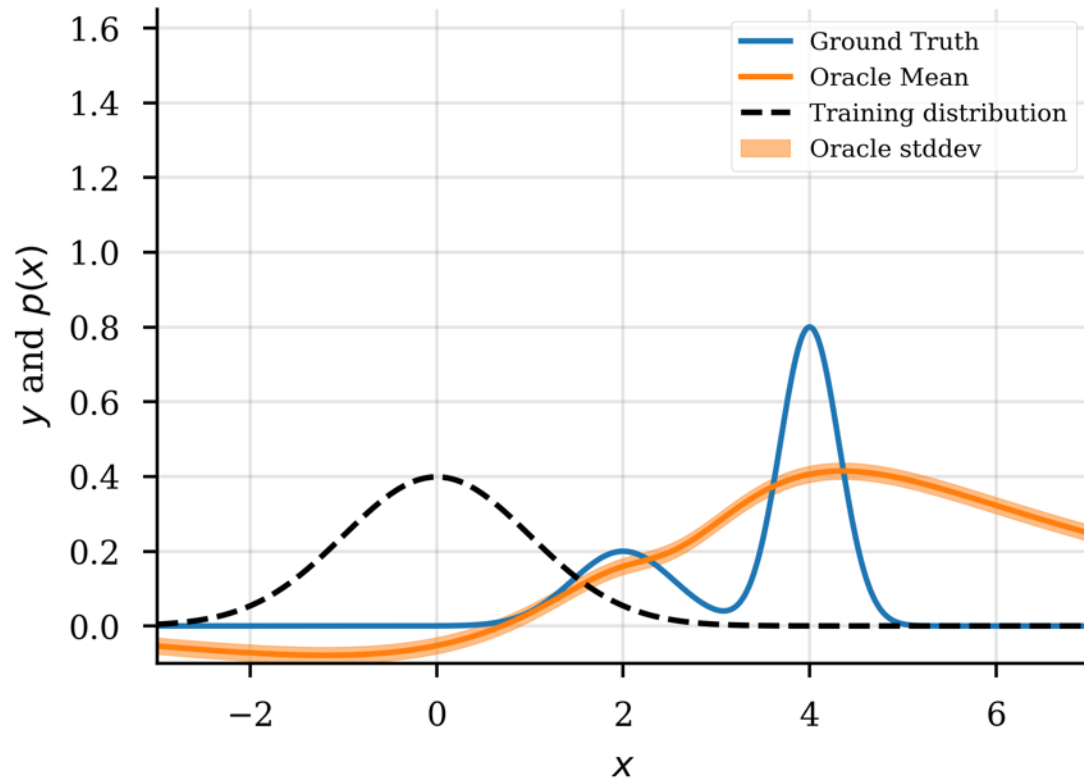
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How pathological oracles lead you astray



