Distributed Weighted Matching via Randomized Composable Coresets

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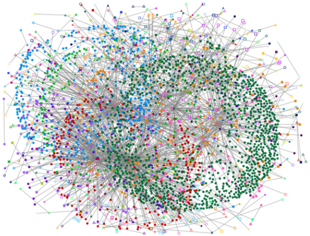
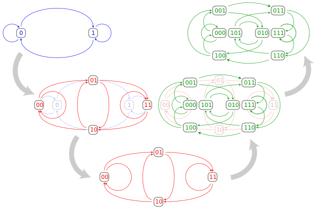
Massive graphs

Everywhere: web graph, social networks, biological networks, etc.





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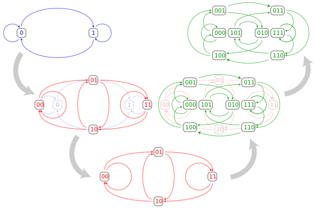
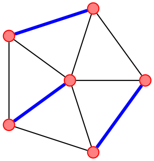


Matching: a collection of vertex-disjoint edges

- Clustering, partitioning
- Finding motifs in bioinformatics
- Trade marketing, online advertisement
- Kidney exchange
- Linear algebra, matrix decomposition



Sequential results

Variants:

- Weighted vs unweighted
- Bipartite vs non-bipartite

Algorithmic results:

- First polytime algorithm: "Blossom" decomposition [Edm65a]
- Extended to weighted case [Edm65b]
- Fastest in time $\tilde{O}(m\sqrt{n})$ [GabTar91]

Approximation algorithms:

- Greedy algorithm: 2-approx in $O(m \log n)$ time
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MapReduce: *de facto* industry standard

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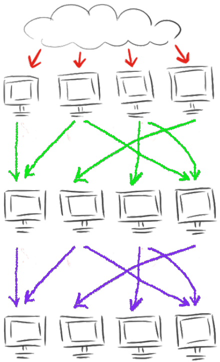
- Split computation into several rounds
- Data is sent to machines based on keys
- Machines produce output for next round



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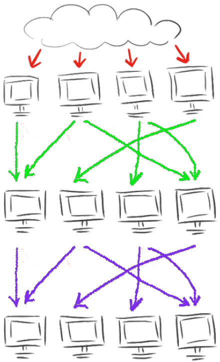

Important:

1. number of rounds
2. number of machines
3. memory on each machine
4. amount of computation on each machine

> "Greedy algorithms are practitioners' best friends—they are intuitive, simple to implement, and often lead to very good solutions. However, implementing greedy algorithms in a distributed setting is challenging since the greedy choice is inherently sequential, and it is not clear how to take advantage of the extra processing power."
>
> — [KumMosVasVat13]

Two typical types of results:

1. Relatively large number of rounds to "faithfully" simulate the greedy algorithm.
2. Very small number of rounds (one or two) for "weak" simulation, $O(1)$ worse than the greedy algorithm.

Credit	Approx	Space	Rounds
[LatMosSurVas11]	8	$\tilde{O}(n)$	$O(\log n)$
[CroStu14]	4	$\tilde{O}(n)$	$O(\log n)$
[AhnGuh15]	$1+\varepsilon$	$\tilde{O}(n)$	$O(\varepsilon^{-1}\log n)$
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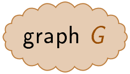
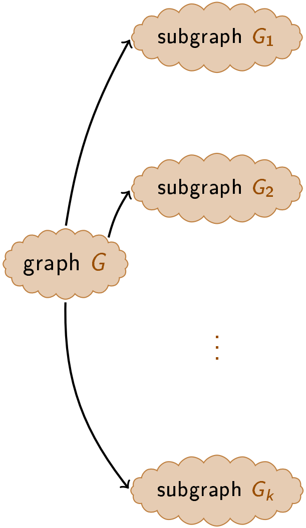
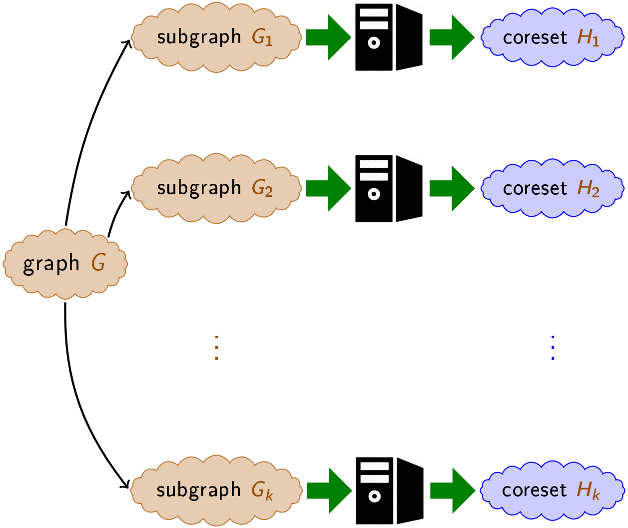

