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Inferring Heterogeneous Causal Effects in Presence of Spatial Confounding

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Causal inference problem

- $y \in \mathbb{R}$: Outcome of interest

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- $y \in \mathbb{R}$: Outcome of interest
- $z \in \mathbb{R}$: Exposure variable

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- $\mathcal{D}_n = \{y_i, z_i, \boldsymbol{s}_i\}_{i=1}^n$, where $\boldsymbol{s} \in \mathbb{R}^d$ is spatial location

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- Target quantity: Average effect of assigning $z = \widetilde{z}$ on y at location $\boldsymbol{s}$

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$$\tau = \frac{d}{d\widetilde{z}} \mathbb{E}\left[y(\widetilde{z}) \mid \boldsymbol{s}\right] \quad (1)$$

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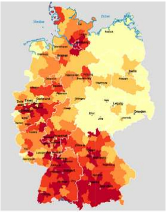

$y = $ income

$z = $ age

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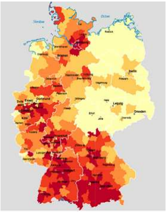

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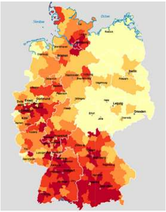

y = income

z = age

c = unemployment

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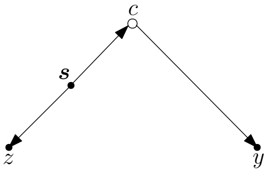
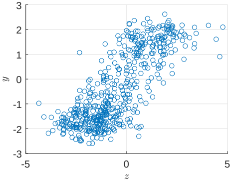
Causal Inference Problem





Example: Here $\tau = 0$ yet $\mathrm{Cov}(z, y) \neq 0$.

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Approach

- Assumptions:

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- $\mathbb{E}\left[y(\widetilde{z}) \mid \boldsymbol{s}\right] = \mathbb{E}\left[y|z = \widetilde{z},\ \boldsymbol{s}\right]$

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- Assumptions:
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$$y = \tau(\boldsymbol{s})z + \beta(\boldsymbol{s}) + \epsilon \tag{2}$$

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$$y = \tau(\boldsymbol{s})z + \beta(\boldsymbol{s}) + \epsilon \tag{2}$$

- $\beta(\boldsymbol{s})$ is a nuisance function correlated with spatially varying exposure z.

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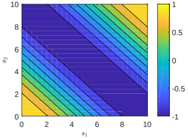

$\tau(\boldsymbol{s})$

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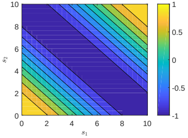

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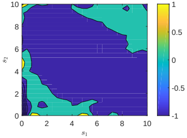

$\widehat{\tau}(\boldsymbol{s})$ via (2)

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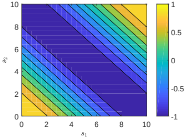

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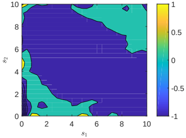

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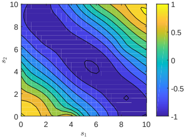

$\widehat{\tau}(\boldsymbol{s})$ proposed

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Error-in-variables model

- Let $w = y - \mathbb{E}\left[y|\boldsymbol{s}\right]$ and $v = z - \mathbb{E}\left[z|\boldsymbol{s}\right]$ [1]

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Error-in-variables model

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- (2) becomes

$$w = \tau(\boldsymbol{s})v + \epsilon \tag{3}$$

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- The effect $\tau(\boldsymbol{s})$ is directly identifiable from (3) which we parameterize as

$$\tau_{\boldsymbol{\theta}}(\boldsymbol{s}) \in \{f(\boldsymbol{s}) : f = \boldsymbol{\phi}(\boldsymbol{s})^\top \boldsymbol{\theta}\},$$

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$$\tau_{\boldsymbol{\theta}}(\boldsymbol{s}) \in \{f(\boldsymbol{s}) : f = \boldsymbol{\phi}(\boldsymbol{s})^\top \boldsymbol{\theta}\},$$

- Residuals w and v are not observed but estimated so that

$$w = \underbrace{\left(y - \widehat{\mathbb{E}[y|\boldsymbol{s}]}\right)}_{\widehat{w}} + \underbrace{\left(\widehat{\mathbb{E}[y|\boldsymbol{s}]} - \mathbb{E}[y|\boldsymbol{s}]\right)}_{\widetilde{w}},$$

$$v = \underbrace{\left(z - \widehat{\mathbb{E}[z|\boldsymbol{s}]}\right)}_{\widehat{v}} + \underbrace{\left(\widehat{\mathbb{E}[z|\boldsymbol{s}]} - \mathbb{E}[z|\boldsymbol{s}]\right)}_{\widetilde{v}},$$

where $\widetilde{w}$ and $\widetilde{v}$ denote errors

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Proposed robust method

- Then (3) becomes

$$\widehat{w} = \left(\widehat{v}\boldsymbol{\phi}(\boldsymbol{s}) + \boldsymbol{\delta}(\boldsymbol{s})\right)^{\top}\boldsymbol{\theta} + \widetilde{\epsilon}$$

where $\boldsymbol{\delta}(\boldsymbol{s}) = \widetilde{v}\boldsymbol{\phi}(\boldsymbol{s})$ is an unobserved random deviation

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- Robust estimator with tolerance against worst-case deviation $\boldsymbol{\delta}(\boldsymbol{s})$

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left\{ \max_{\boldsymbol{\delta} \in \Delta} \sqrt{\mathbb{E}_{\mathrm{n}}\left[|\widehat{w} - (\widehat{v}\boldsymbol{\phi}(\boldsymbol{s}) + \boldsymbol{\delta})^{\top}\boldsymbol{\theta}|^2 \right]} \right\} \quad (4)$$

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  where

$$\Delta = \left\{ \boldsymbol{\delta} : \mathbb{E}_{\mathrm{n}}\left[|\delta_k|^2\right] \leq n^{-1}\, \mathbb{E}_{\mathrm{n}}\left[|\widehat{v}\phi_k(\boldsymbol{s})|^2\right], \forall k \right\}$$

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- (4) is a convex problem and can be solved using coordinate descent.

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Real data

- y : Number of crimes, z : number of poor families across states $\boldsymbol{s} = \{1, \ldots, 50\}$

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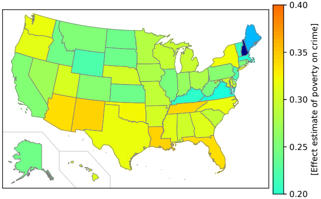
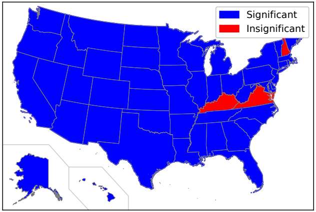


(a) Estimate $\widehat{\tau}(\boldsymbol{s})$



(b) Significance at 5% level

- Results consistent with previous findings [2]

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Conclusion

- We propose an orthogonalization-based strategy for estimating heterogeneous effects from spatial data in presence of spatially varying confounding variables
- Our proposed method is robust to errors-in-variables
- Visit poster $\# \ 80$ at Pacific Ballroom $6.30pm - 9pm$

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References

Chernozukhov et al., *Double machine learning for treatment and causal parameters*, cemmap working paper, Centre for Microdata Methods and Practice, 2016.

Ellis et al., *Crime, delinquency, and social status: A reconsideration*, Journal of Offender Rehabilitation, 2001.

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