How pathological oracles lead you astray

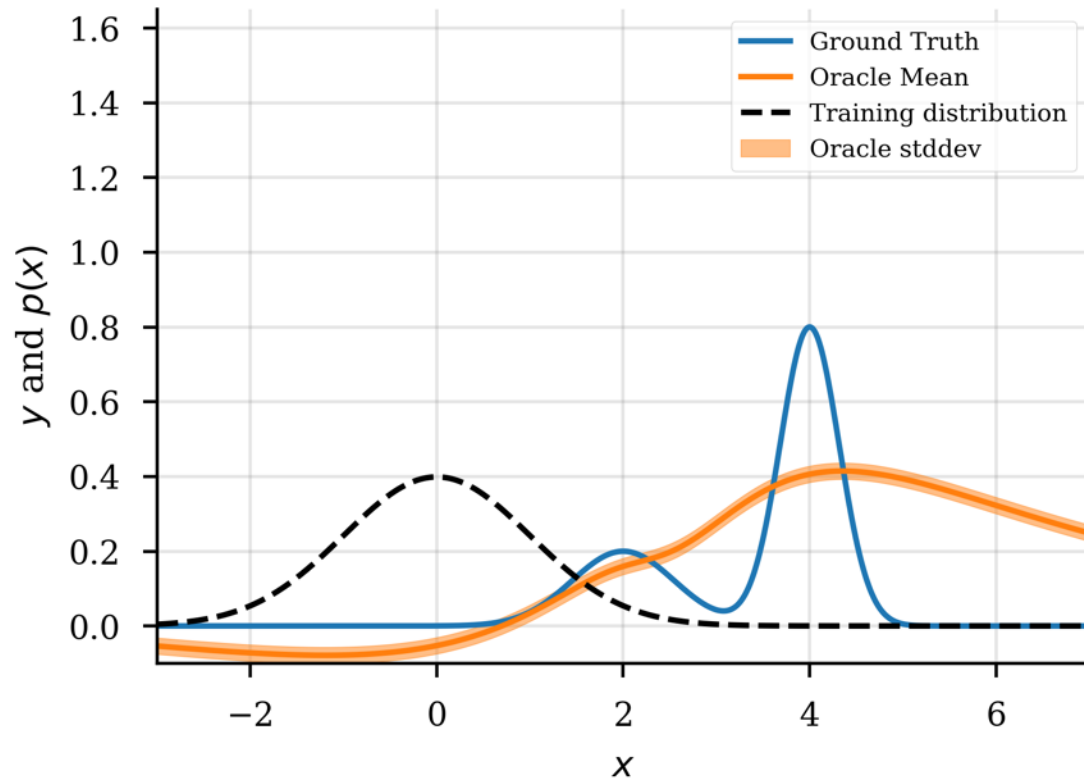
Acceptable



Many training examples

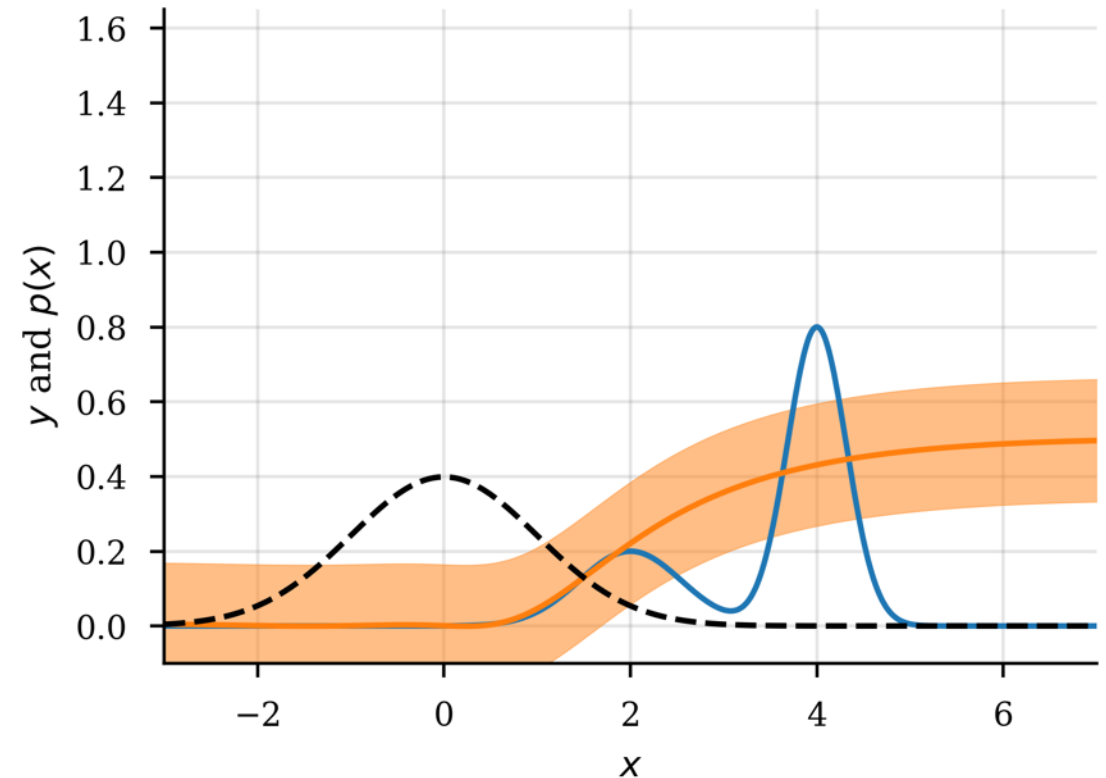
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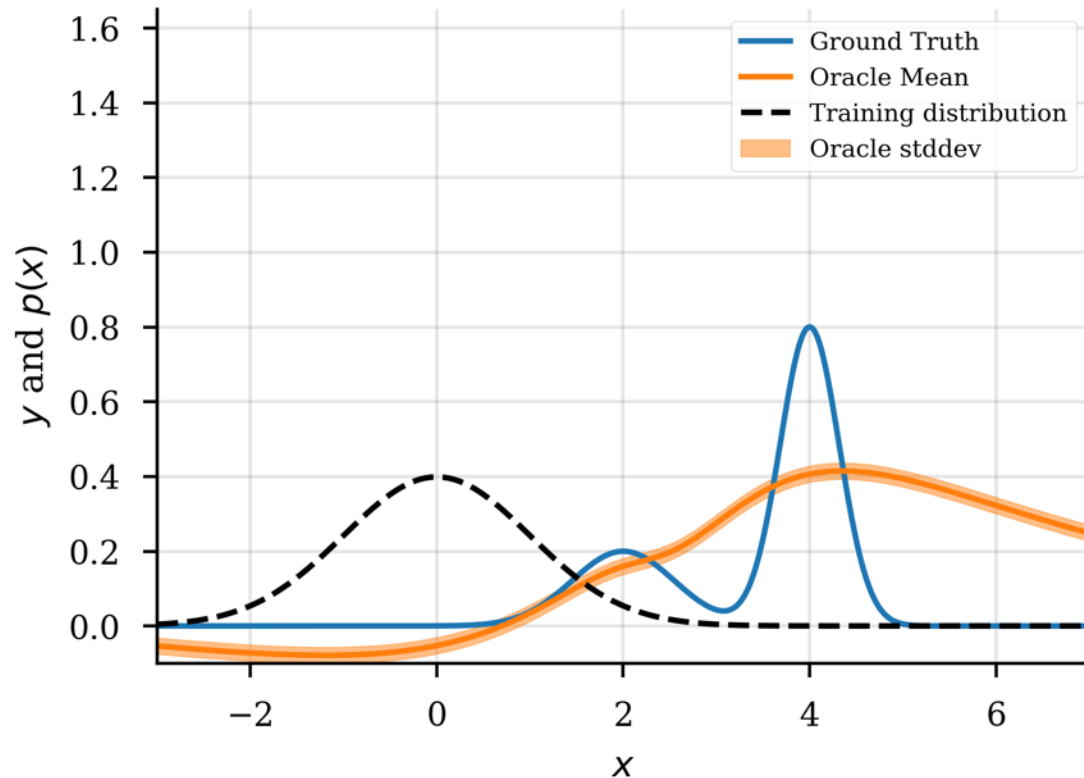
Pathological