graph G


subgraph G_1
subgraph G_2
graph G
$\vdots$
subgraph G_k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coreset H_1
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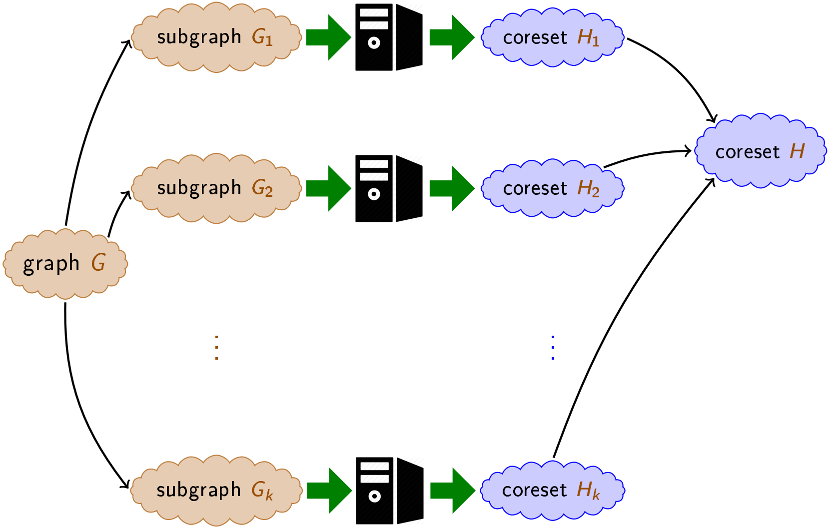

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⋮
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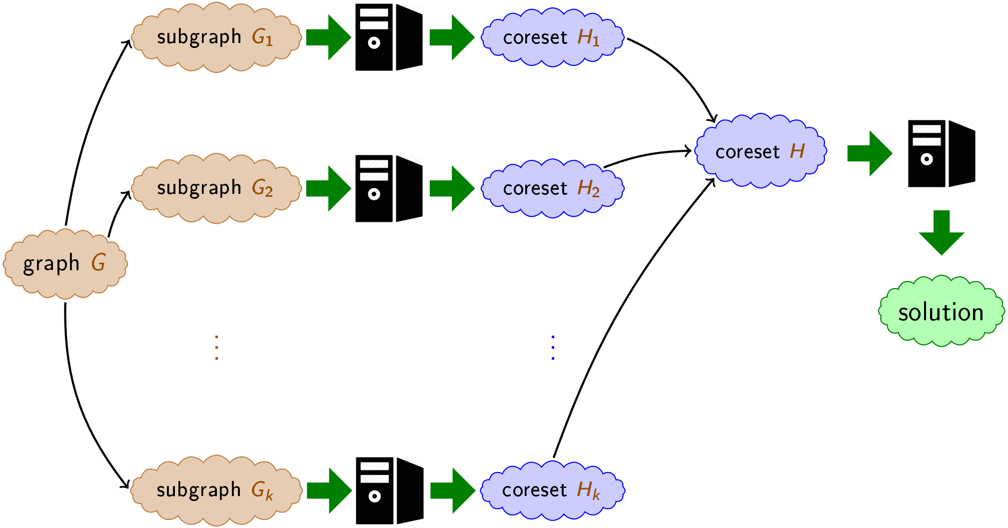

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subgraph G_1
subgraph G_2
subgraph G_k
coreset H_1
coreset H_2
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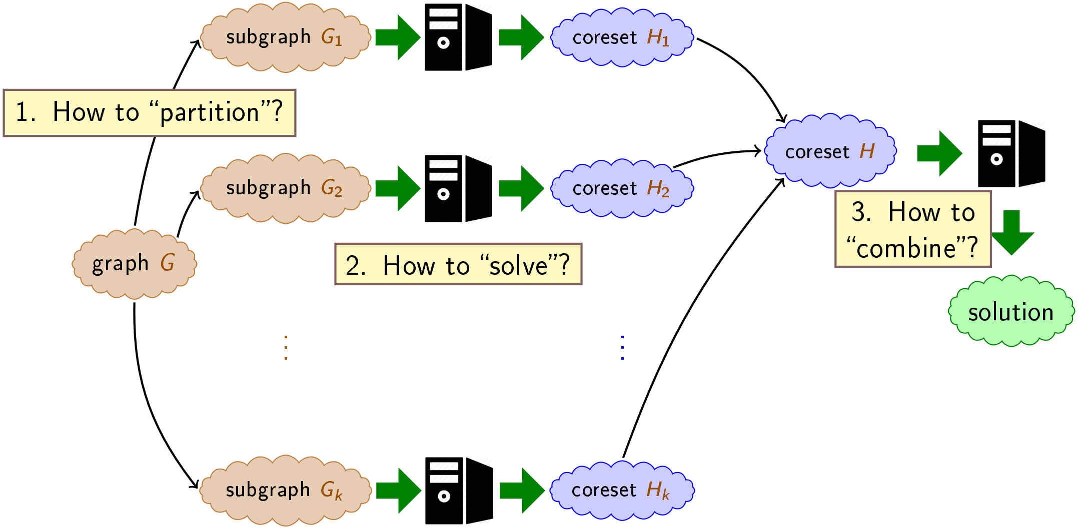
coreset H
solution

