On Scalable and Efficient Computation of Large Scale Optimal Transport

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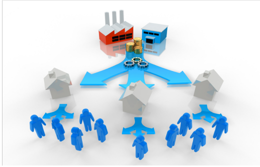
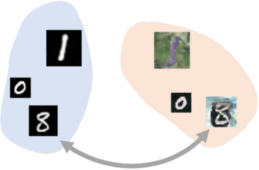
Optimal Tranport (OT)

The OT problem aims to **align** data from multiple sources.

Resource Allocation: We want to assign a set of assets to a set of receivers so that an optimal economic benefit is achieved.



Domain Adaptaion: We collect multiple datasets from different domains, and we need to learn a model from a source dataset, which can be further adapted to target datasets.

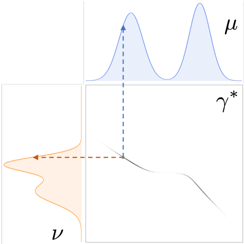


Both applications can be formulated as OT problems.

Optimal Tranport

Formulation OT aims to find an optimal joint distribution γ^* of μ and ν , which minimizes the expectation on some cost function c, i.e.,

$$\gamma^* = \arg\min_{\gamma} \mathbb{E}_{(X,Y)\sim\gamma}[c(X,Y)],$$
$$\text{subject to} \quad X \sim \mu, \quad Y \sim \nu.$$



γ^* is referred as the **optimal transport plan**, suggesting the way to transport between μ and ν with minimum cost.

Existing Methods Discretization + Linear Programming
The number of grids needs to **scale exponentially** w.r.t. dimension

SPOT

- OT: $\gamma^* = \arg\min_\gamma \mathbb{E}_{(X,Y)\sim\gamma}[c(X,Y)], \quad \text{s.t.} \quad \mathrm{X} \sim \mu, \mathrm{Y} \sim \nu.$
- Approximate γ^* by an implicit generative model $G(Z)$,

$$G(Z) = \left[\frac{G_X(Z)}{G_Y(Z)}\right] \approx \left[\frac{X}{Y}\right],$$

where $\quad Z \sim \rho, X \sim \mu, Y \sim \nu.$

- Substitute $G(Z)$ into OT problem, we can rewrite the problem as

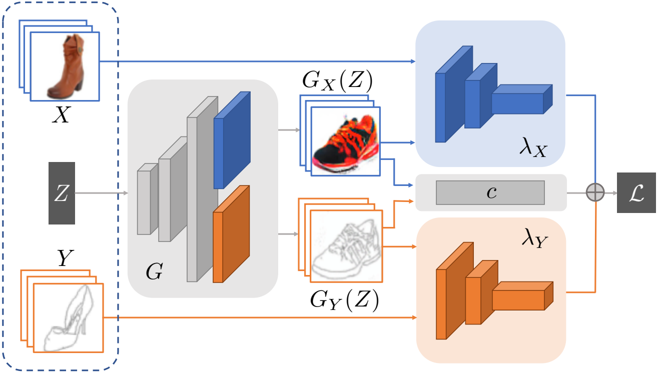
$$\underset{G}{\arg\min} \;\; \mathbb{E}_{Z\sim\rho}[c(G_X(Z), G_Y(Z))],$$

$$\text{subject to} \;\; \mathcal{W}_1(G_X(Z), \mu) = 0, \;\; \mathcal{W}_1(G_Y(Z), \nu) = 0.$$

where $\mathcal{W}_1(G_X(Z), \mu)$ denotes the standard Wasserstein metric between a random vector $G_X(Z)$ and a distribution μ. Here we use the fact that $\mathcal{W}_1(G_X(Z), \mu) = 0$ indicates $G_X(Z) \sim \mu$.

SPOT

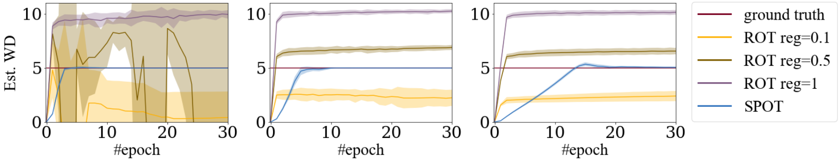
$$\min_{G \in \mathcal{G}} \max_{\lambda_X \in \mathcal{F}_X^1, \lambda_Y \in \mathcal{F}_Y^1} \mathbb{E}_{Z \sim \rho}[c(G_X(Z), G_Y(Z))] + \eta\big(\lambda_X(G_X(Z), X) + \lambda_Y(G_Y(Z), Y)\big),$$



Computing Wasserstein Distance (WD)

WD is the expected cost of optimal transport plan,

$$\mathcal{W} = \mathbb{E}_{(X,Y)\sim\gamma^*}[c(X,Y)].$$



LR $=10^{-3}$ LR $=10^{-4}$ LR $=10^{-5}$

Here, ROT is the state-of-the-art method (Seguy, 2018).

Generate Paired Samples

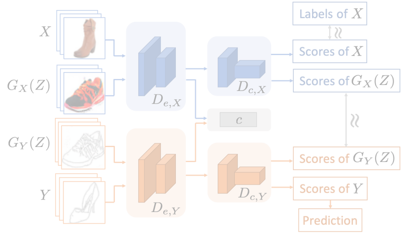
Photos-Monet

Domain Adaptation (DA)

Setting:

Goal: predict the labels of $\{y_j\}$.

New DA method – **DASPOT**



Source Target	MNIST USPS	USPS MNIST	SVHN MNIST	MNIST MNISTM
ROT (Seguy, 2018)	72.6%	60.5%	62.9%	–
StochJDOT (Damodaran, 2018)	93.6%	90.5%	67.6%	66.7%
DeepJDOT (Damodaran, 2018)	95.7%	96.4%	**96.7%**	92.4%
DASPOT	**97.5%**	**96.5%**	96.2%	**94.9%**

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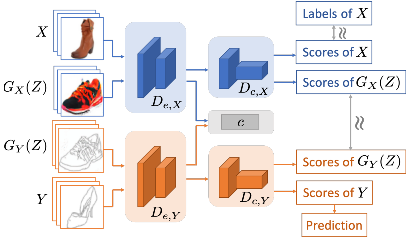

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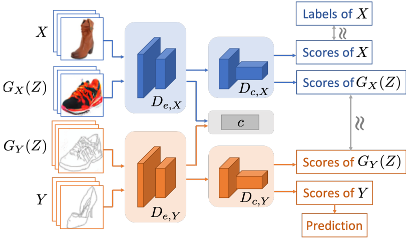

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Thank you!