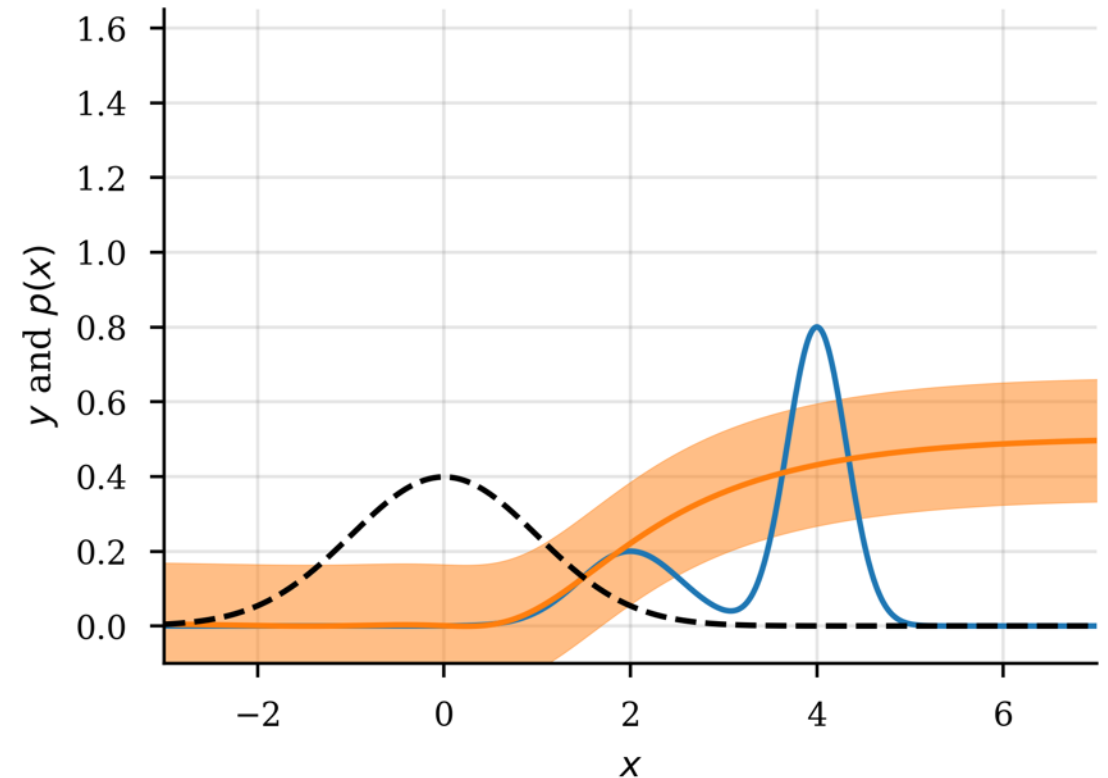
Fewer training examples

How pathological oracles lead you astray

Acceptable

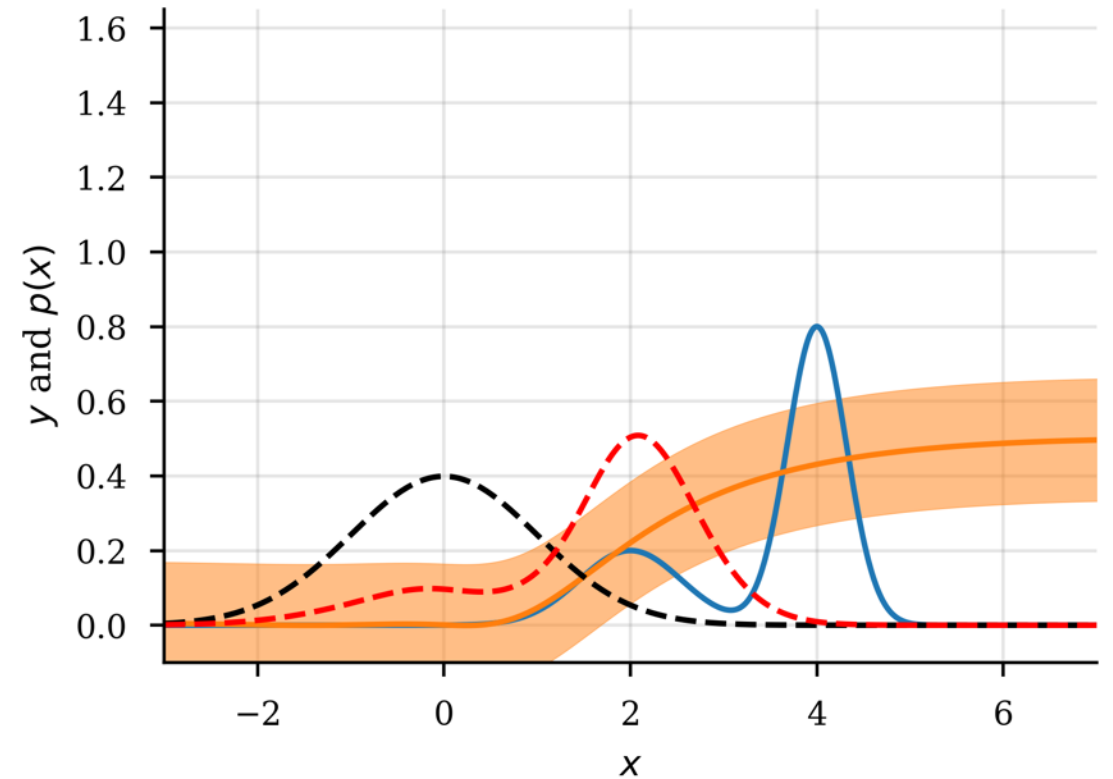
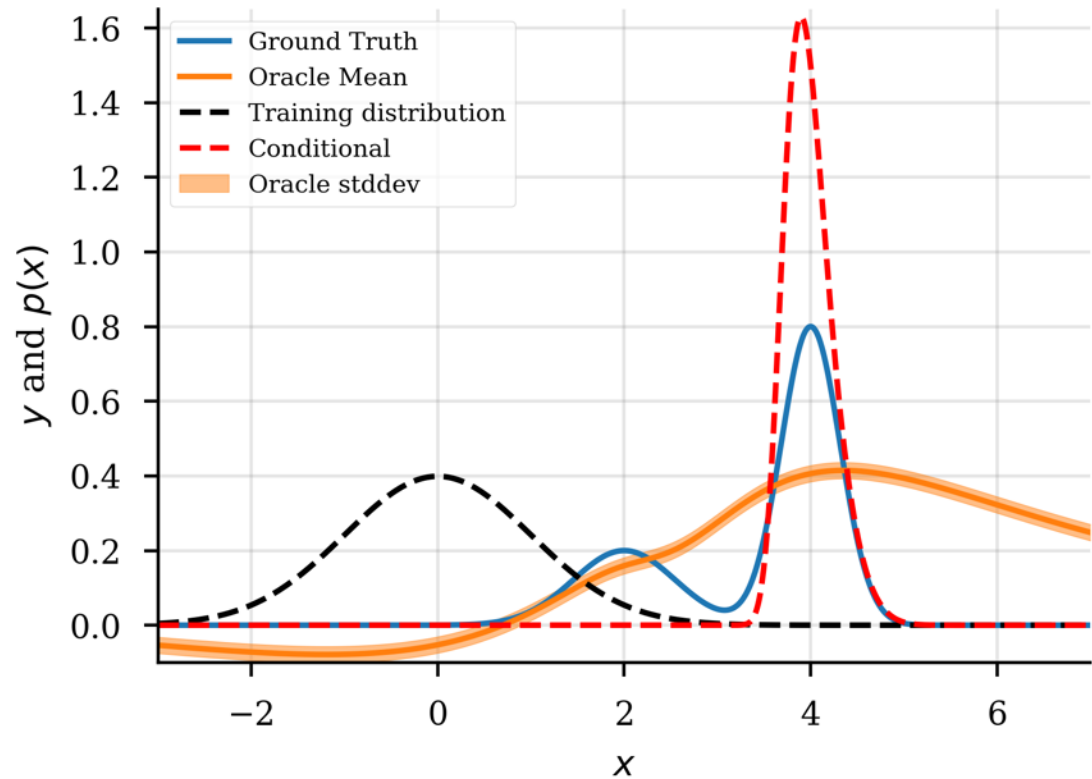


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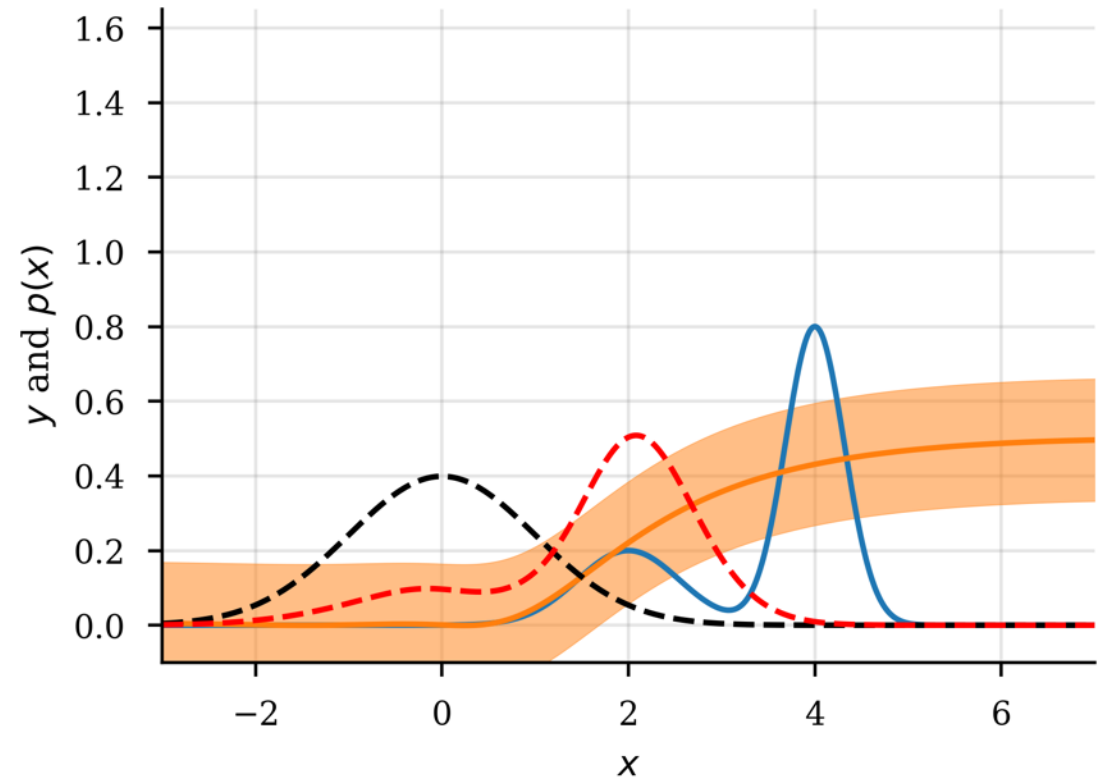
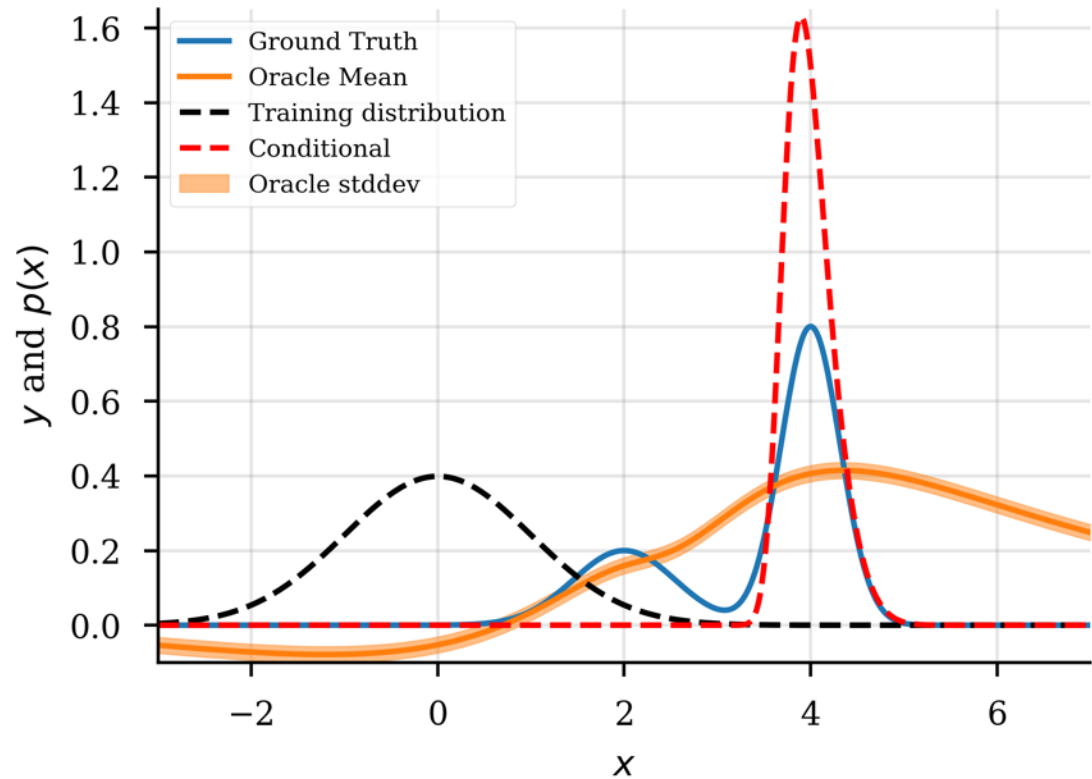
Idea: estimate training distribution of x *conditioned* on high values of oracle