Composable coresets

Basic idea



Definition

- Let $G_1, \ldots, G_k$ be a partitioning of G; send each edge $e \in G$ to a subgraph G_i arbitrarily.
- Consider an algorithm ALG that given G_i outputs a subgraph H_i of G_i with s edges.
- ALG outputs an α-approx composable coreset of size s for a problem P iff $P(\mathrm{ALG}(G_1) \cup \ldots \cup \mathrm{ALG}(G_k))$ is an α-approx to $P(G)$.

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- $n^{o(1)}$ approx requires $n^{2-o(1)}$ space [AssKhaLiYar16].

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- Randomized coresets were introduced by [MirZad15] for submodular maximization.

Definition

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We present a simple $(2 + \epsilon)$-approximation randomized coreset with mulitplicity $\mu = O(\frac{1}{\epsilon} \log \frac{1}{\epsilon})$ for maximum-weight matching.

1. Send each edge to $\mu = O\left(\frac{\log 1/\epsilon}{\epsilon}\right)$ machines.
2. Sort edges according to weight on each machine (with consistent tie-breaking), and greedily find a maximal matching (the coresets).
3. Find a maximum matching M of the union of coresets H.

- This gives a $2 + \epsilon$ approx.
- Simple, scalable implementation except for Step 3.
  - Replacing Step 3 with a greedy maximal matching algorithm is provably a $3 + \epsilon$ approx (not a $4 + \epsilon$ approx).
- Better analysis uses "consistency" of maximal (vs. maximum) matching coresets.

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Proof idea

- **Machine** i works on subgraph G_i and contributes maximal matching M_i to coreset $H = \cup_i M_i$.
- Compare to a reference maximum-weight matching M^*.
- Let permutation π of edges be the consistent sorting order.
- Edge $e \in M^*$ is free on machine i iff its endpoints are available when e "arrives."
  - Otherwise $e \in M^*$ is blocked on machine i.
  - Maybe e is missing on machine i.

1. Subgraphs G_i have the same distribution; focus on G_1 and M_1.
2. A blocked edge has a heavier edge in M_i as its "certificate."
   - Use it in the charging argument for M_1.
3. Free edge part of G_i will make it to M_i and H.
   - Free edges in $M_2 \cup M_3 \cup \dots$ compensate the free edges in M_1.
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Dataset	$\lvert V \rvert$	$\lvert E \rvert$	Δ	Speed-up [AssKha17]	Speed-up This work
Friendster	66M	550B	2.1M	4.7x	130x
Orkut	3M	21B	900K	1.4x	17x
LiveJournal	4.8M	3.9B	444K	2.5x	6x
DBLP	1.5M	5.9M	1961	1.2x	3x

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Dataset				Quality	
	$\lvert V\rvert$	$\lvert E\rvert$	Δ	[AssKha17]	This work
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Orkut	3M	21B	900K	92.4%	98.9%
LiveJournal	4.8M	3.9B	444K	96.5%	99.9%
DBLP	1.5M	5.9M	1961	92.4%	99.6%

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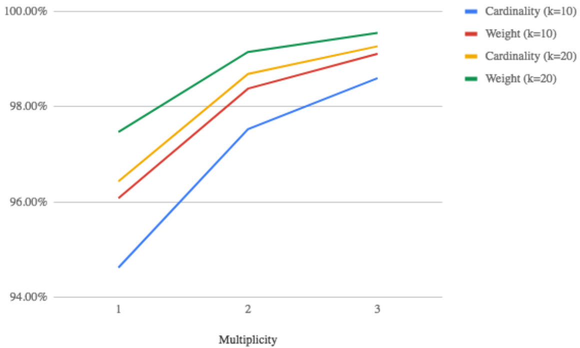
- Multiplicity 2 or 3 is sufficient



Summary

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    - Uses greedy algorithm for coreset construction.
    - Uses multiplicity.
- Good performance in practice.
    - Small multiplicity suffices.
    - Beats prior algorithms.
    - Huge speed-up of sequential algorithm, but faithful simulation.

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  - Small multiplicity suffices.
  - Beats prior algorithms.
  - Huge speed-up of sequential algorithm, but faithful simulation.

Thanks!

Poster #161
Pacific Ballroom

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