Fixing pathological oracles w/ conditioning



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Fixing pathological oracles w/ conditioning



Idea: estimate training distribution of x *conditioned* on high values of oracle

Don't have access to training distribution, but can build a model $p(\mathbf{x}|\theta^{(0)})$ to approximate it

Conditioning by Adaptive Sampling (CbAS)

Previous formulation:

$$\operatorname{argmax}_{\boldsymbol{\theta}} \log \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} [P(S|\mathbf{x})]$$



$$\operatorname{argmax}_{\boldsymbol{\theta}} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta}^{(t)})} [P(S|\mathbf{x}) \log p(\mathbf{x}|\boldsymbol{\theta})]$$



Anneal and MC

$$\boldsymbol{\theta}^{(t+1)} = \operatorname{argmax}_{\boldsymbol{\theta}} \sum_{i=1}^M P(S^{(t)}|\mathbf{x}_i^{(t)}) \log p(\mathbf{x}_i^{(t)}|\boldsymbol{\theta})$$

New formulation:

$$\operatorname{argmin}_{\boldsymbol{\theta}} D_{KL} \left(p(\mathbf{x}|S, \boldsymbol{\theta}^{(0)}) || p(\mathbf{x}|\boldsymbol{\theta}) \right)$$



$p(\mathbf{x}|\boldsymbol{\theta}^{(0)})$ models the training distribution

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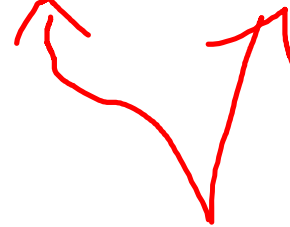
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Can't anneal when sampling dist. doesn't change!

Conditioning by Adaptive Sampling (CbAS)

Previous formulation:

$$\begin{aligned} & \underline{\underline{\quad}} \\ & \downarrow \geq \\ & \operatorname{argmax}_{\boldsymbol{\theta}} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta}^{(t)})} [P(S|\mathbf{x}) \log p(\mathbf{x}|\boldsymbol{\theta})] \\ & \downarrow \text{Anneal and MC} \\ & \boldsymbol{\theta}^{(t+1)} = \operatorname{argmax}_{\boldsymbol{\theta}} \sum_{i=1}^M P(S^{(t)}|\mathbf{x}_i^{(t)}) \log p(\mathbf{x}_i^{(t)}|\boldsymbol{\theta}) \end{aligned}$$

New formulation:

$$\begin{aligned} & \operatorname{argmin}_{\boldsymbol{\theta}} D_{KL} \left(p(\mathbf{x}|S, \boldsymbol{\theta}^{(0)}) || p(\mathbf{x}|\boldsymbol{\theta}) \right) \\ & \downarrow = \\ & \operatorname{argmax}_{\boldsymbol{\theta}} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta}^{(0)})} [P(S|\mathbf{x}) \log p(\mathbf{x}|\boldsymbol{\theta})] \\ & \downarrow = \\ & \operatorname{argmax}_{\boldsymbol{\theta}} \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta}^{(t)})} \left[\frac{p(\mathbf{x}|\boldsymbol{\theta}^{(0)})}{p(\mathbf{x}|\boldsymbol{\theta}^{(t)})} P(S|\mathbf{x}) \log p(\mathbf{x}|\boldsymbol{\theta}) \right] \\ & \quad \text{Importance sampling proposal dist.} \end{aligned}$$

Conditioning by Adaptive Sampling (CbAS)

Previous formulation:

$$\operatorname{argmax}_{\boldsymbol{\theta}} \log \mathbb{E}_{p(\mathbf{x}|\boldsymbol{\theta})} [P(S|\mathbf{x})]$$

↓ ≥

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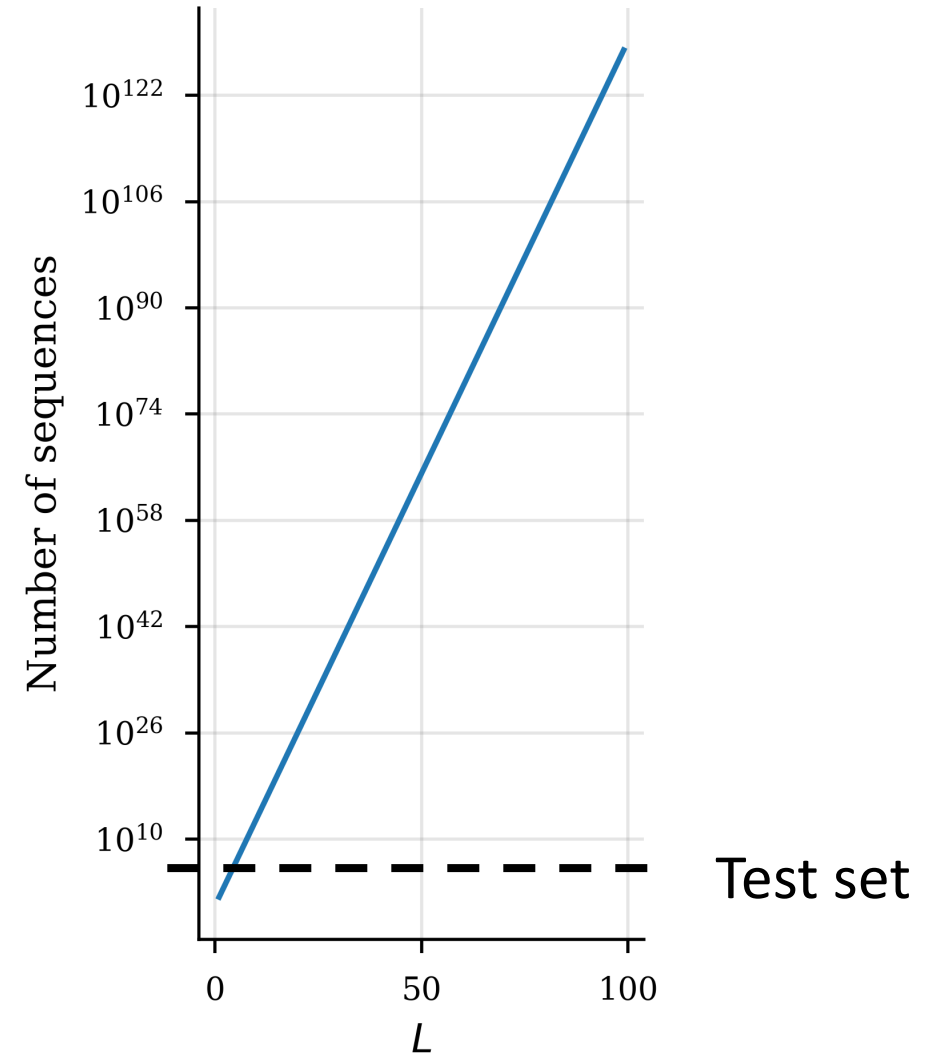
$$\operatorname{argmax}_{\boldsymbol{\theta}} \sum_{i=1}^M \frac{p(\mathbf{x}_i^{(t)}|\boldsymbol{\theta}^{(0)})}{p(\mathbf{x}_i^{(t)}|\boldsymbol{\theta}^{(t)})} P(S^{(t)}|\mathbf{x}_i^{(t)}) \log p(\mathbf{x}_i^{(t)}|\boldsymbol{\theta})$$

Testing is fundamentally different

- We don't trust our oracle and generally can't query the ground truth

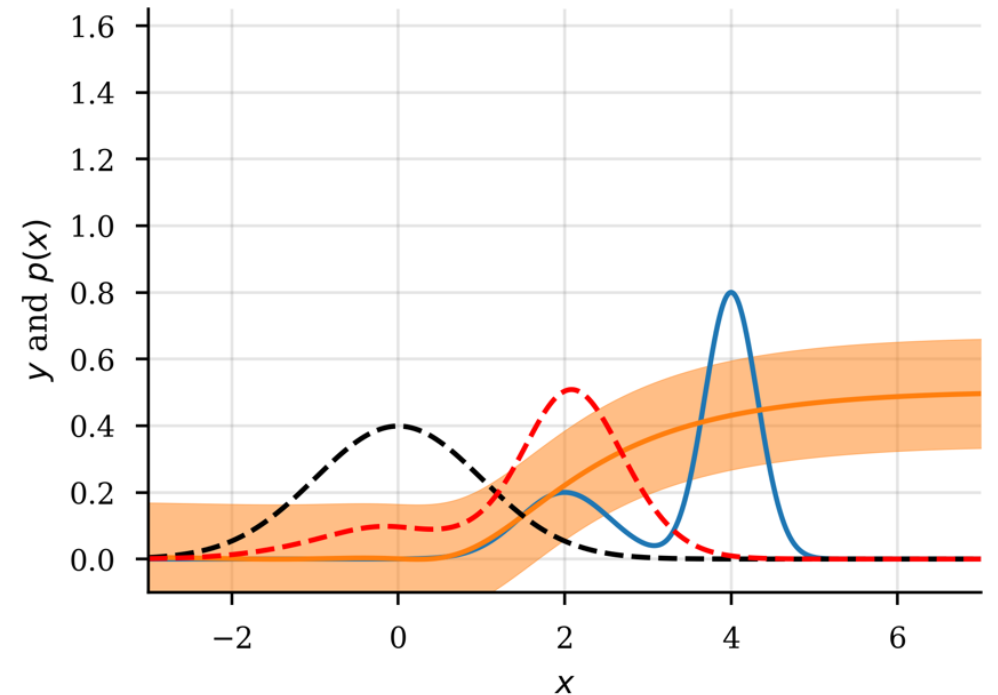
Testing is fundamentally different

- We don't trust our oracle and generally can't query the ground truth
- We can't hold-out a test set of good sequences
 - Near-zero chance of any of these sequences being found by the method



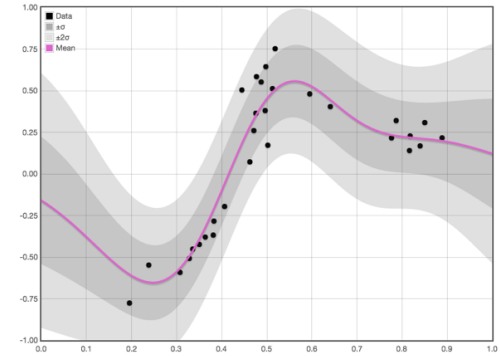
Testing is fundamentally different

- We don't trust our oracle and generally can't query the ground truth
- We can't hold-out a test set of good sequences
 - Near-zero chance of any of these sequences being found by the method
- We can't use some canonical test function as the oracle
 - In our problem it is untrustworthy



Testing strategy

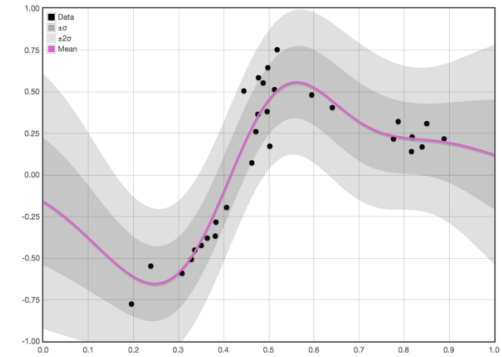
- Simulate a ground truth based on real data
→ “Ground truth” is a GP mean function



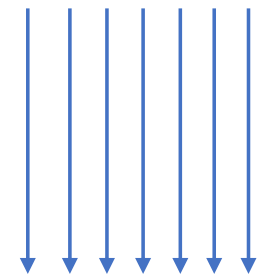
Ground
truth GP

Testing strategy

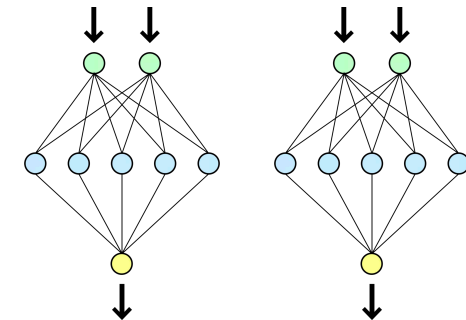
- Simulate a ground truth based on real data
 - “Ground truth” is a GP mean function
- Ground truth values are sampled from the GP for given sequences
- Use these input-output pairs to train oracles.



Ground truth GP



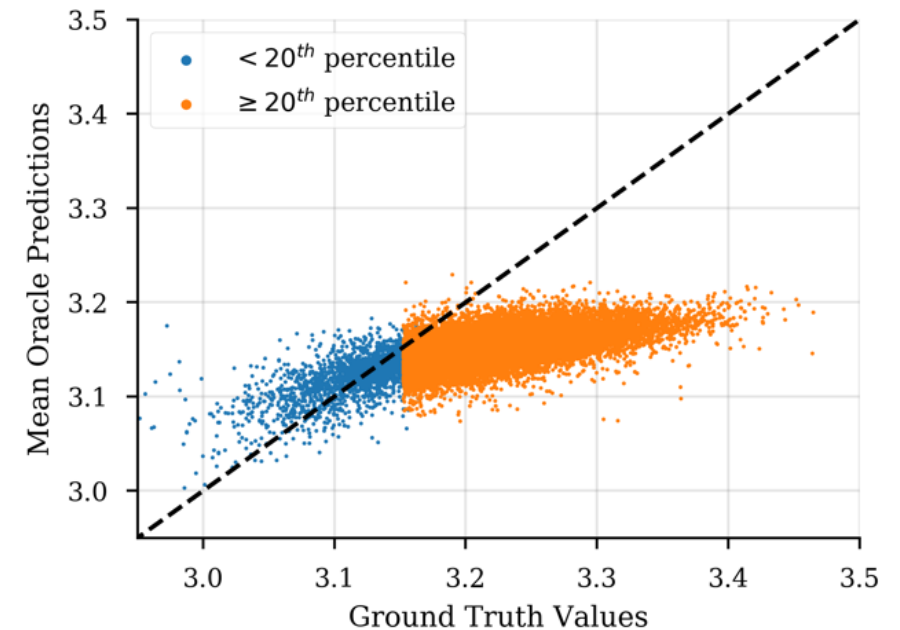
Training data



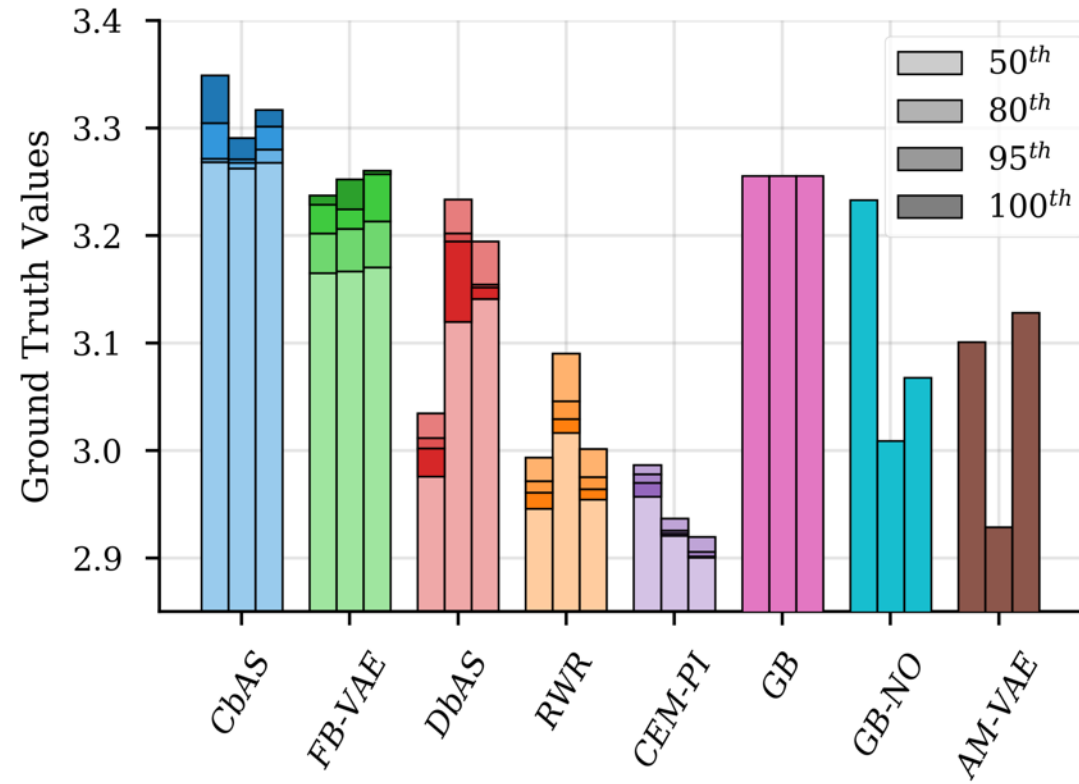
Oracles

Testing strategy

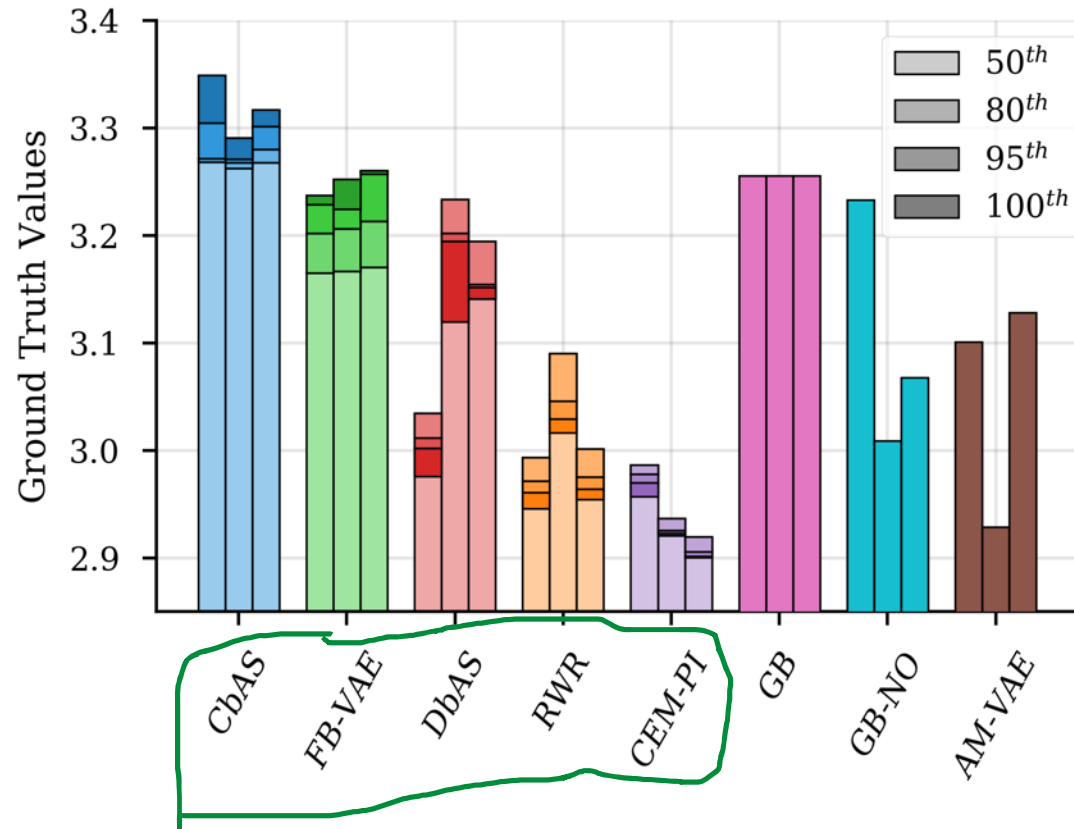
- Simulate a ground truth based on real data
 - “Ground truth” is a GP mean function
- Ground truth values are sampled from the GP for given sequences
- Use these input-output pairs to train oracles
- Coerce training set so these oracles exhibit pathologies



Results



Results

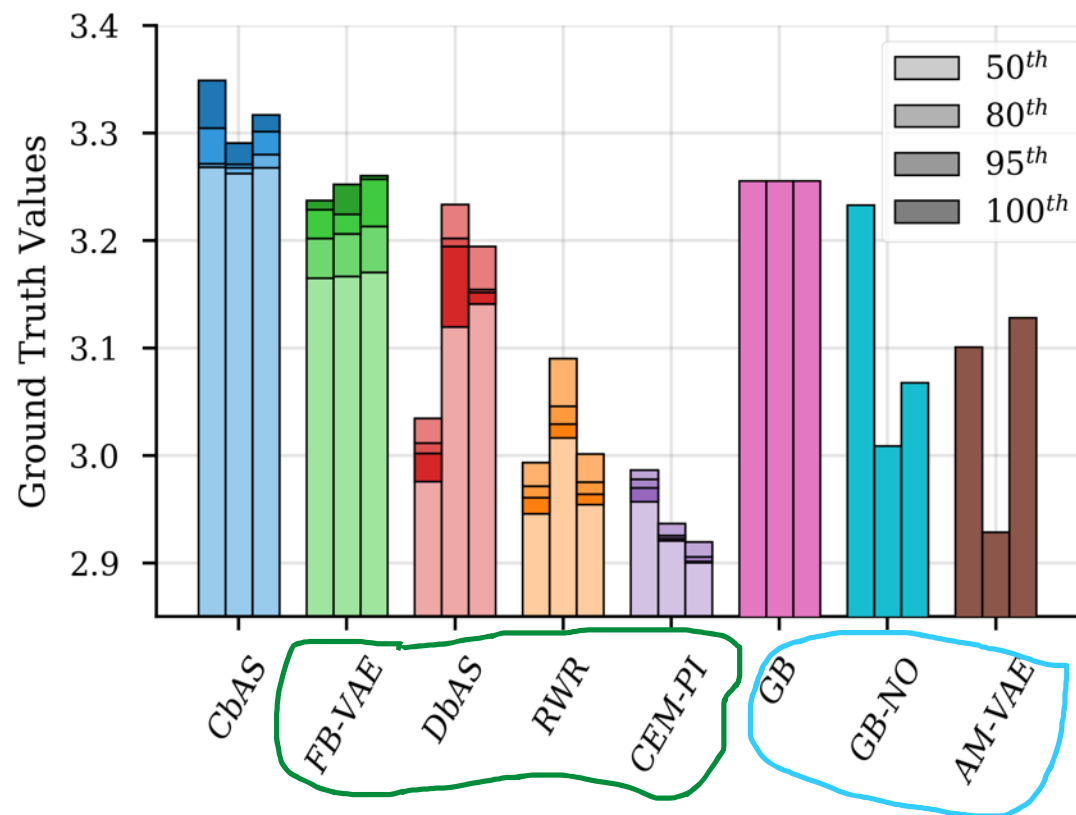


Model-based optimizations

Use weighted ML updates with weights:

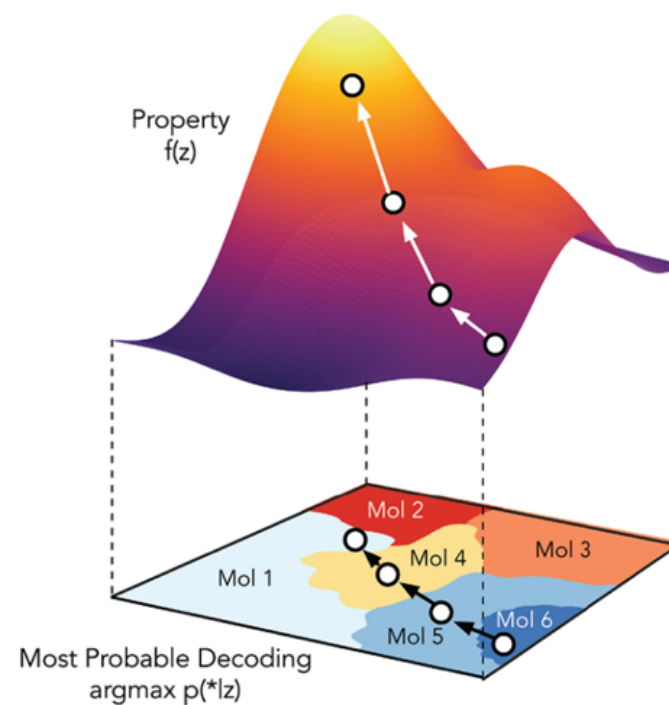
- *CbAS*: $\frac{p(x|\theta^{(0)})}{p(x|\theta^{(t)})} P(S^{(t)}|x)$
- *DbAS*: $P(S^{(t)}|x)$
- *RWR*: $e^{\alpha f(x)}$
- *CEM-PI*: $\mathbb{1}_{\{PI(x) > \gamma^{(t)}\}}(x)$
- *FB-VAE*: $\mathbb{1}_{\{f(x) > \gamma^{(t)}\}}(x)$ w/ additional considerations

Results

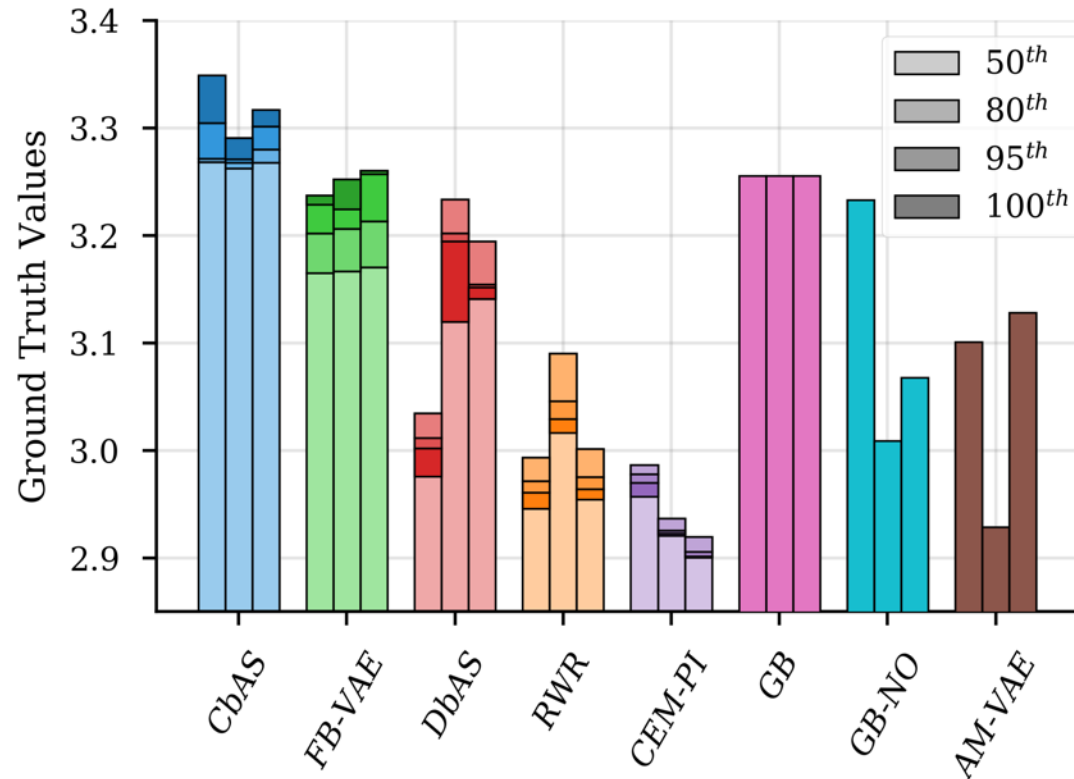


Model-based optimizations

Gradient descent on latent spaces



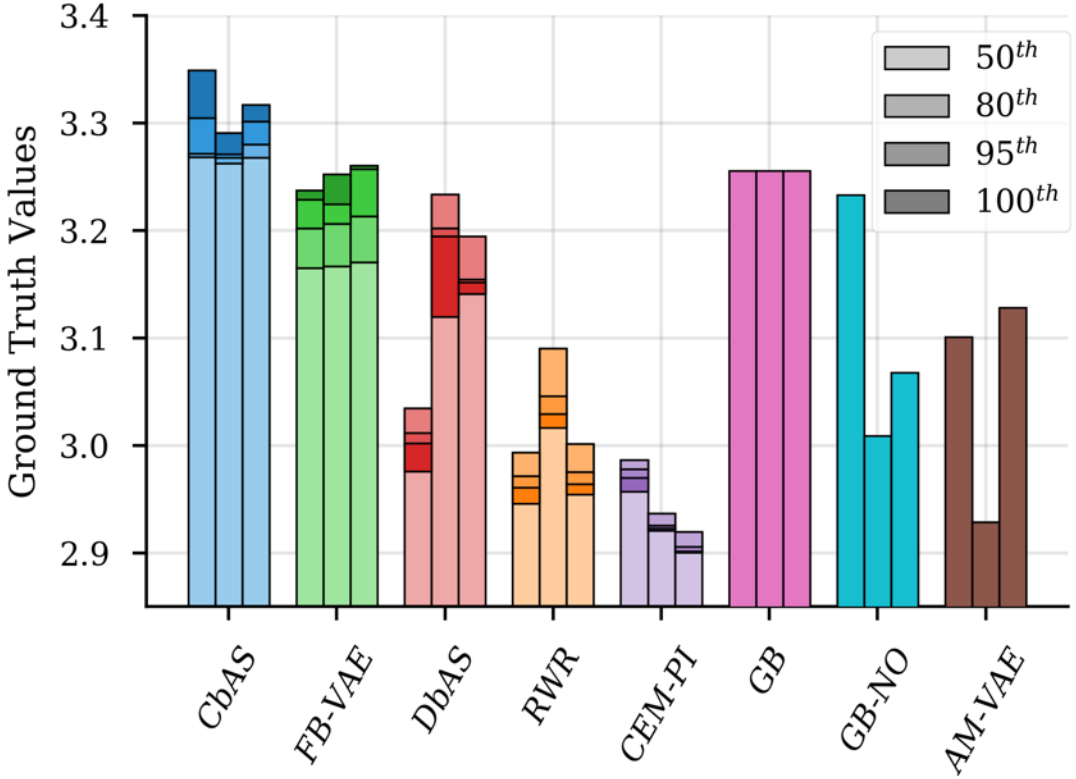
Results



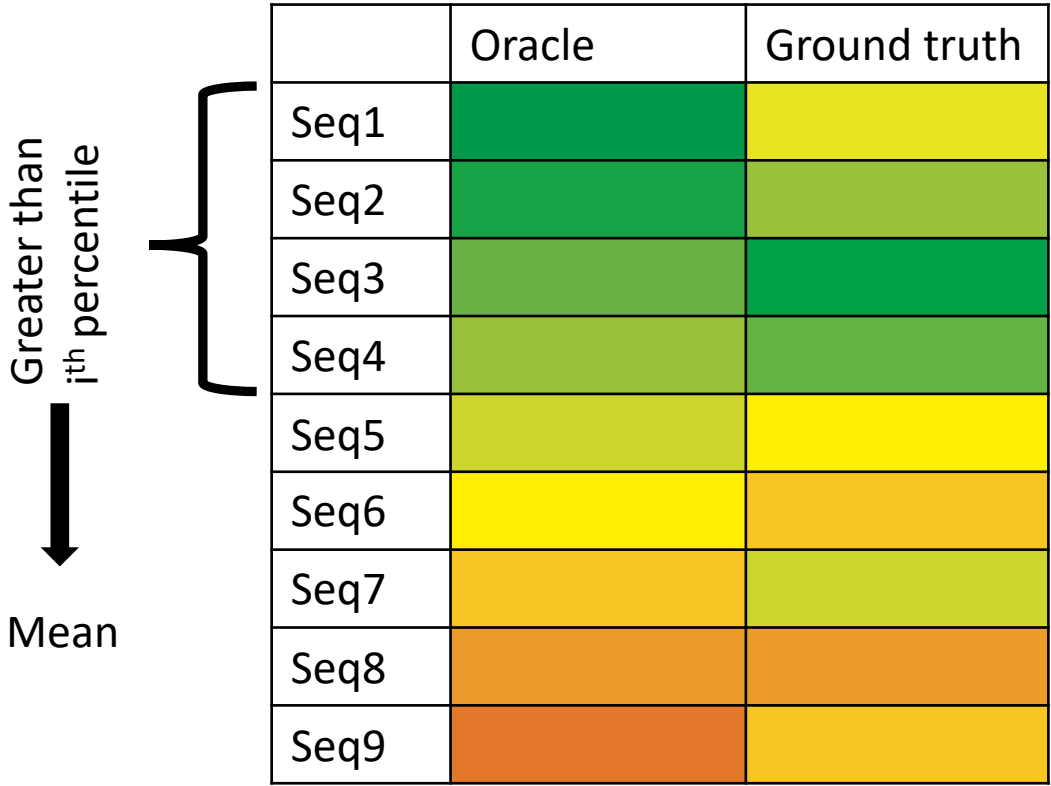
What does each bar represent?

	Oracle	Ground truth
Seq1	Green	Yellow
Seq2	Green	Light Green
Seq3	Light Green	Green
Seq4	Light Green	Light Green
Seq5	Light Green	Yellow
Seq6	Yellow	Orange
Seq7	Orange	Light Green
Seq8	Orange	Orange
Seq9	Orange	Yellow

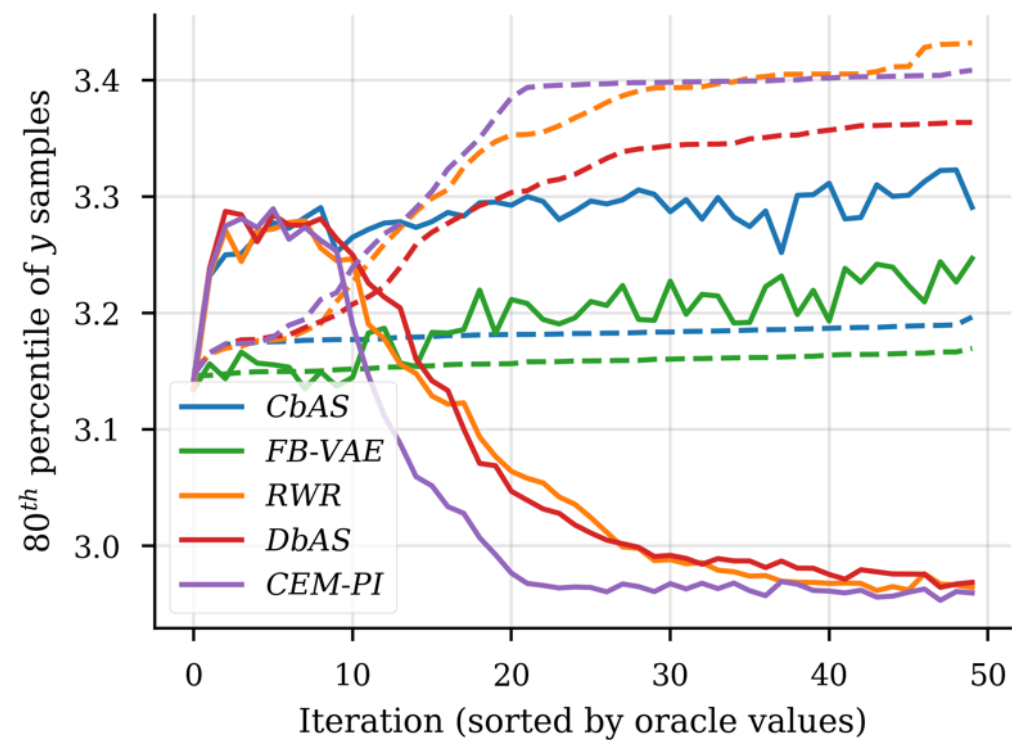
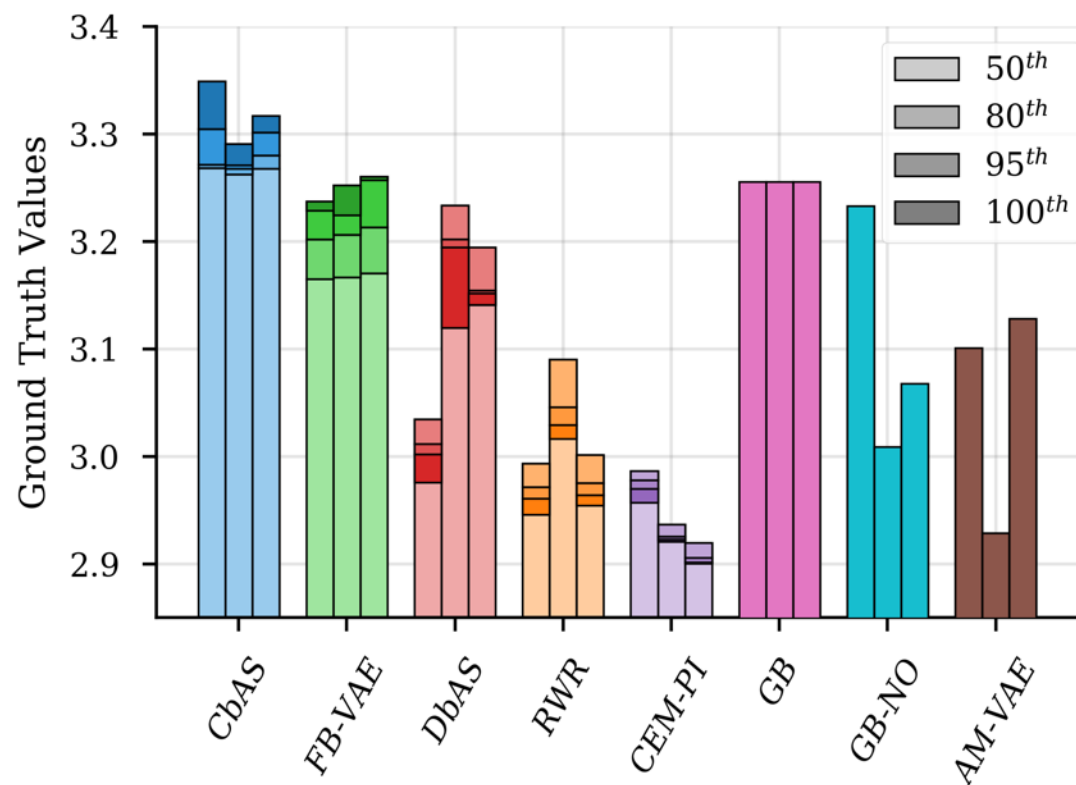
Results



What does each bar represent?



Results



Wrap-up

- Introduced a new model-based optimization method that is robust to pathological oracles
- Specifically targeted for discrete design problems
- Ongoing work to move beyond proof-of-principle:
 - Collaboration with wet-lab to perform end-to-end validation

Thanks!

Funding:

