Band-limited Training and Inference for Convolutional Neural Networks

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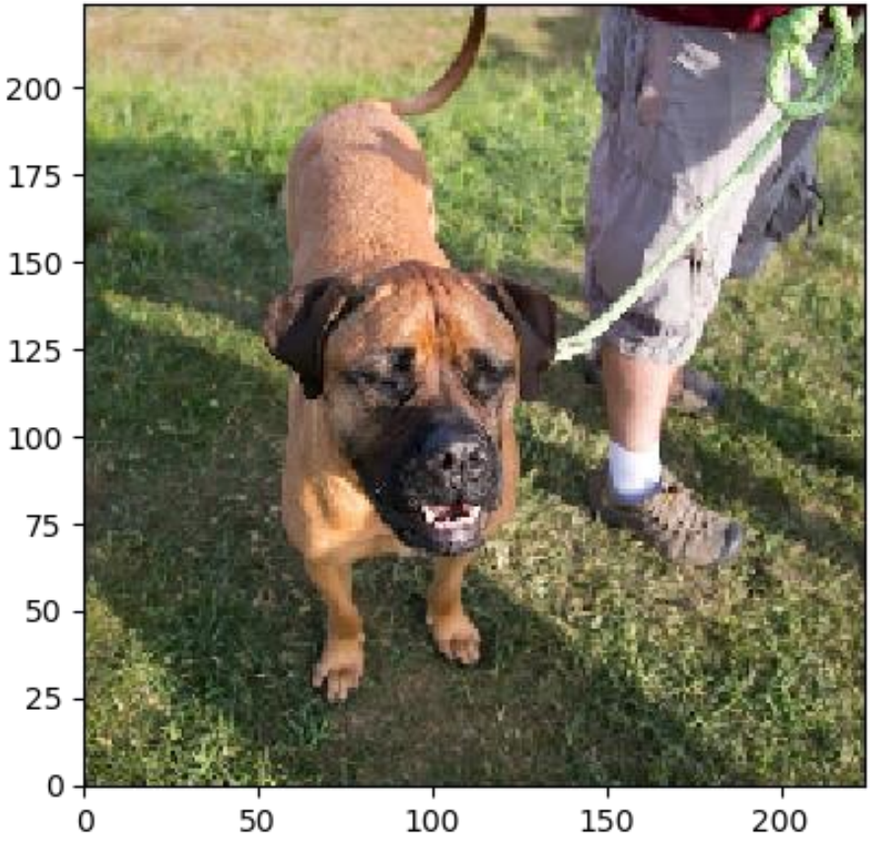
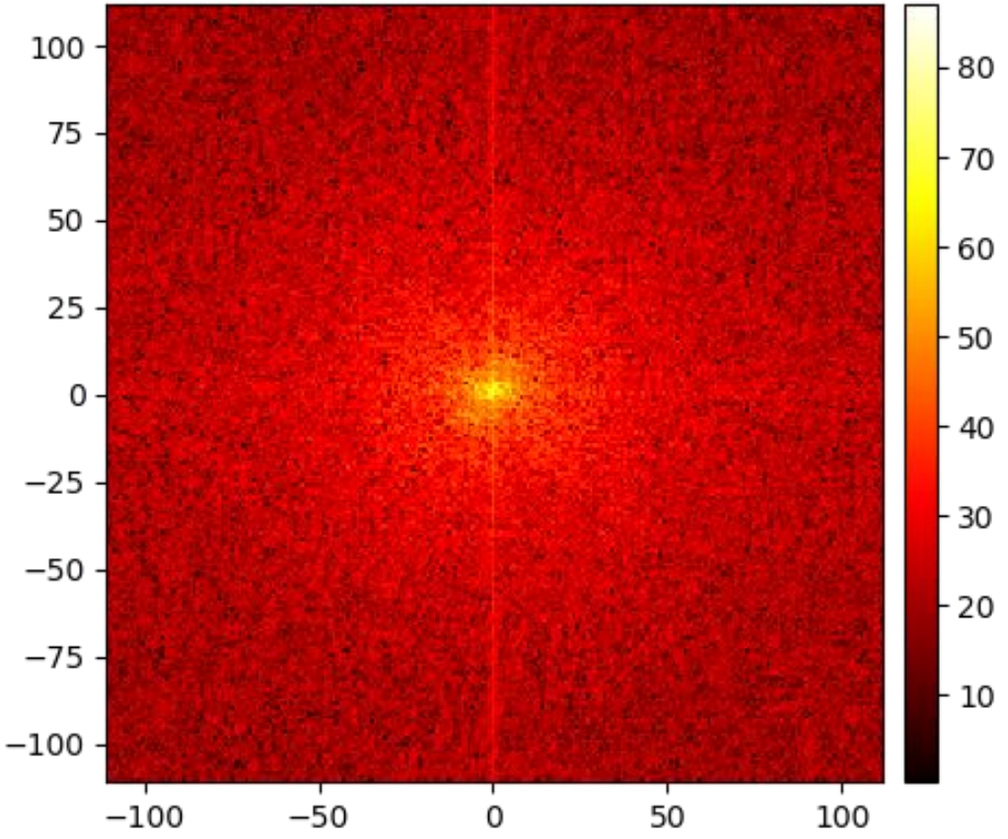
Natural images
More information put in lower frequencies

Original image



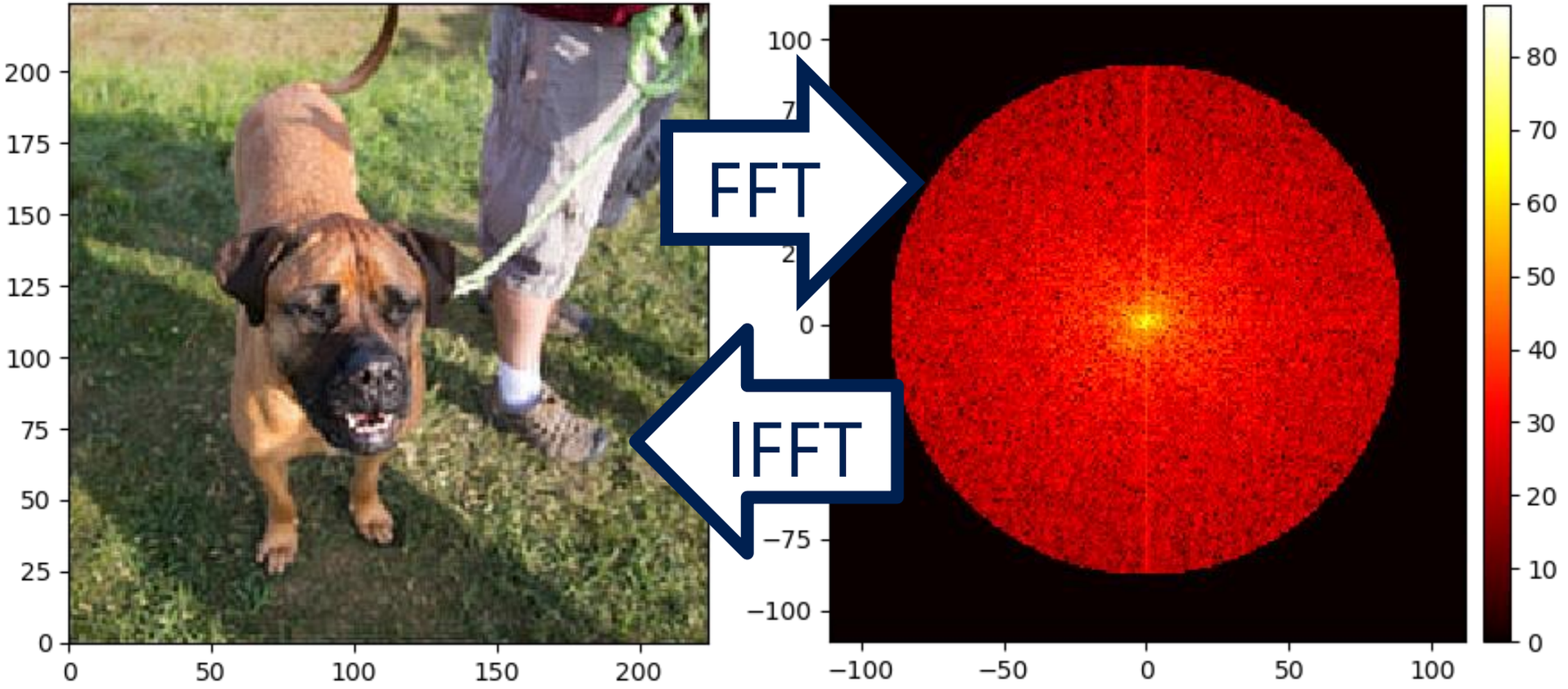
Spatial domain



Frequency domain

Natural images
Transformations between the domains

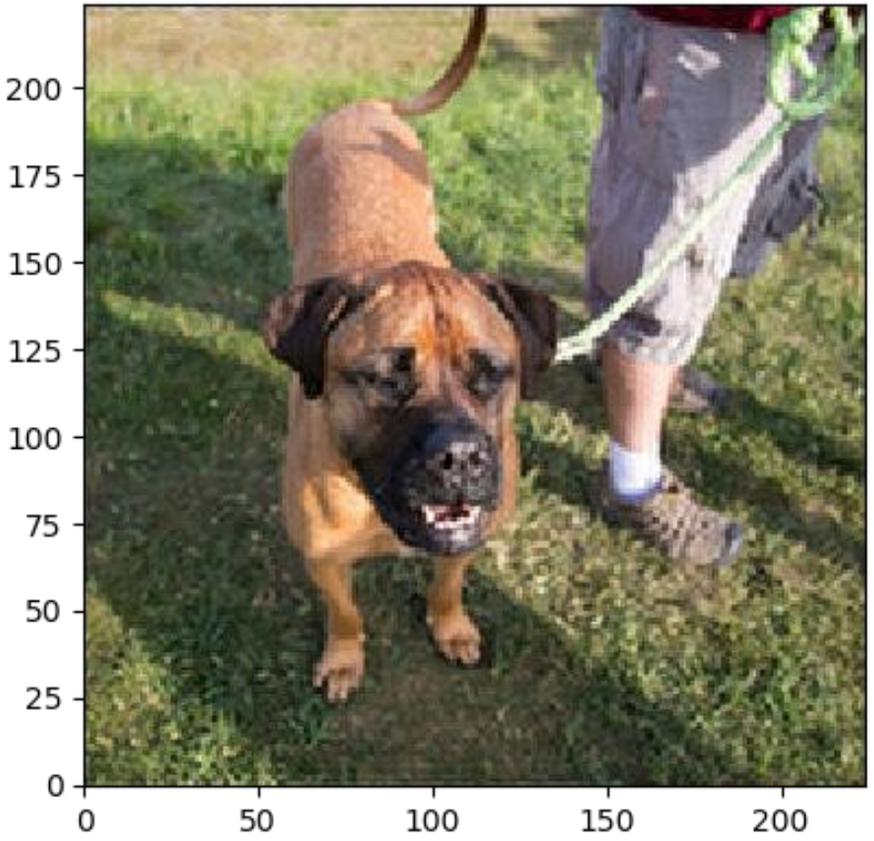
Compression 50%



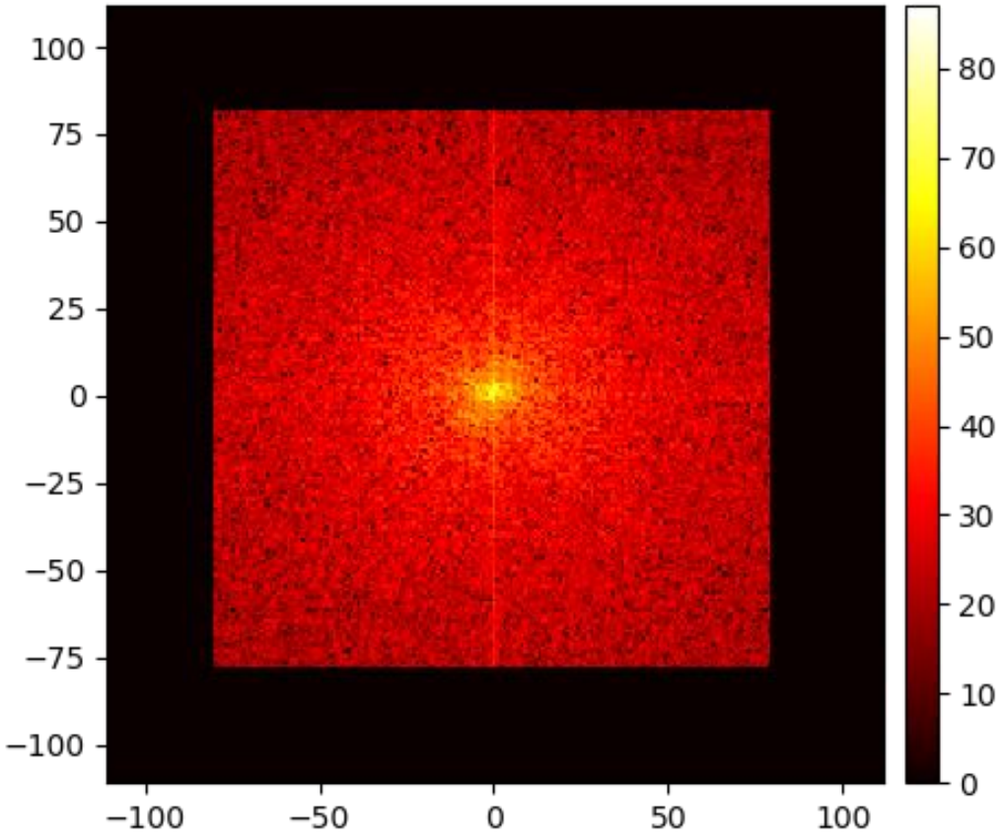
Spatial domain **Frequency domain**

Method for ConvNets to constrain the frequency band in convolution operation for efficiency

Compression 50% in practice



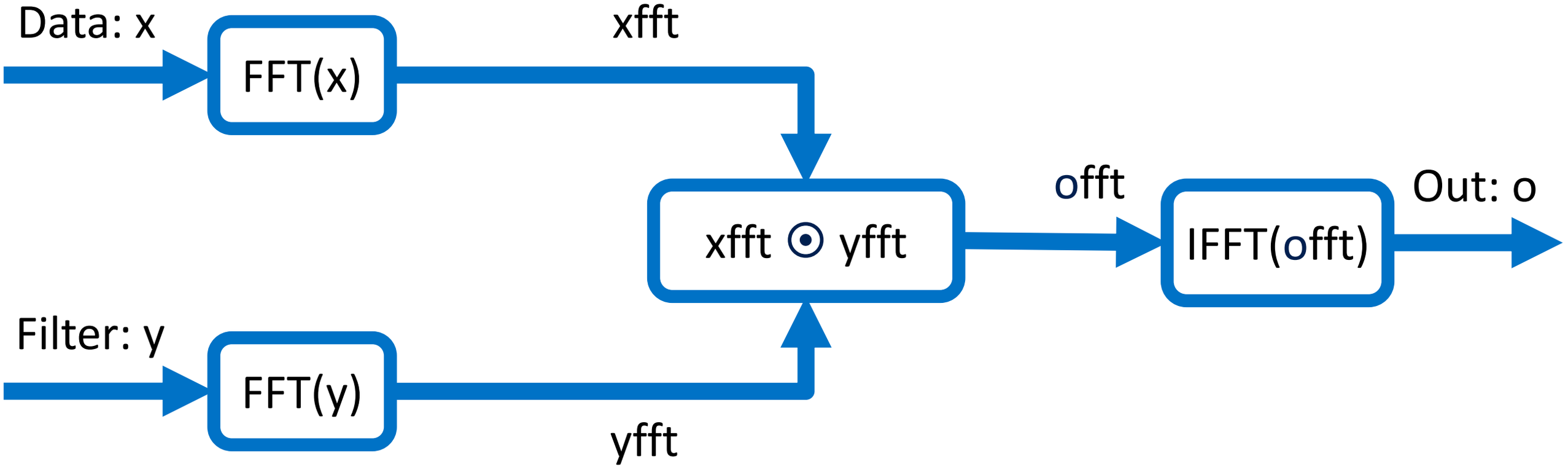
Spatial domain



Frequency domain

FFT based convolution

Mathieu et al.: "Fast Training of Convolutional Networks through FFTs"
Vasilache et al.: "Fast Convolutional Nets With fbfft: A GPU Performance Evaluation"



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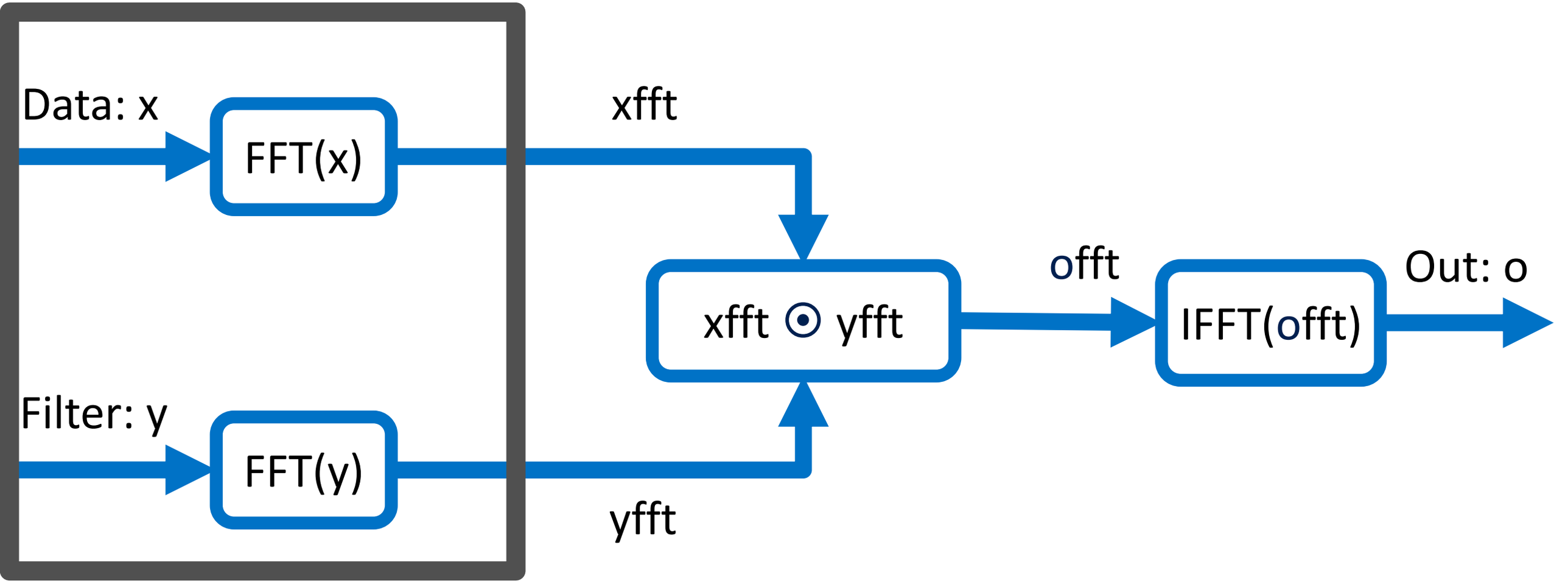

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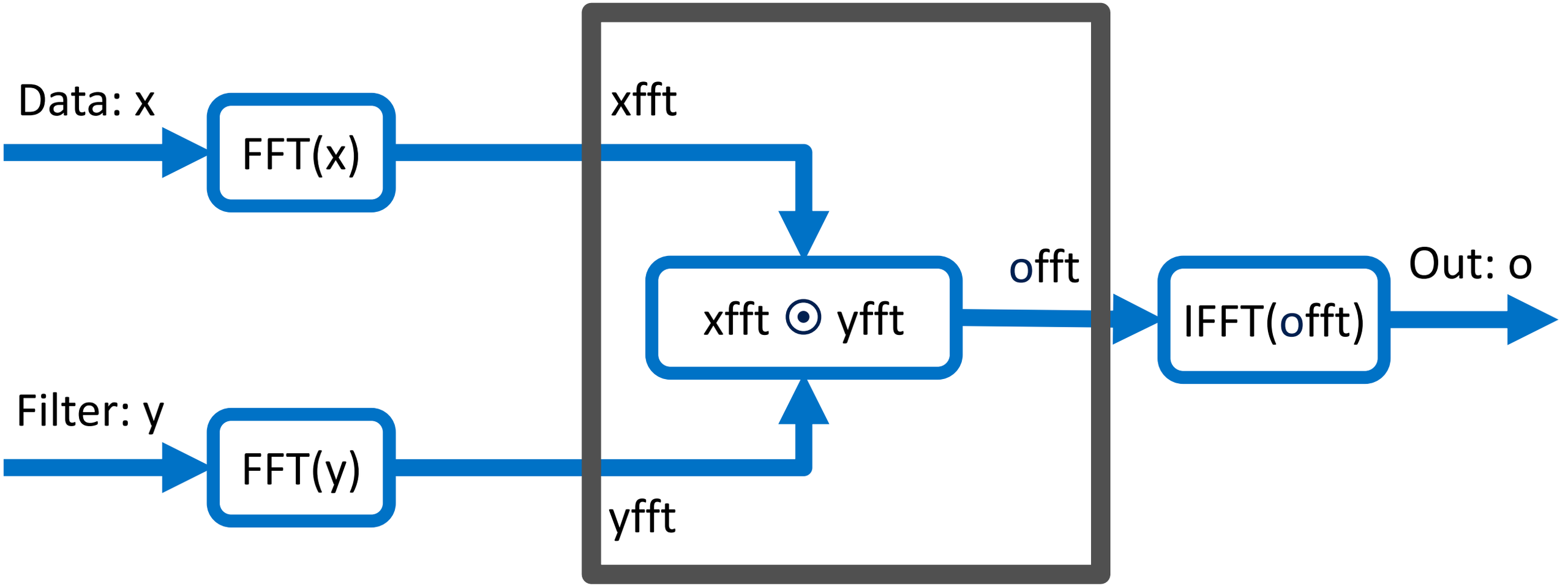

FFT based convolution

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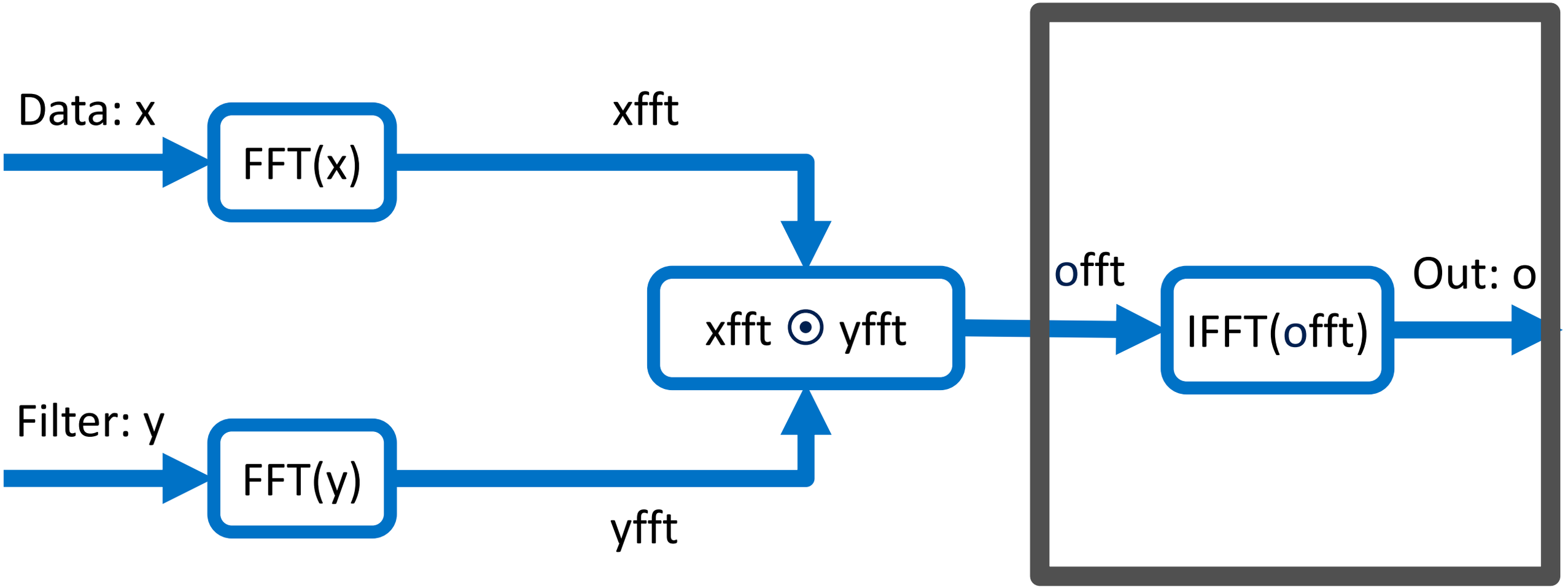

FFT based convolution

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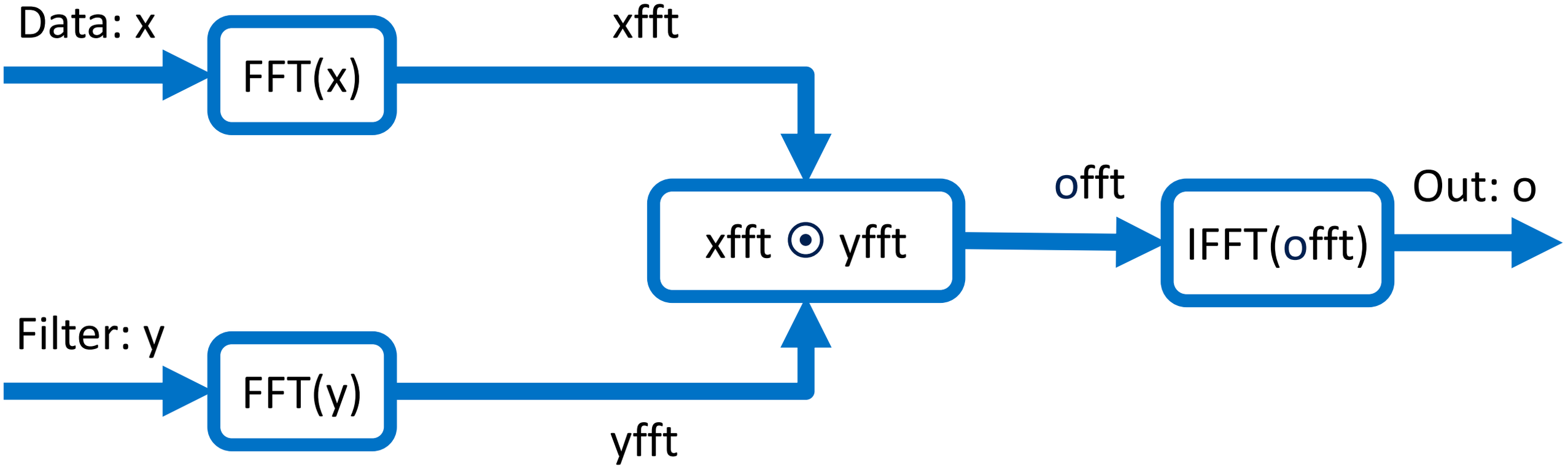
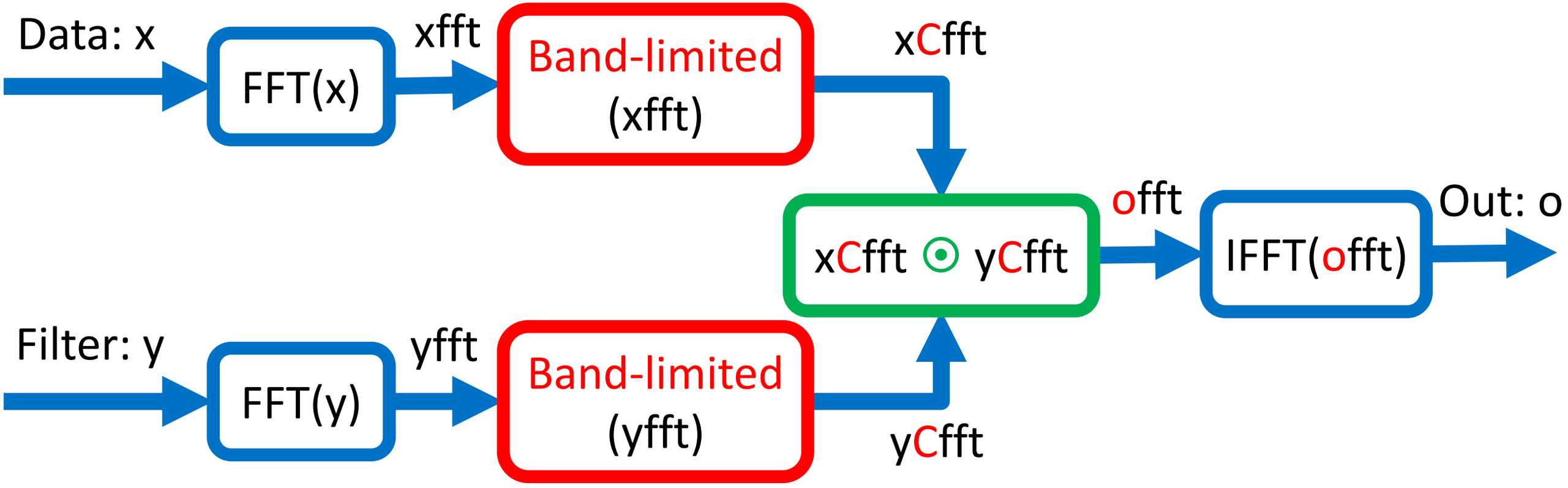
cuDNN: Substantial memory workspace needed for intermediate results.

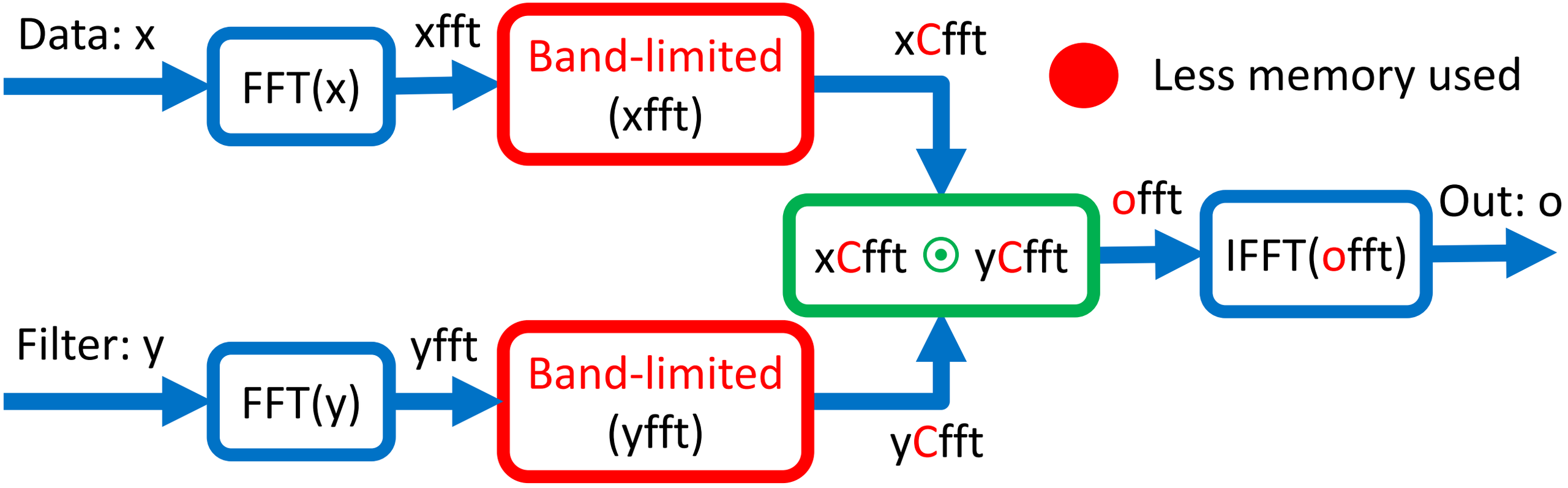


Band-limited FFT based convolution

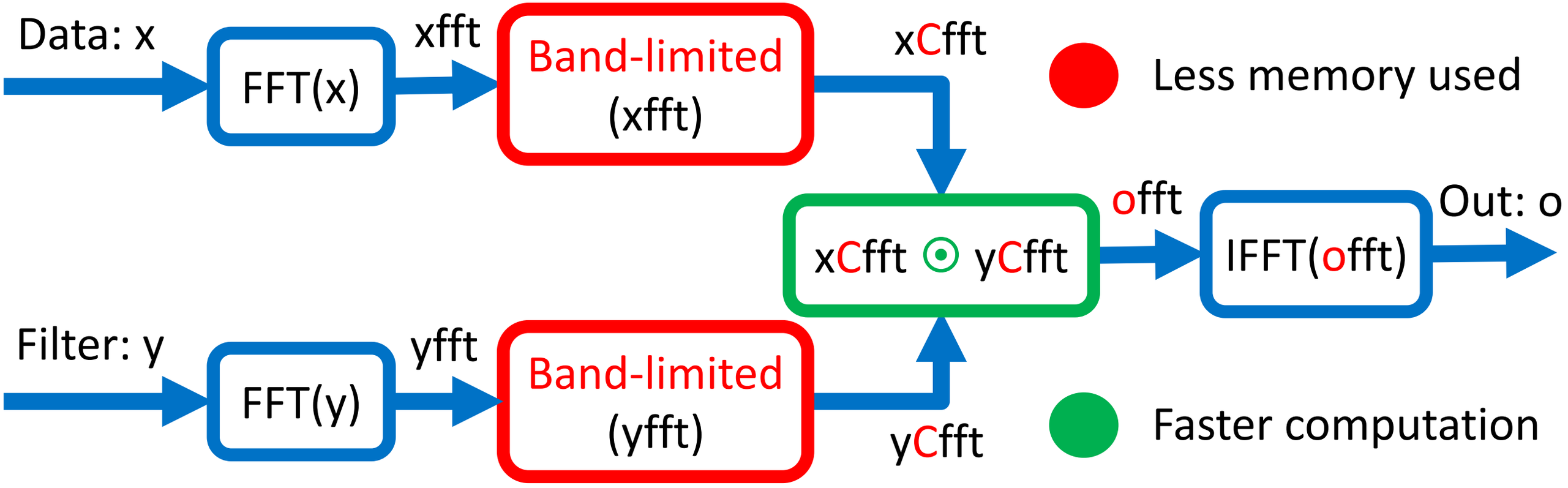
Band-limiting = masking out high frequencies



Band-limited FFT based convolution

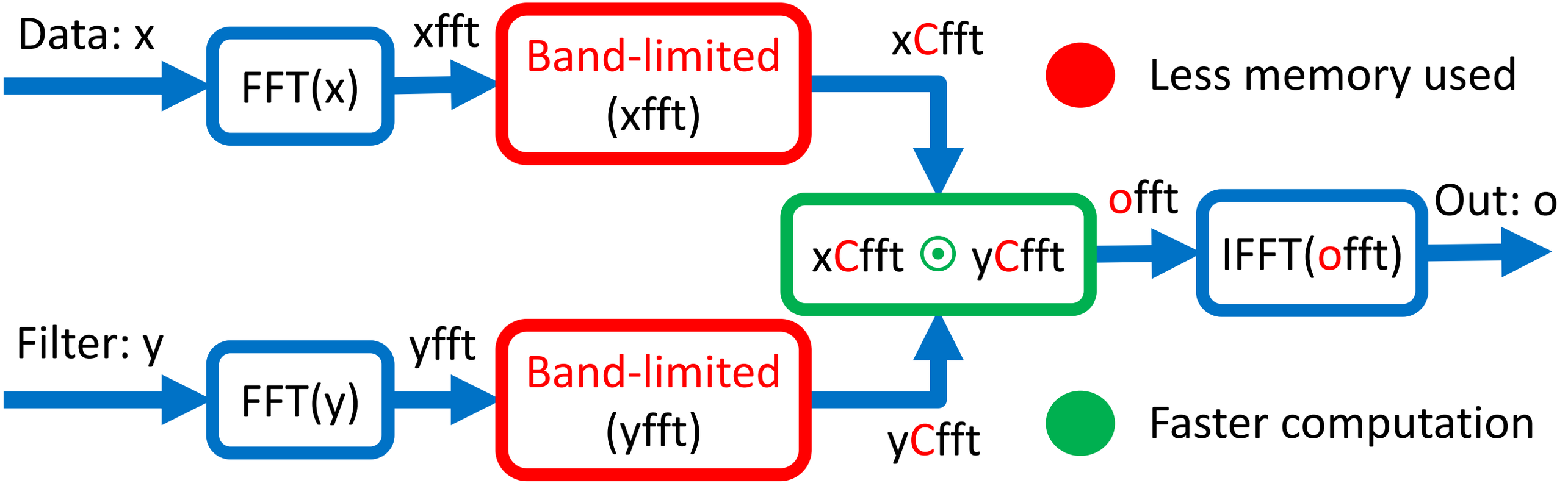


Band-limited FFT based convolution



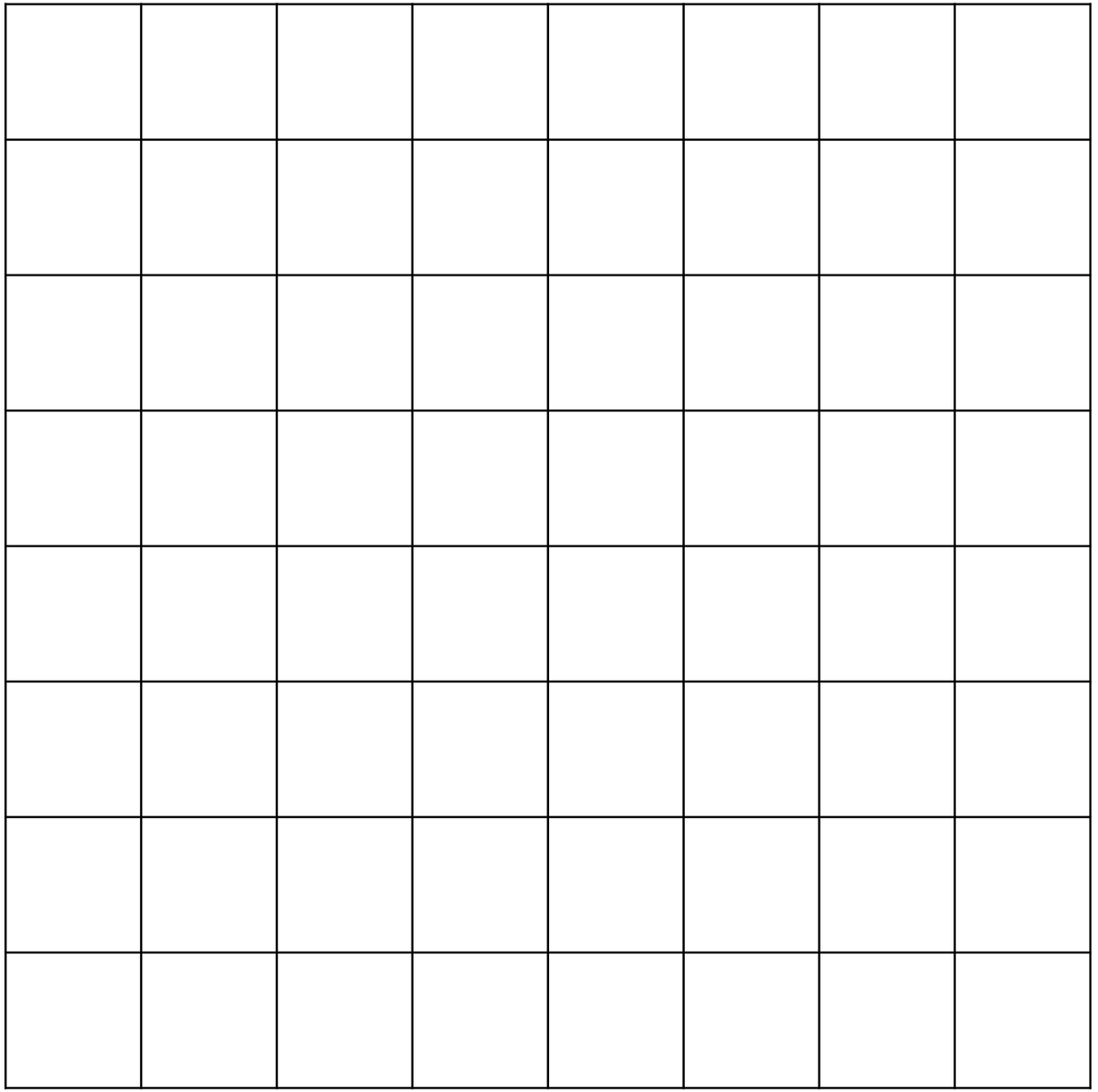
Band-limited FFT based convolution

Preserve enough of the spectrum to retain high accuracy of models.



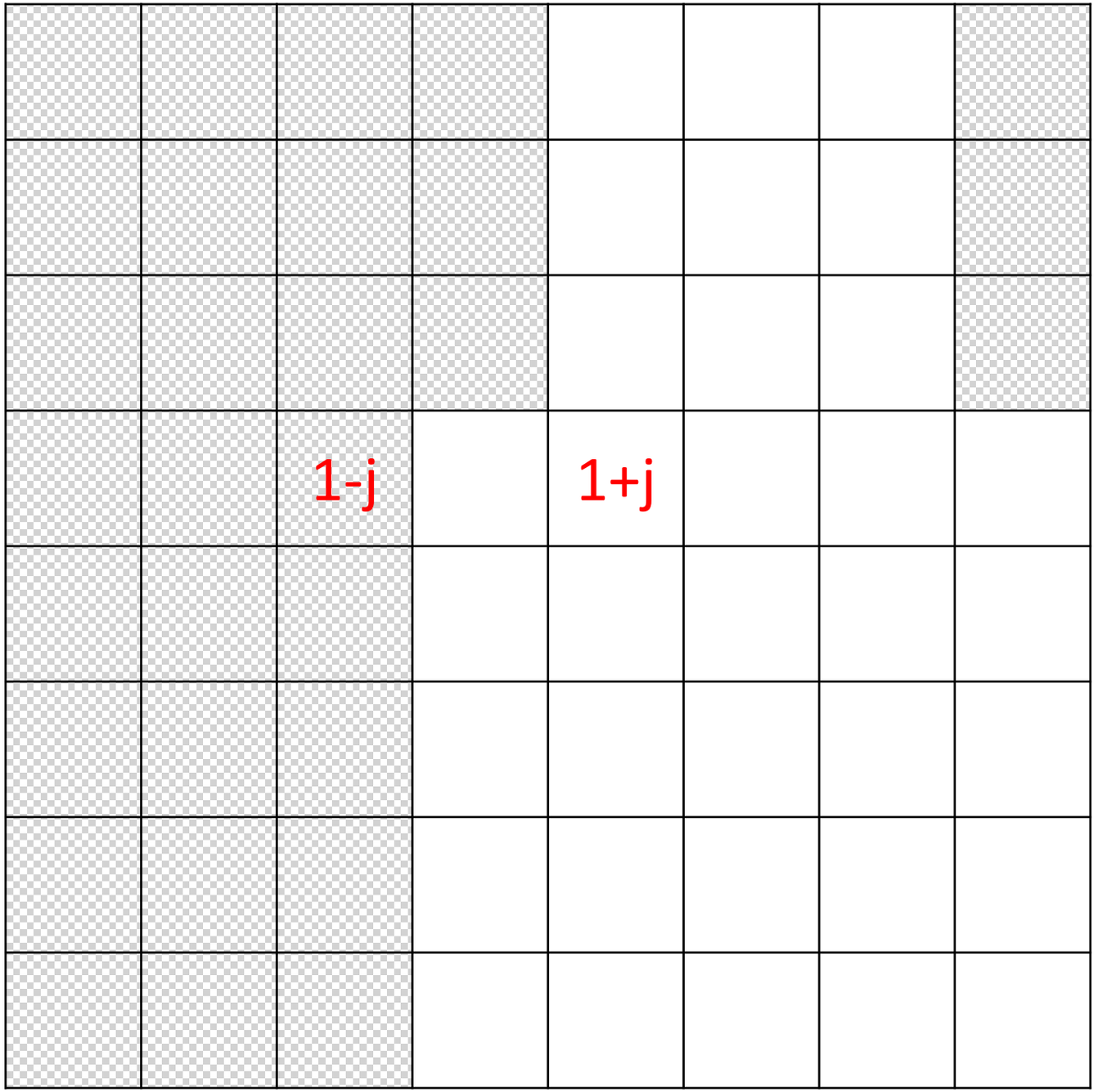
Band-limiting Technique

1. FFT of an input data



Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry



Band-limiting Technique

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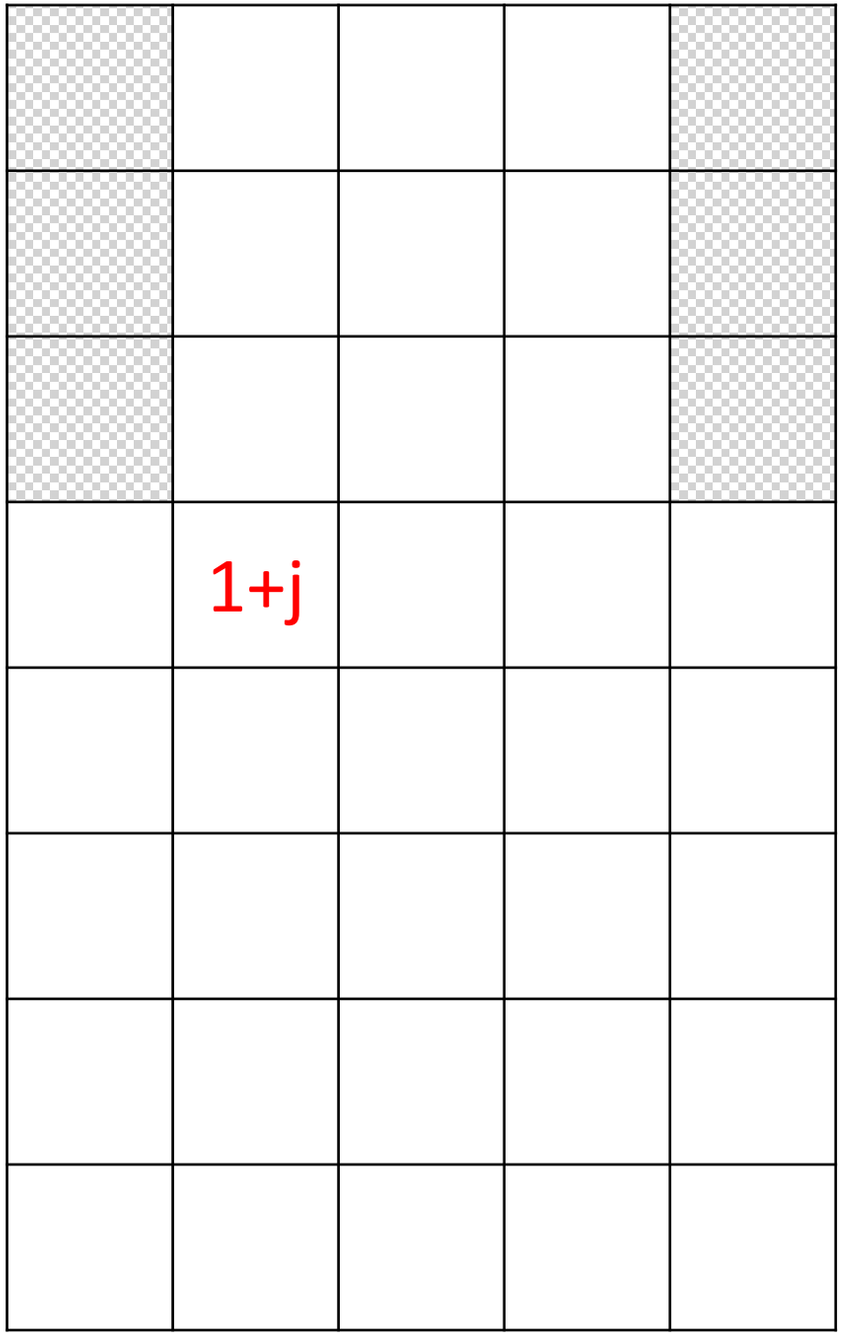
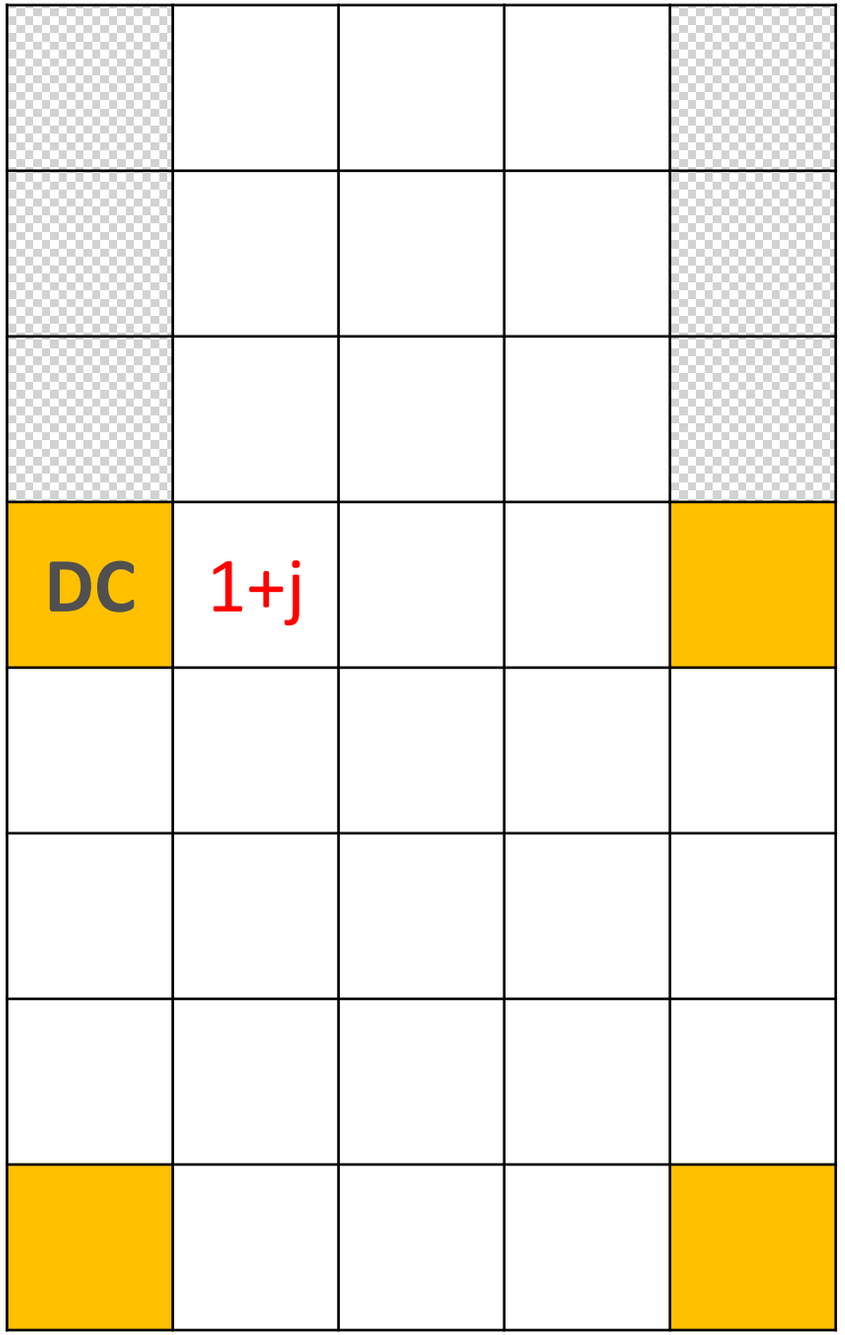


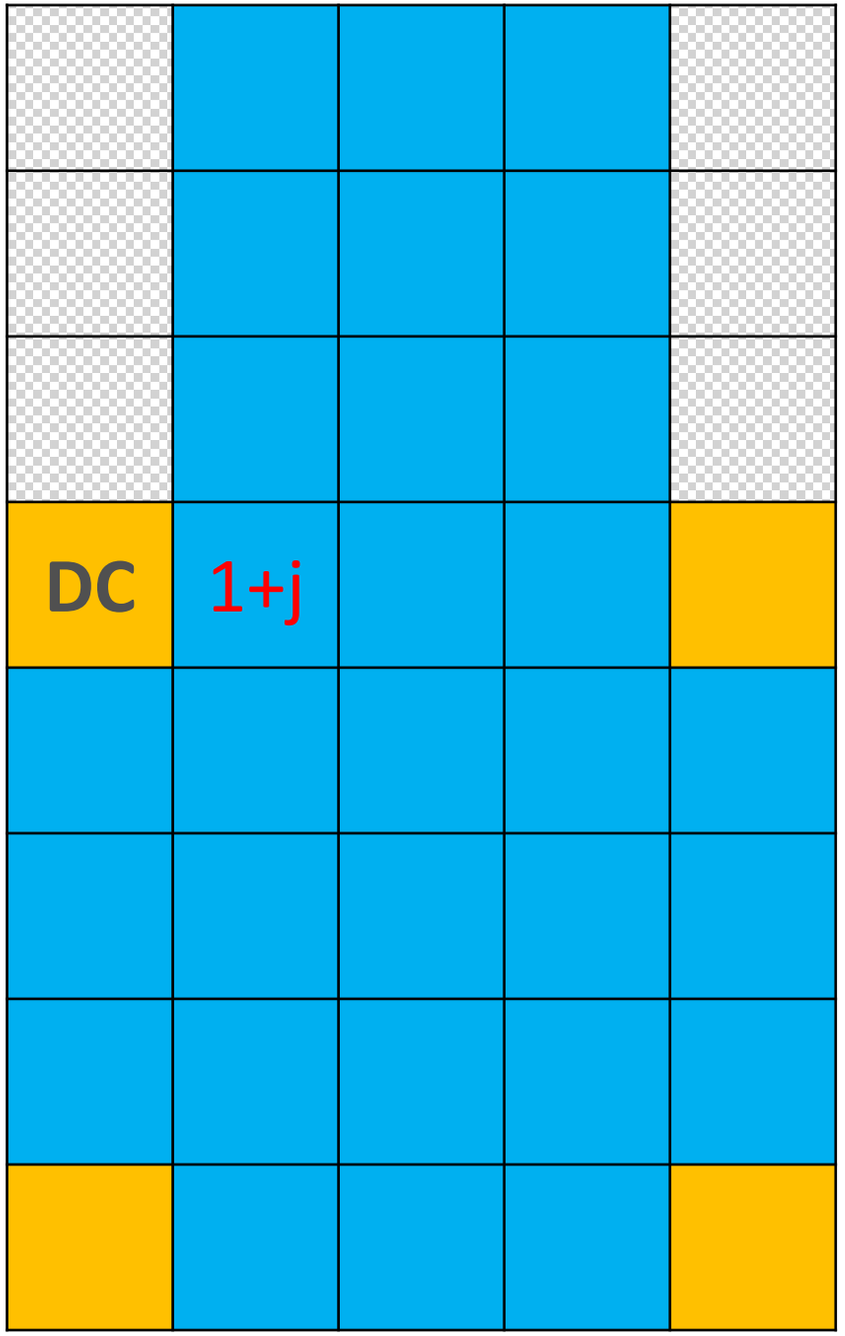
Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values



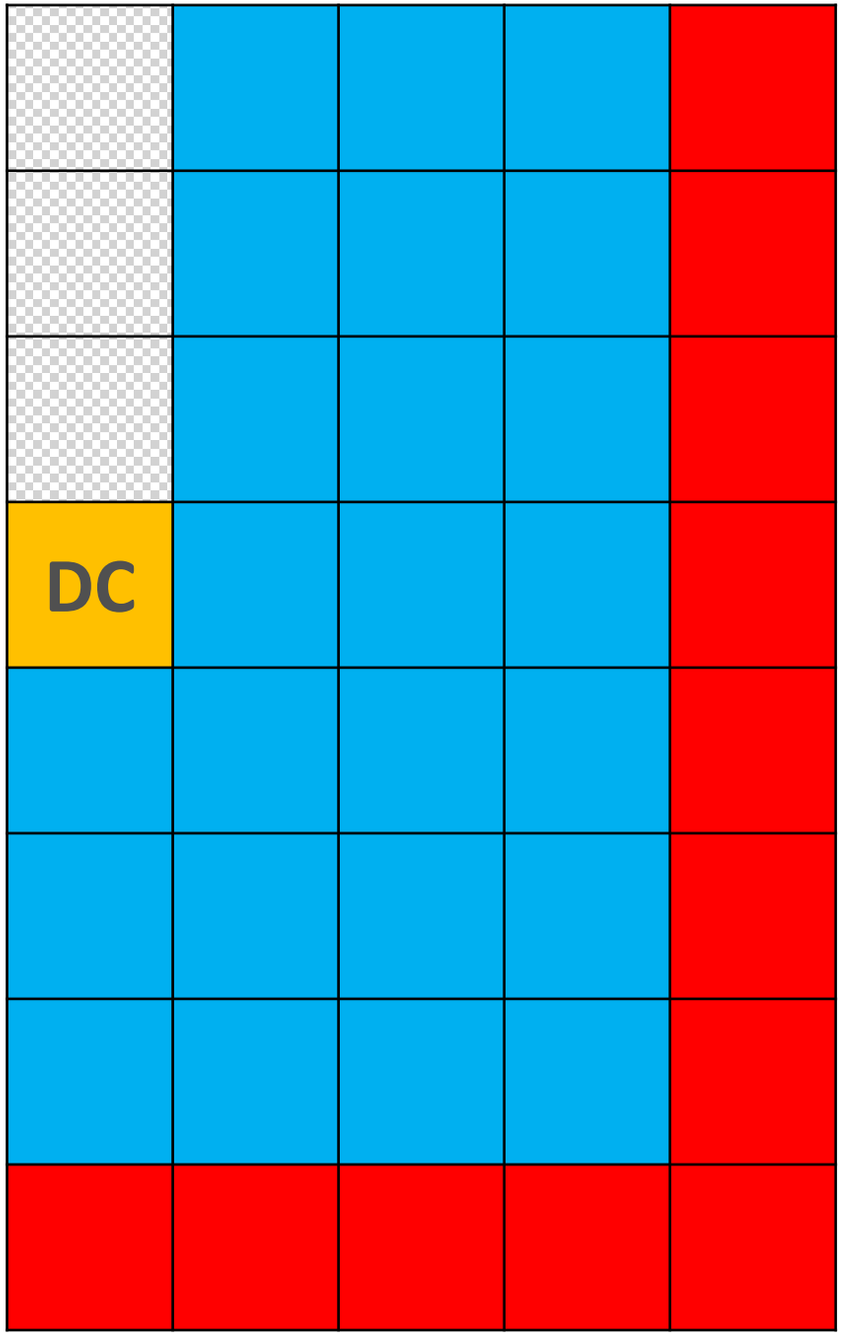
Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values
4. No constraints



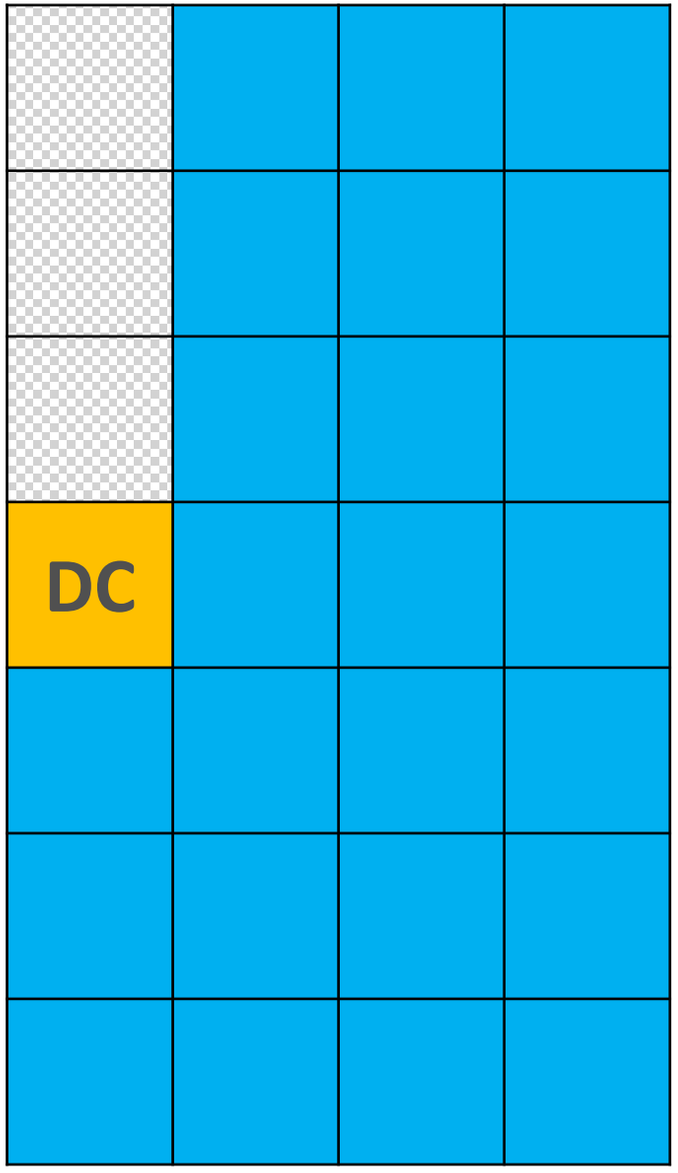
Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values
4. No constraints
5. 1st compression



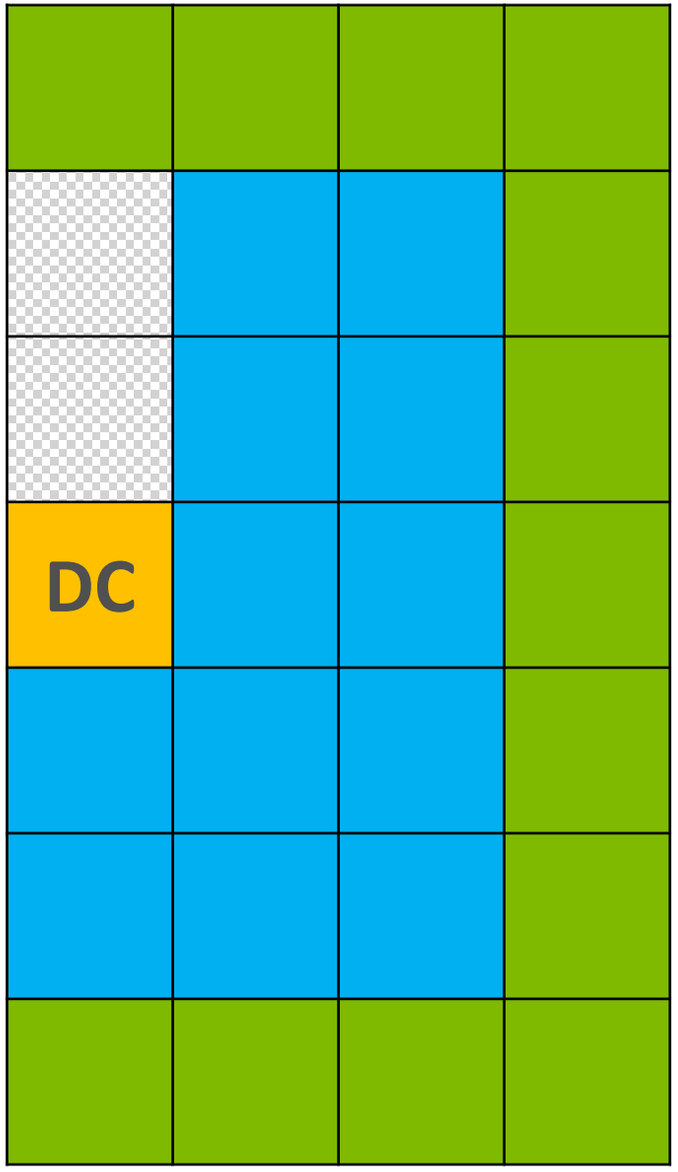
Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values
4. No constraints
5. 1st compression



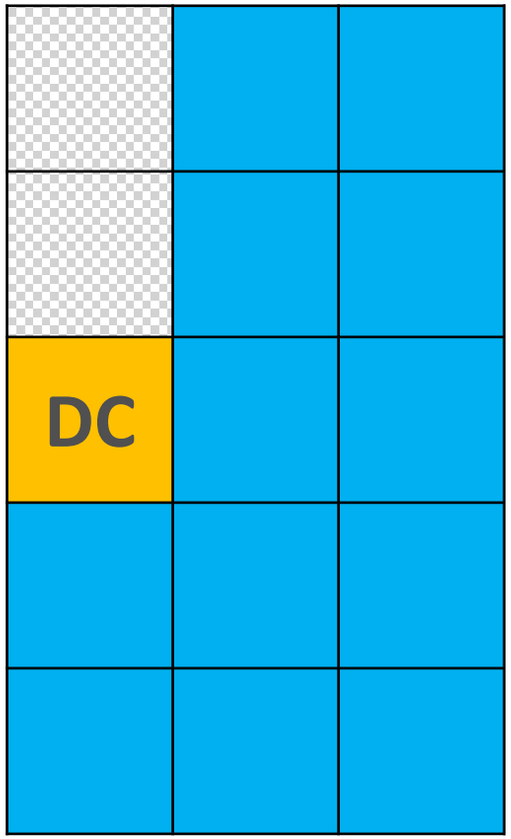
Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values
4. No constraints
5. 1st compression
6. 2nd compression

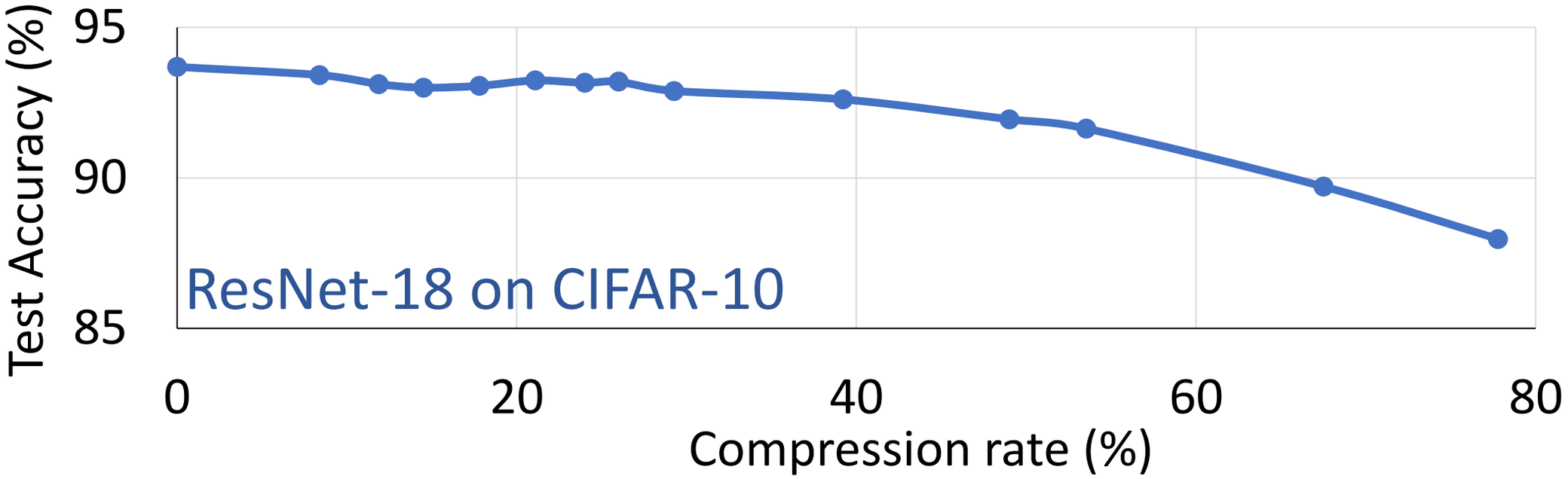


Band-limiting Technique

1. FFT of an input data
2. Conjugate symmetry
3. Real values
4. No constraints
5. 1st compression
6. 2nd compression



Effects of band-limiting on accuracy



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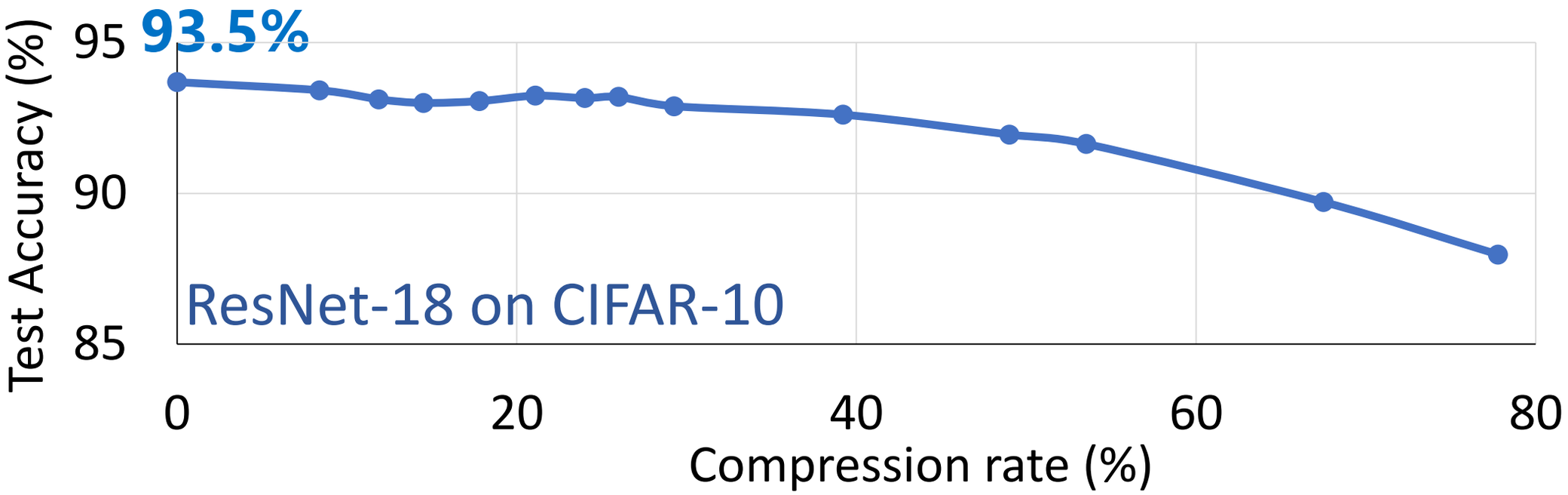
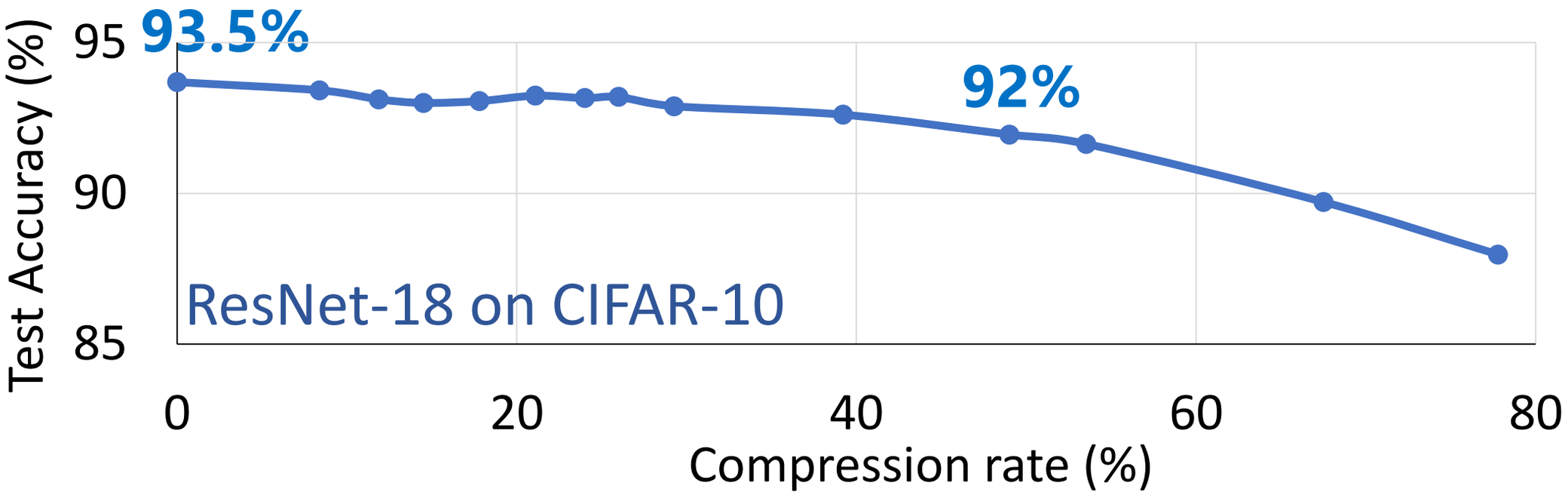
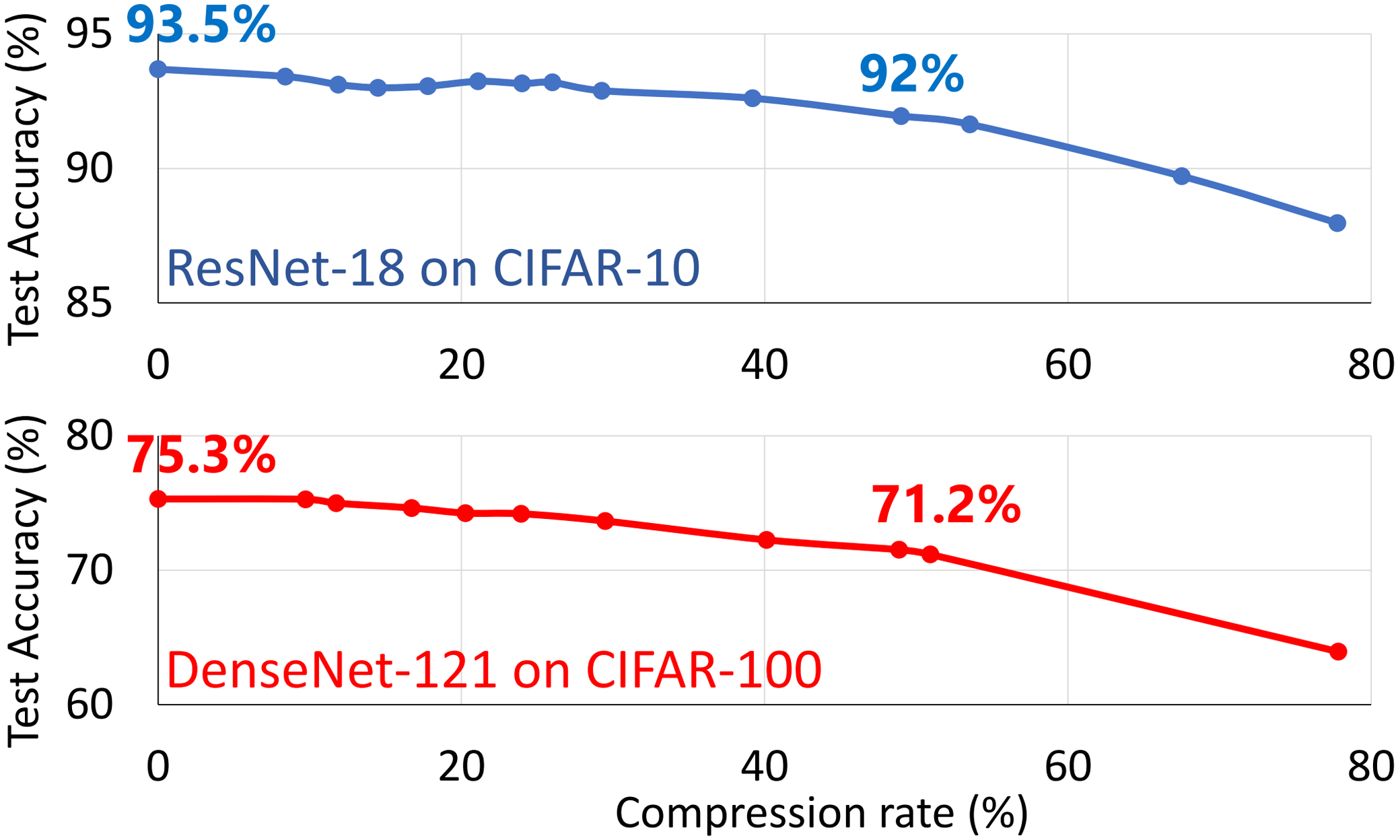


Effects of band-limiting on accuracy



Effects of band-limiting on accuracy



Main take-aways from Band-limited CNNs

- Method to constrain the frequency band in convolution.
- Models trained with band-limiting **gracefully degrade** the accuracy as the function of the compression rate.
- Effectively **control resource usage** (GPU/CPU and memory).
- The **low frequency** coefficients learned first during training.
- The **same compression rate** applied to training and inference.
- The more band-limited model, the more **robust to attacks**.
- Applicable to **other domains:** time-series & speech data.

Thank you

Poster: 6:30-9:00 PM @ Pacific Ballroom #132

github.com/adam-dziedzic/bandlimited-cnns

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Backup

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Why is FFT based convolution important?

- The theoretical properties of the Fourier domain are well understood. No such properties in other domains (Winograd).
- ResNet and DenseNet architectures use 7x7 filters in first layers.
- FFT based convolution can be combined with spectral pooling.
- Band-limiting in the first layer serves as a simple defense.
- A standard algorithm included in popular frameworks (cuDNN).
- Gradient acts as a large filter in the backward pass.
- Zlateski et al. suggest using FFT based convolution on CPUs.
- The 1D FFT convolution for DSP where large filters are used.

Band-limited FFT based convolution *formally*

Cross-correlate input data and filter: $x *_c y$

$$F_x[\omega] = F(x[n]) \qquad F_y[\omega] = F(y[n])$$

$$x *_c y = F^{-1}(F_x[\omega] \odot F_y[\omega])$$

Spectrum of convolution: $S[\omega] = F_x[\omega] \odot F_y[\omega]$

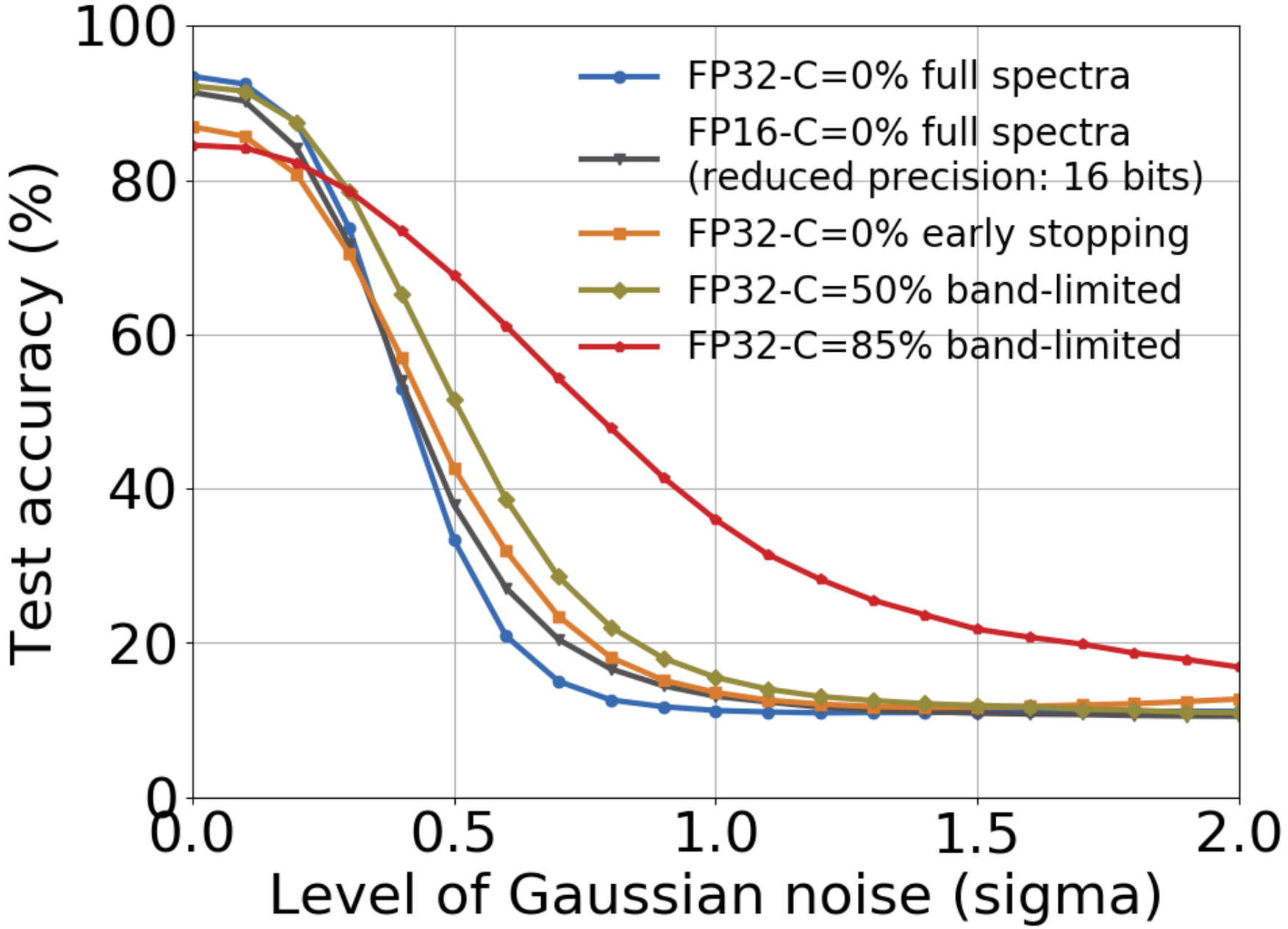
$$\mathbf{M_c[\omega]} = \begin{cases} \mathbf{1}, \boldsymbol{\omega} \leq \mathbf{c} \\ \mathbf{0}, \boldsymbol{\omega} > \mathbf{c} \end{cases}$$

$$x *_c y = F^{-1}[(F_x[\omega] \odot M_c[\omega]) \odot (F_y[\omega] \odot M_c[\omega])]$$

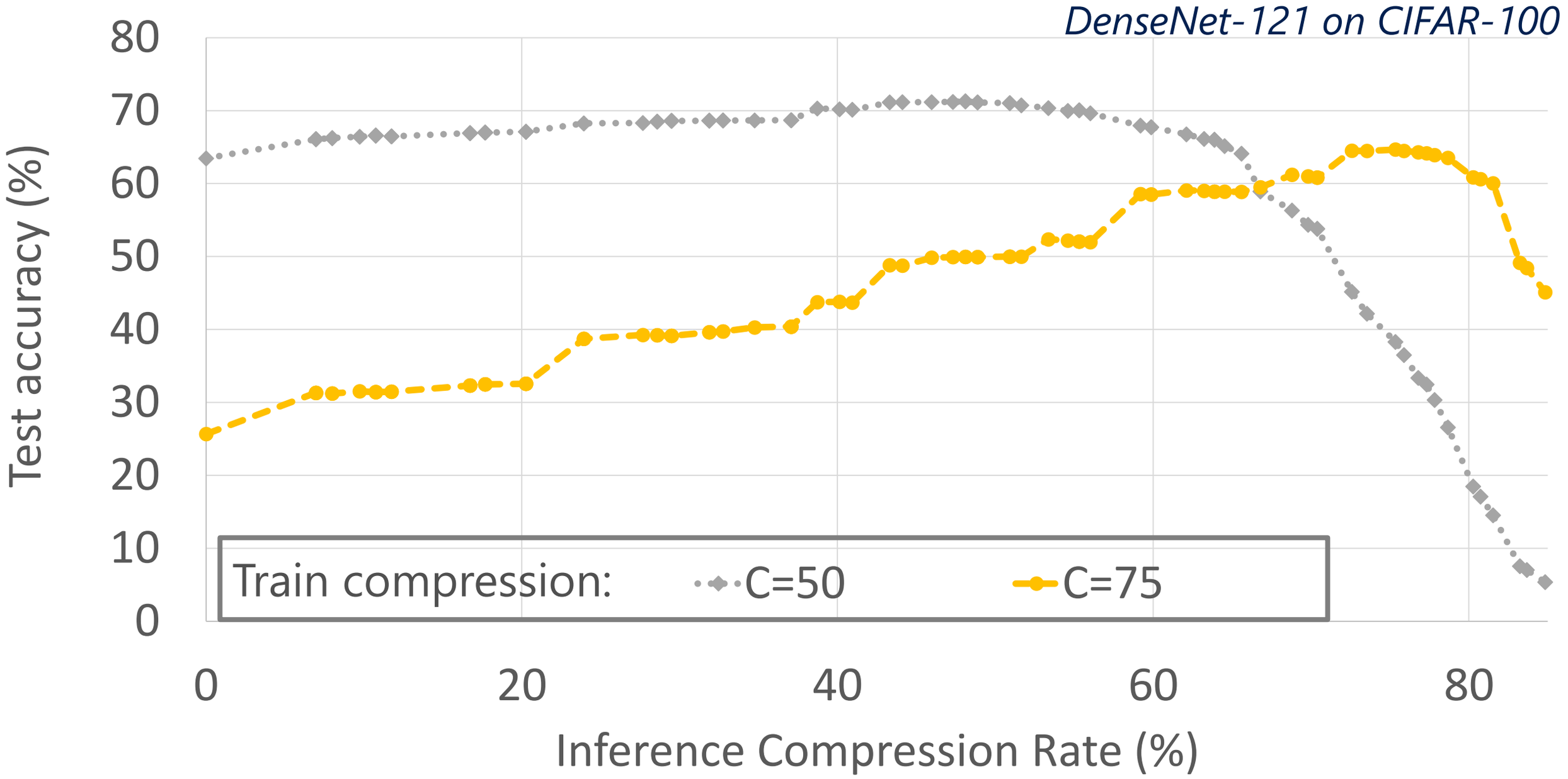
$$x *_c y = F^{-1}(S[\omega] \odot M_c[\omega])$$

Energy (Parseval's theorem): $\sum_{n=0}^{N-1} |x[n]|^2 = \sum_{\omega=0}^{2\pi} |F_x(\omega)|^2$

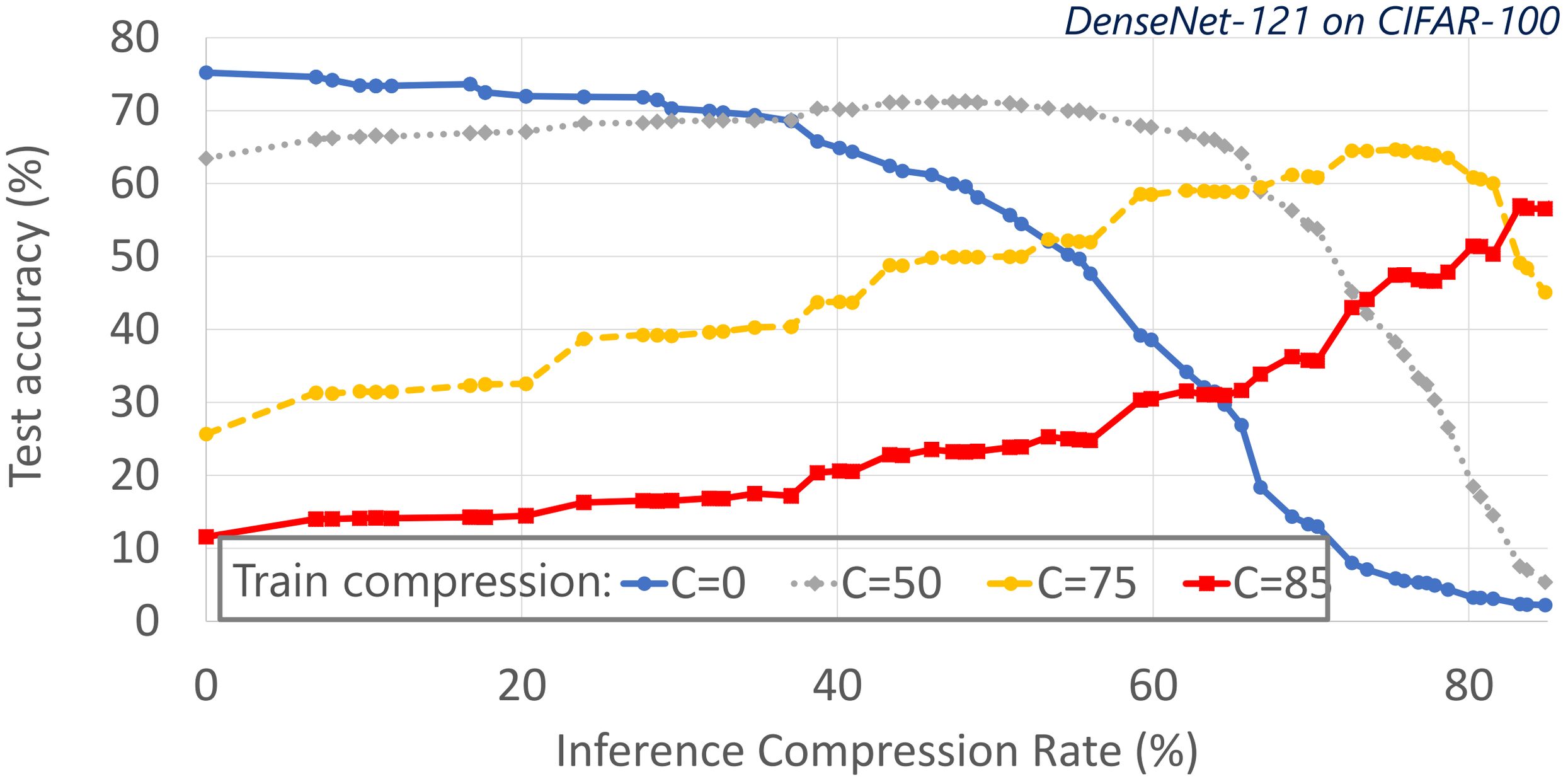
Robustness to noise



Compression Rate for Training vs Inference

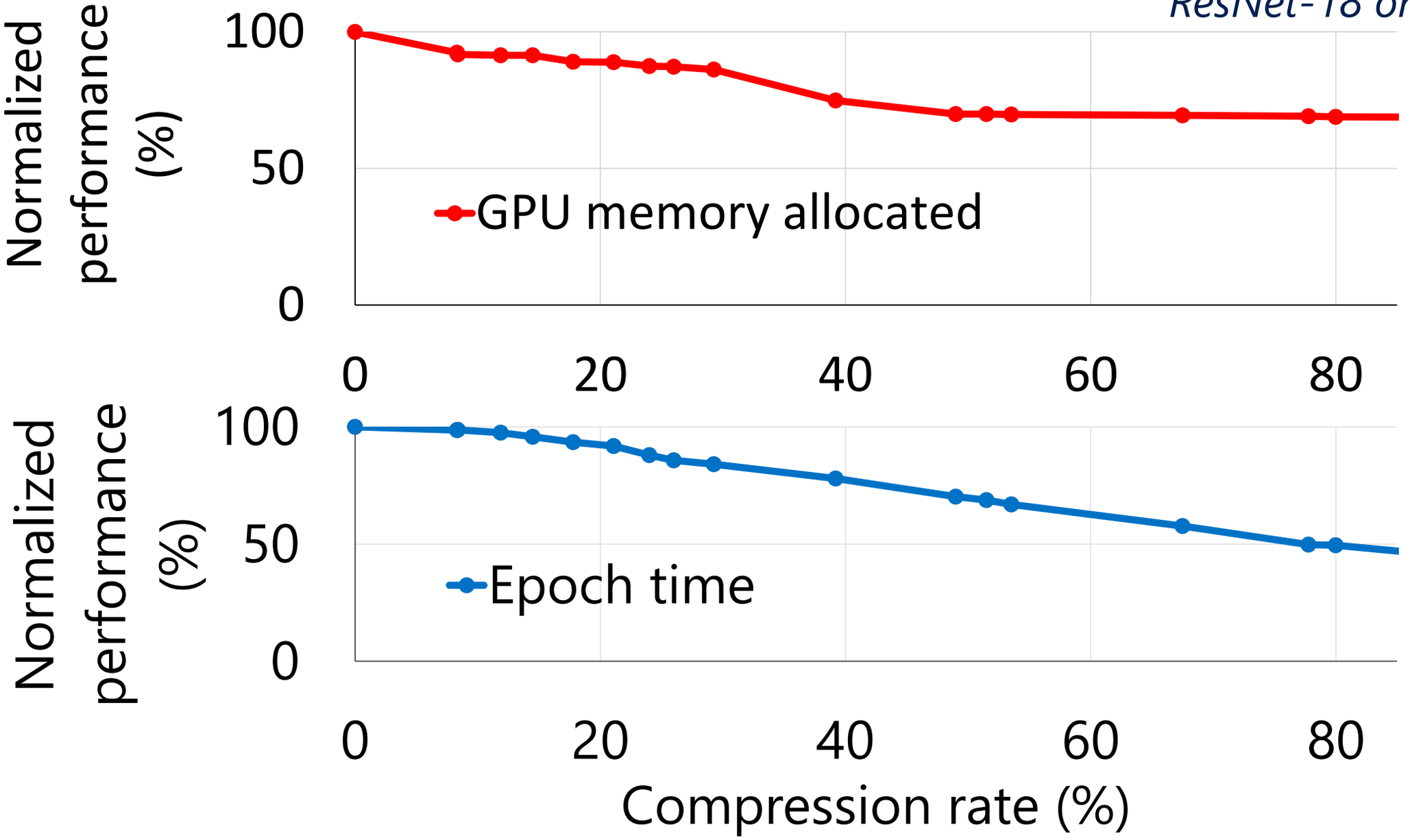


Compression Rate for Training vs Inference

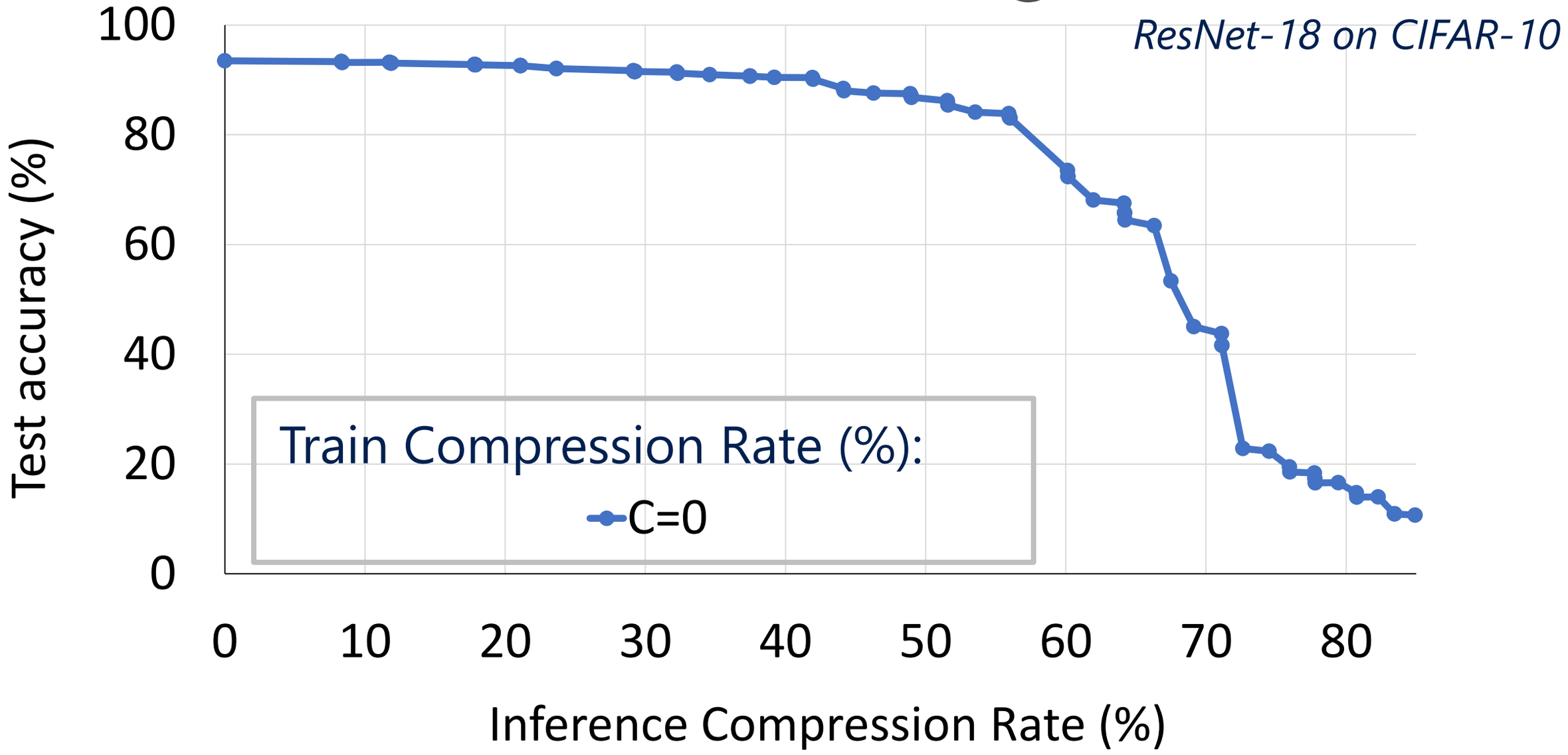


Effectively control resource usage

ResNet-18 on CIFAR-10



Compression Rate for Training vs Inference



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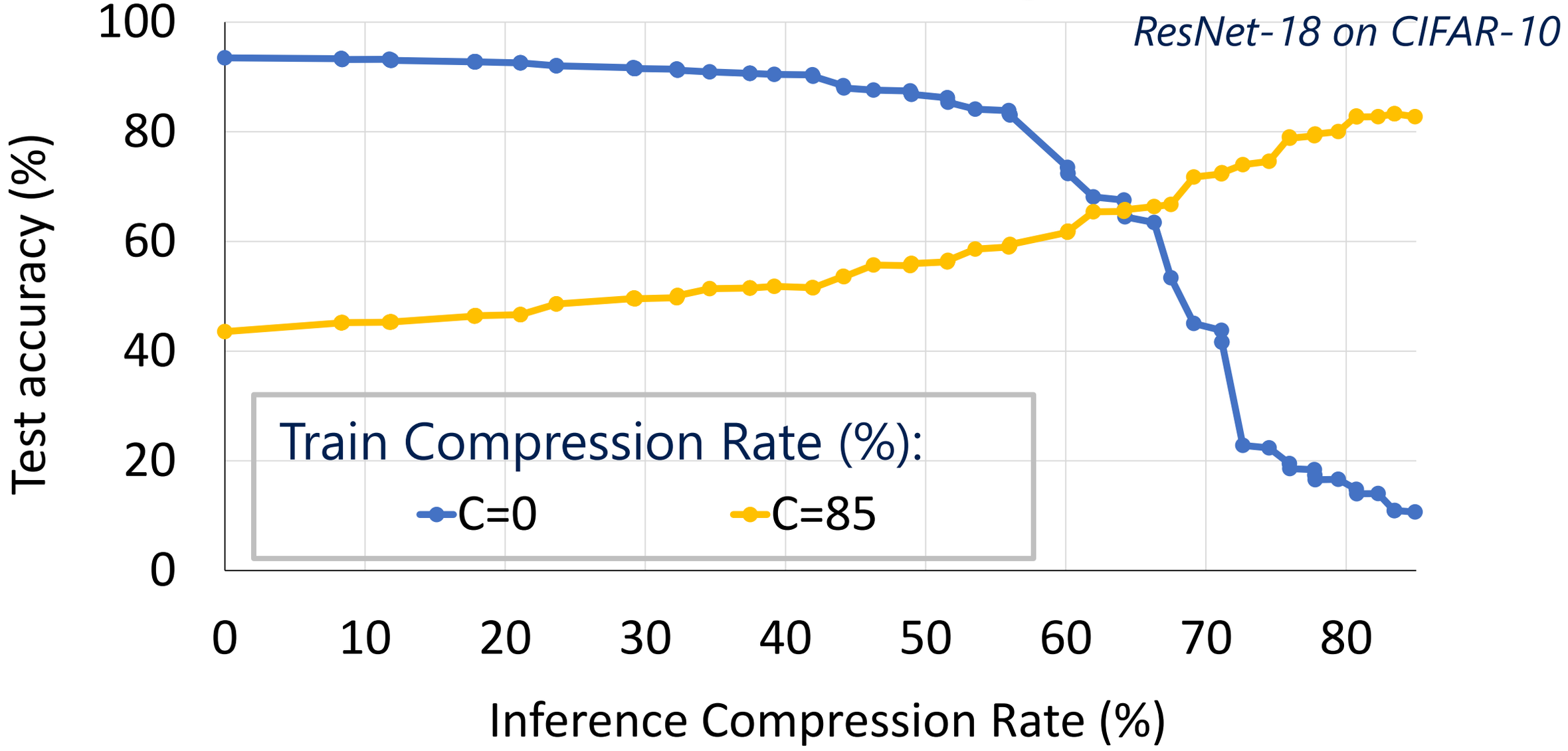
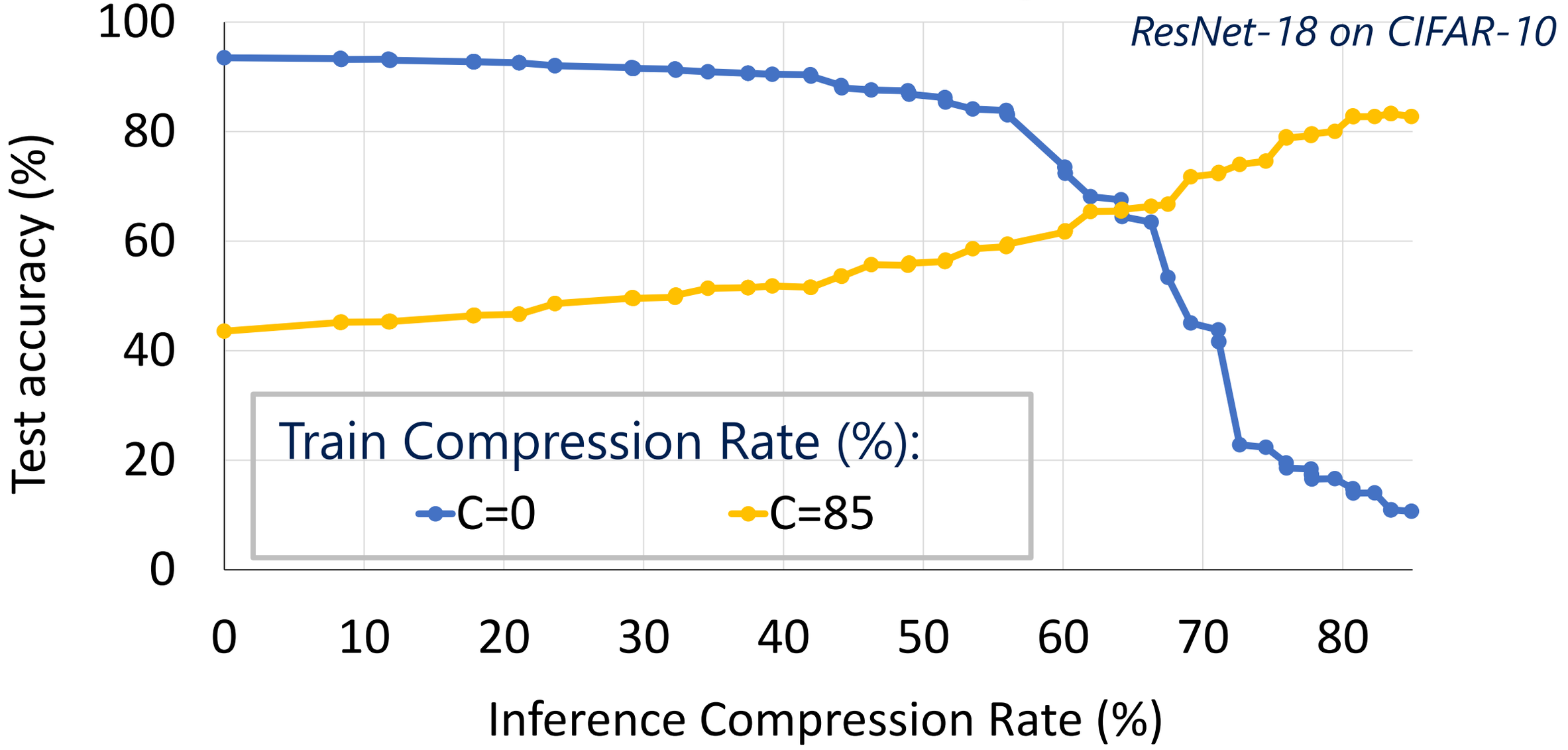
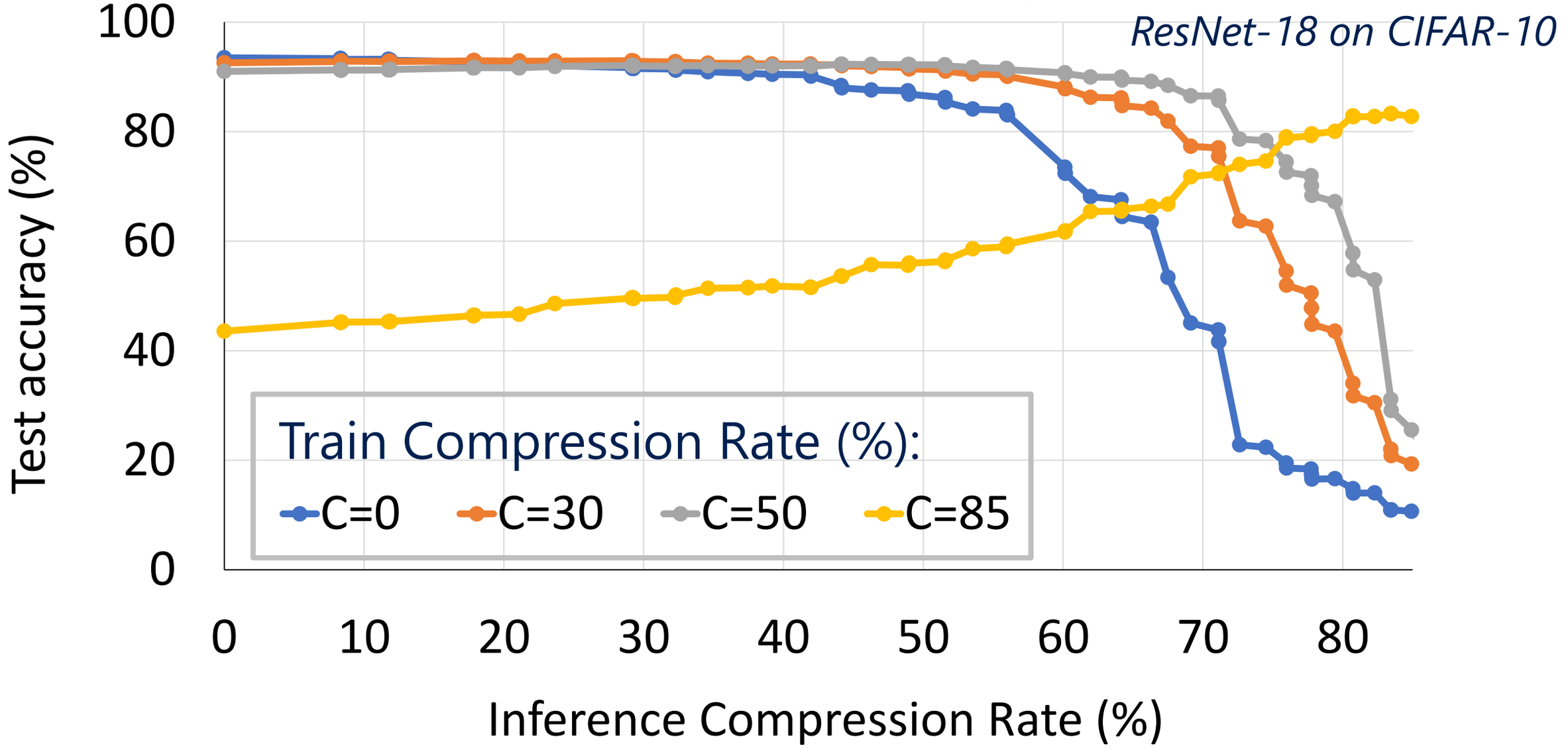


Compression Rate for Training vs Inference



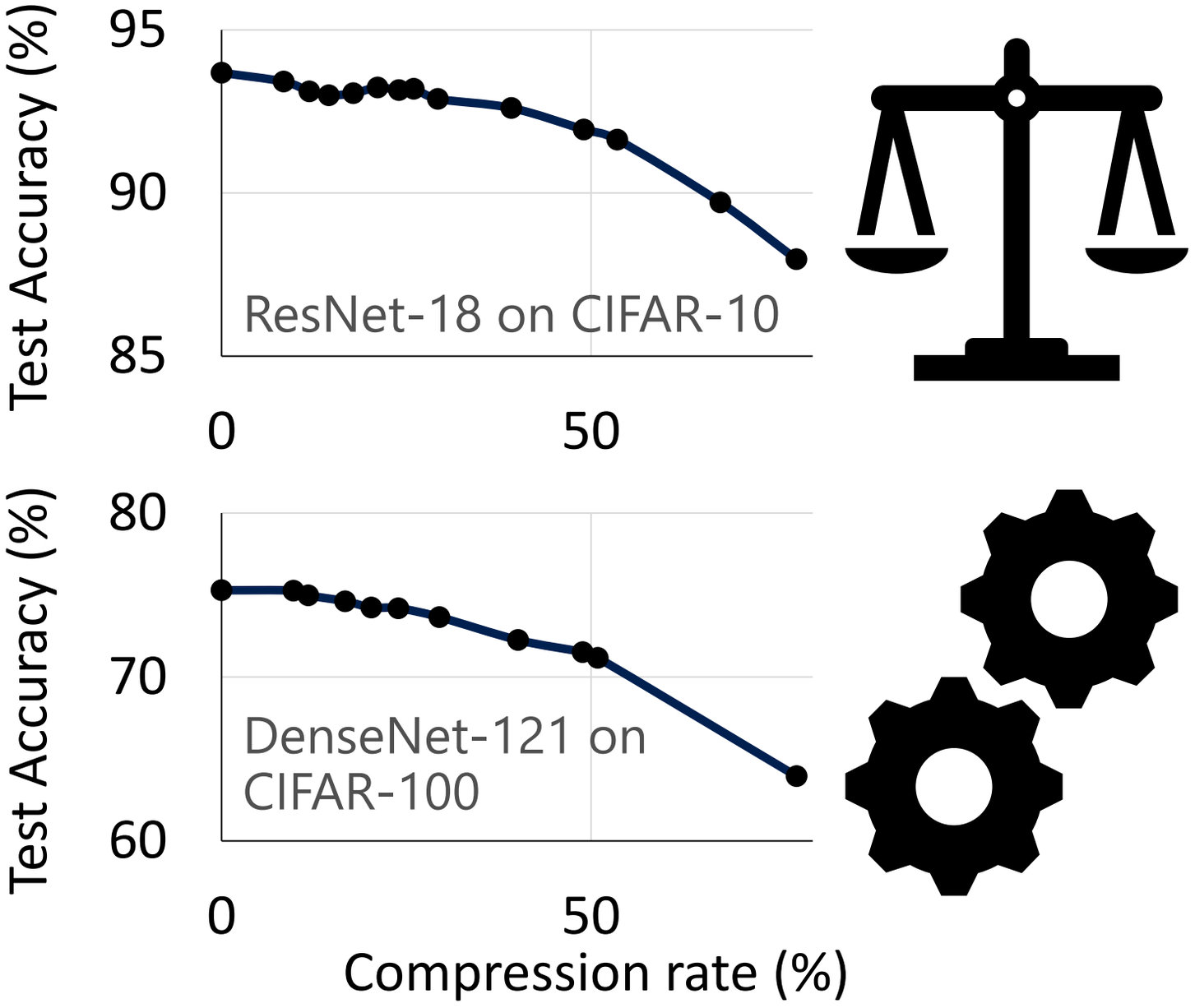
Smooth degradation of accuracy during inference

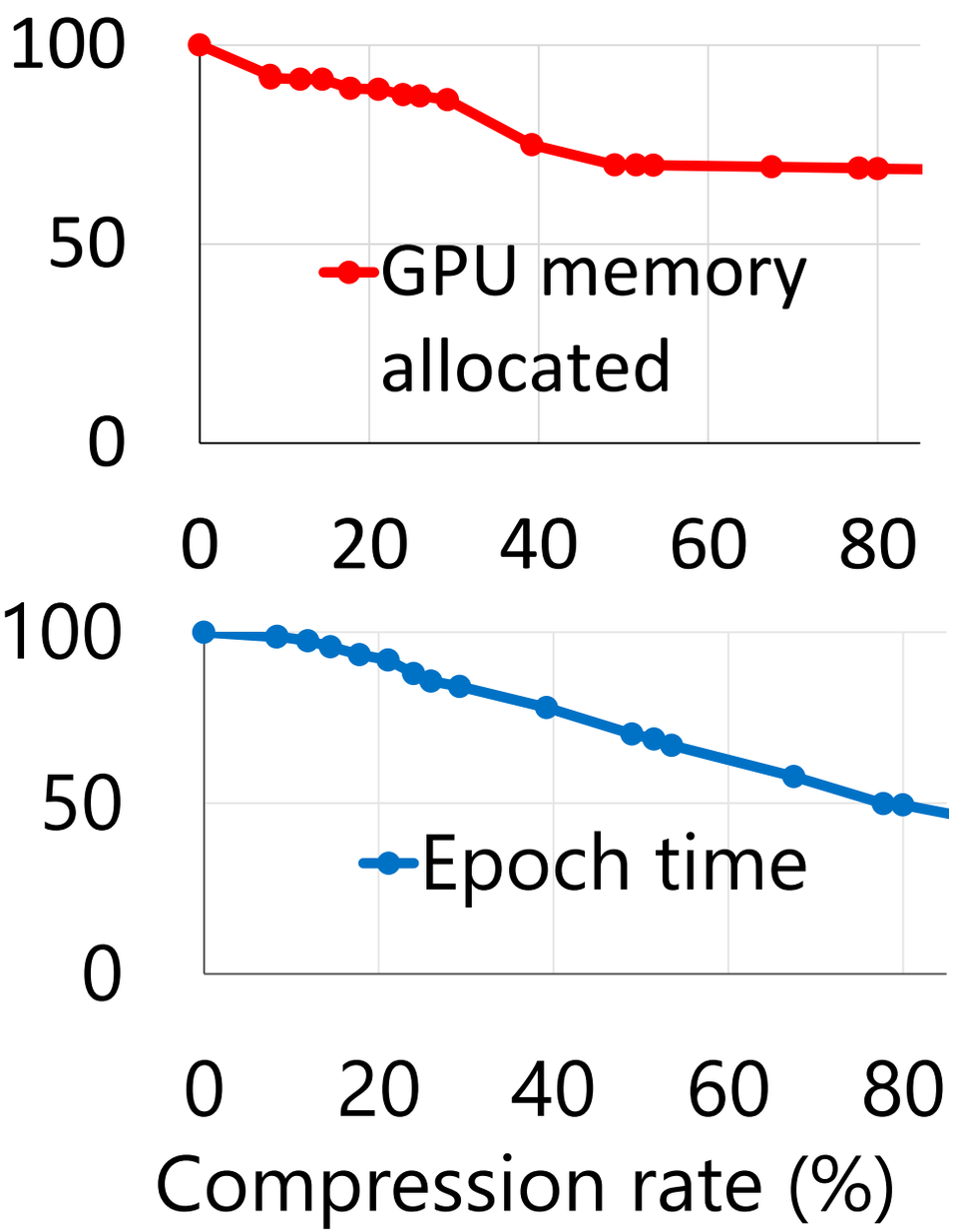
Compression Rate for Training vs Inference

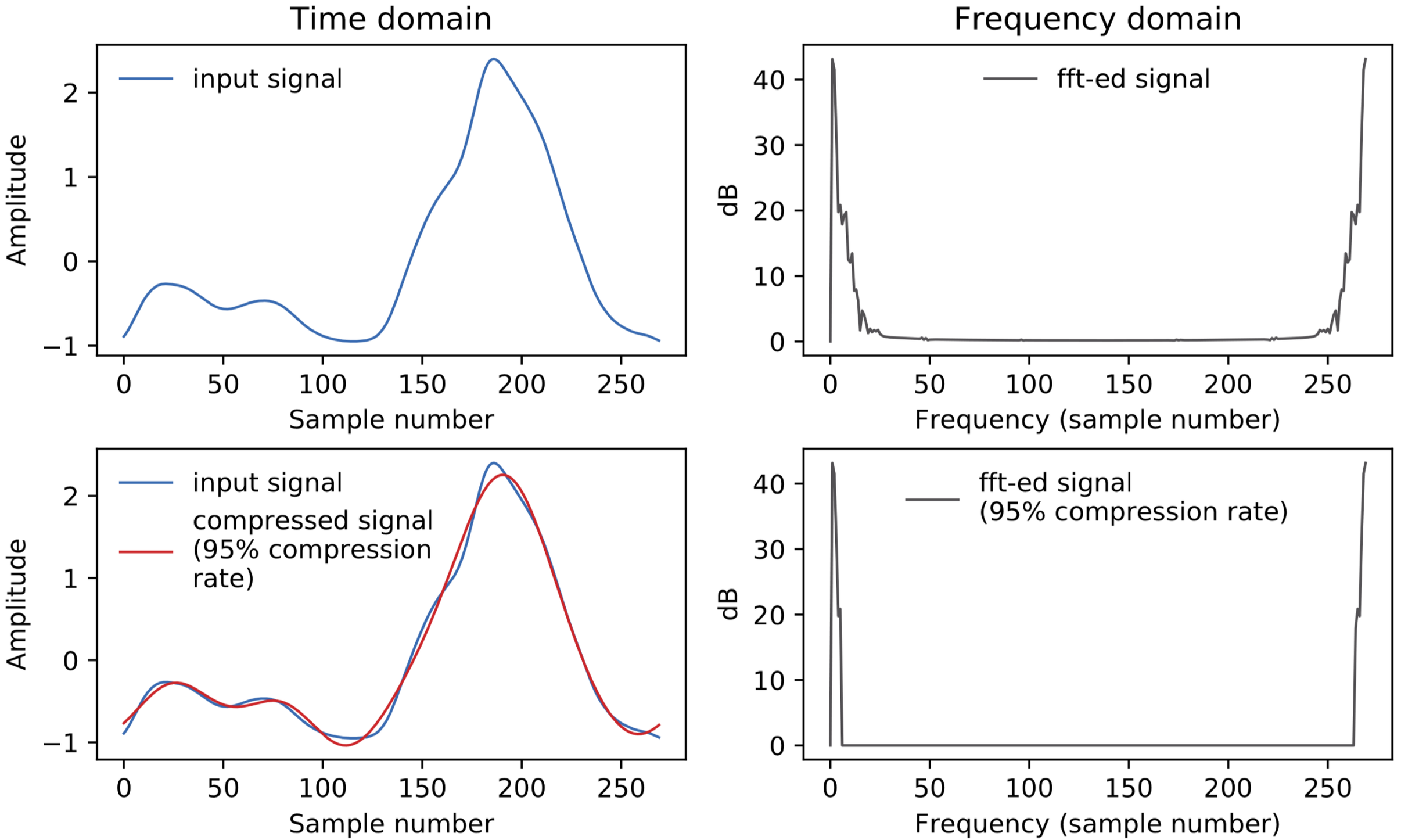


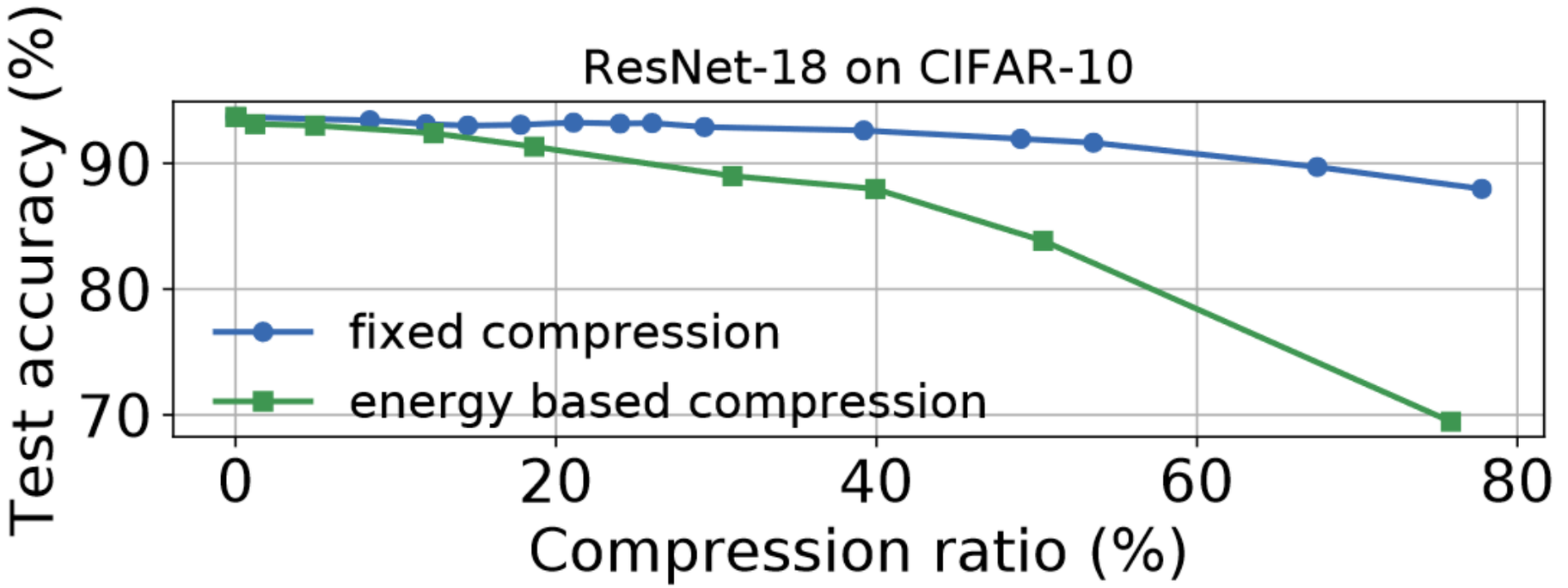
Apply the same compression rate to training and inference

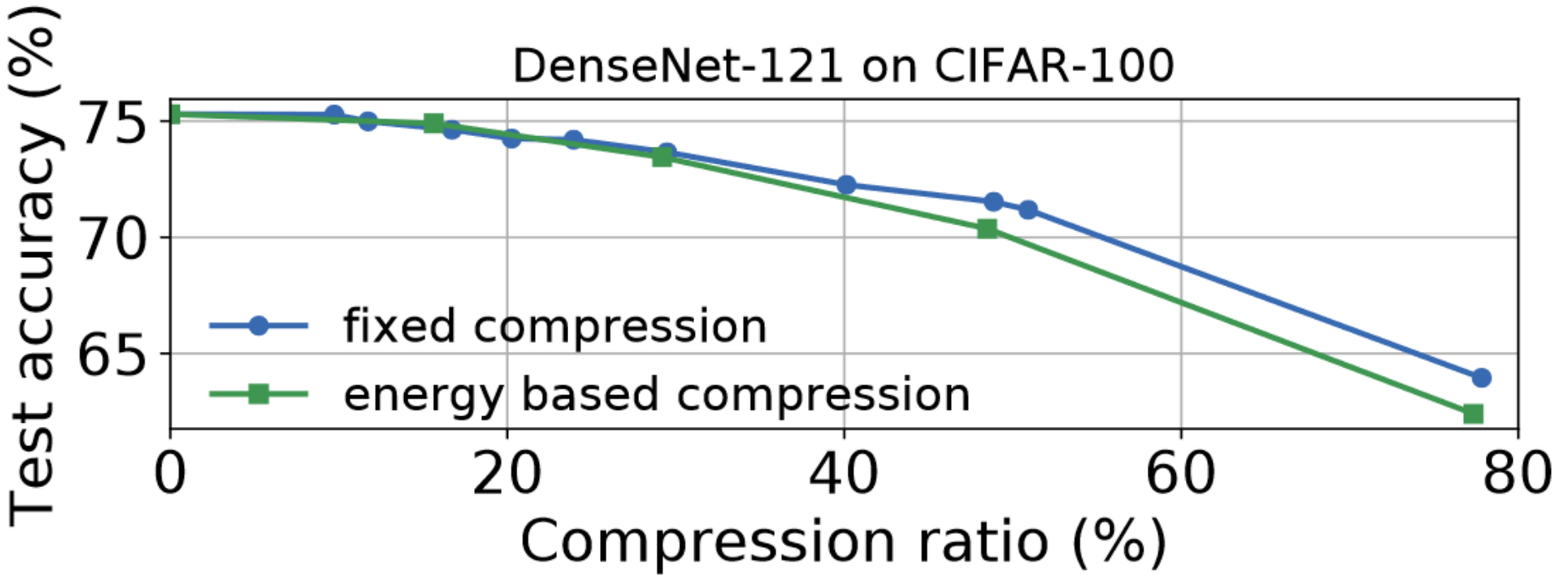
Tuning: Accuracy vs Higher Performance

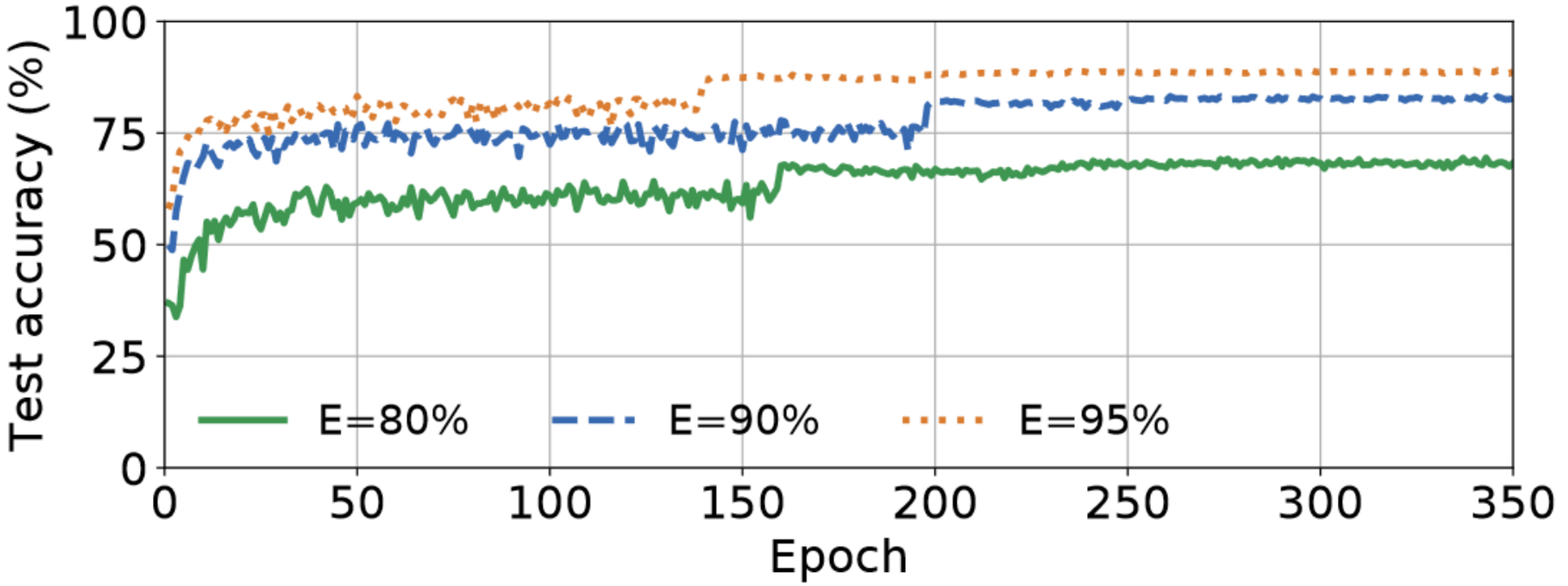


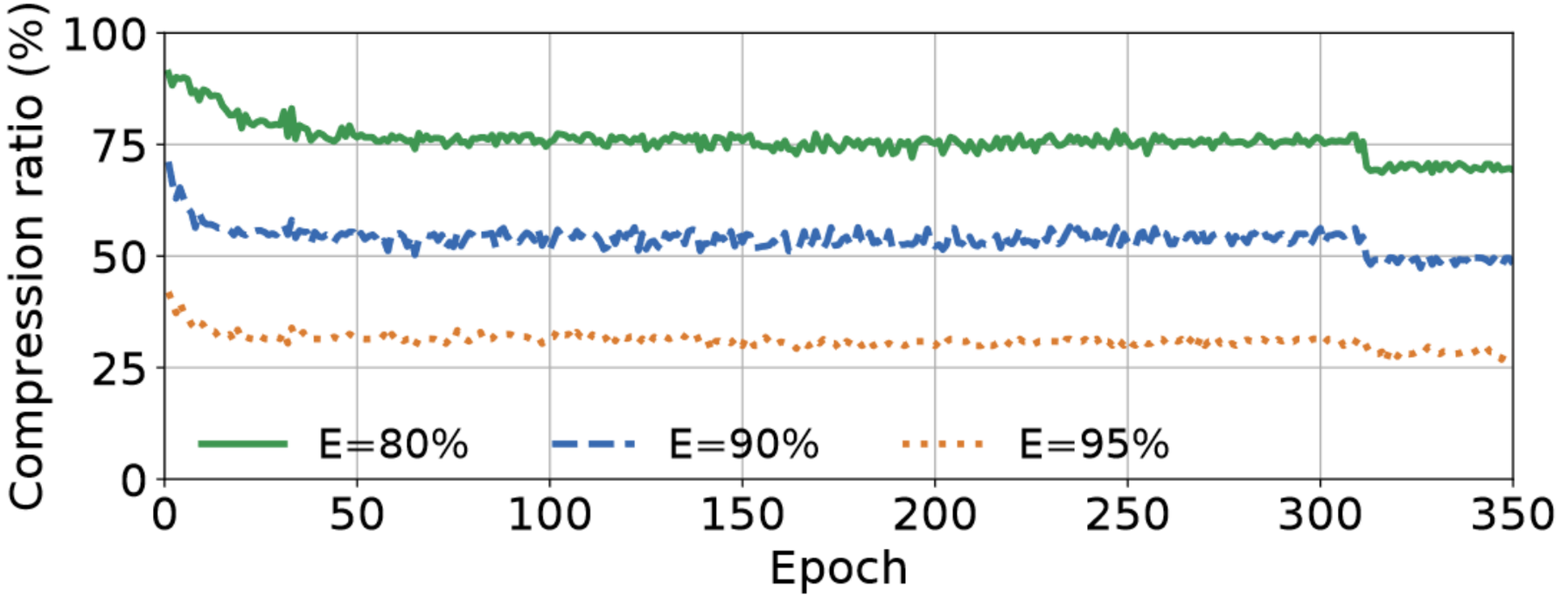


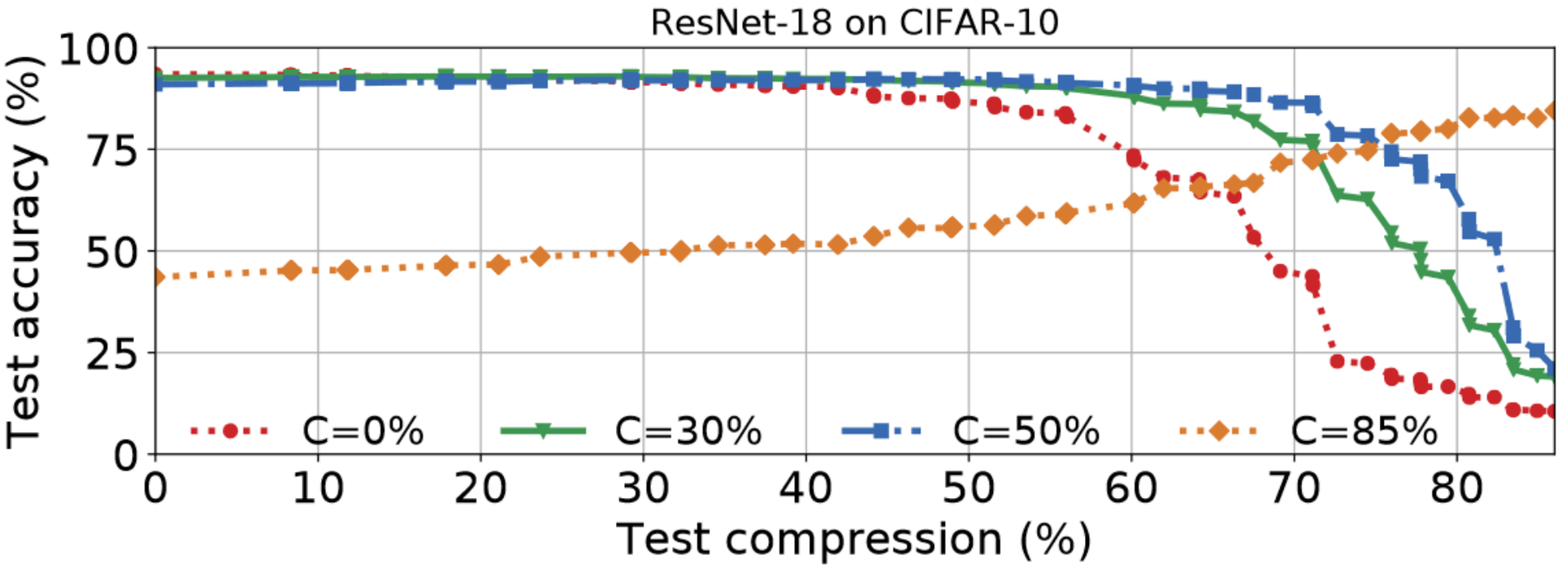

Time domain
Frequency domain
input signal
fft-ed signal
Amplitude
dB
Sample number
Frequency (sample number)
input signal
compressed signal
(95% compression
rate)
fft-ed signal
(95% compression rate)

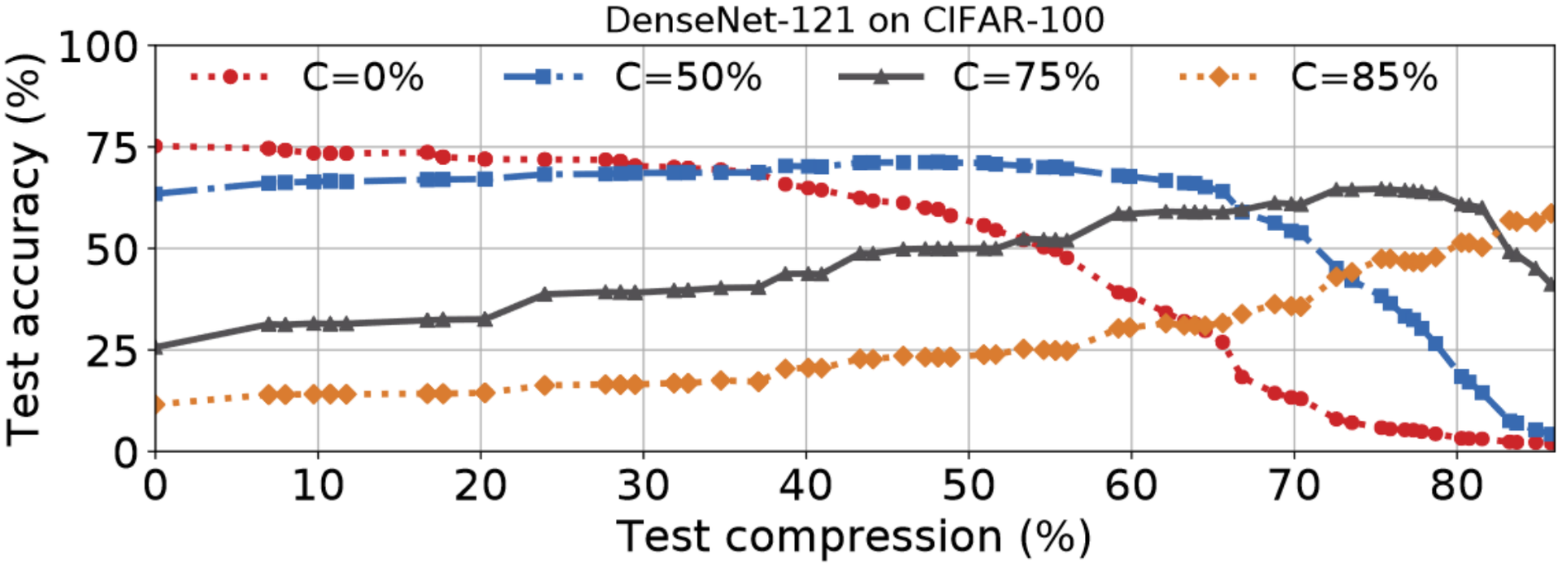

ResNet-18 on CIFAR-10
Test accuracy (%)
90
80
70
0
20
40
60
80
Compression ratio (%)
fixed compression
energy based compression

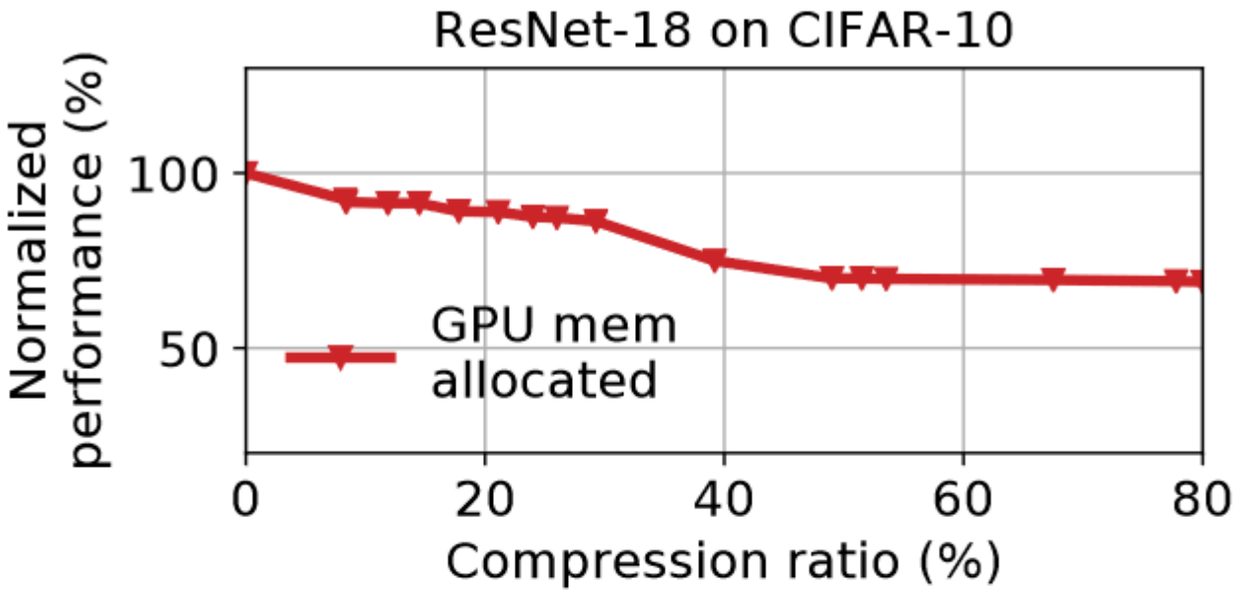

DenseNet-121 on CIFAR-100
Test accuracy (%)
75
70
65
0
20
40
60
80
Compression ratio (%)
fixed compression
energy based compression

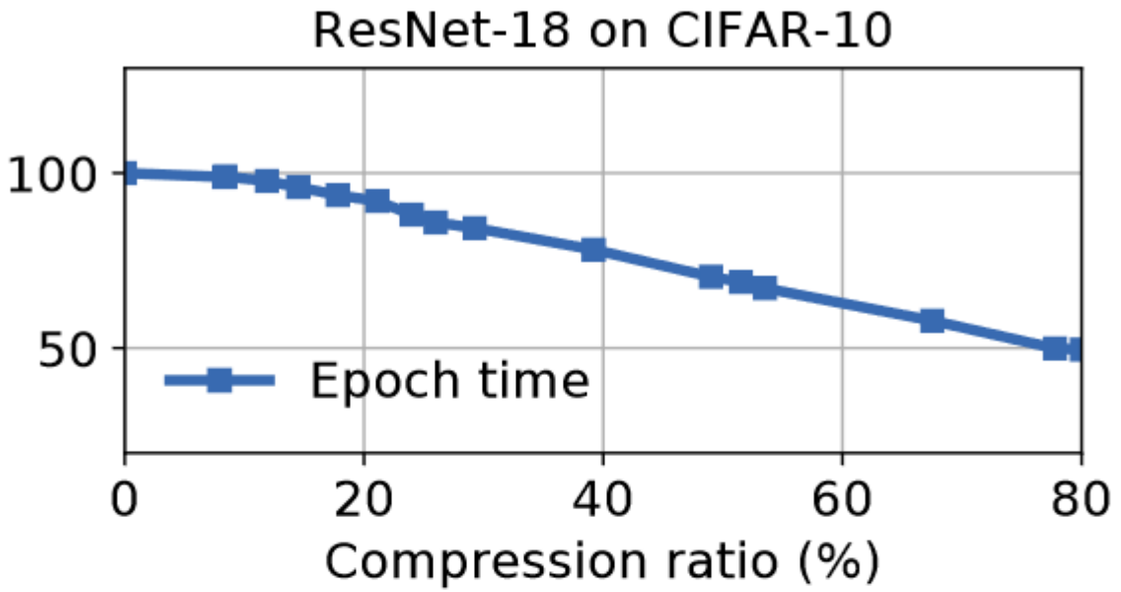

Test accuracy (%)
100
75
50
25
0
0
50
100
150
200
250
300
350
Epoch
E=80%
E=90%
E=95%

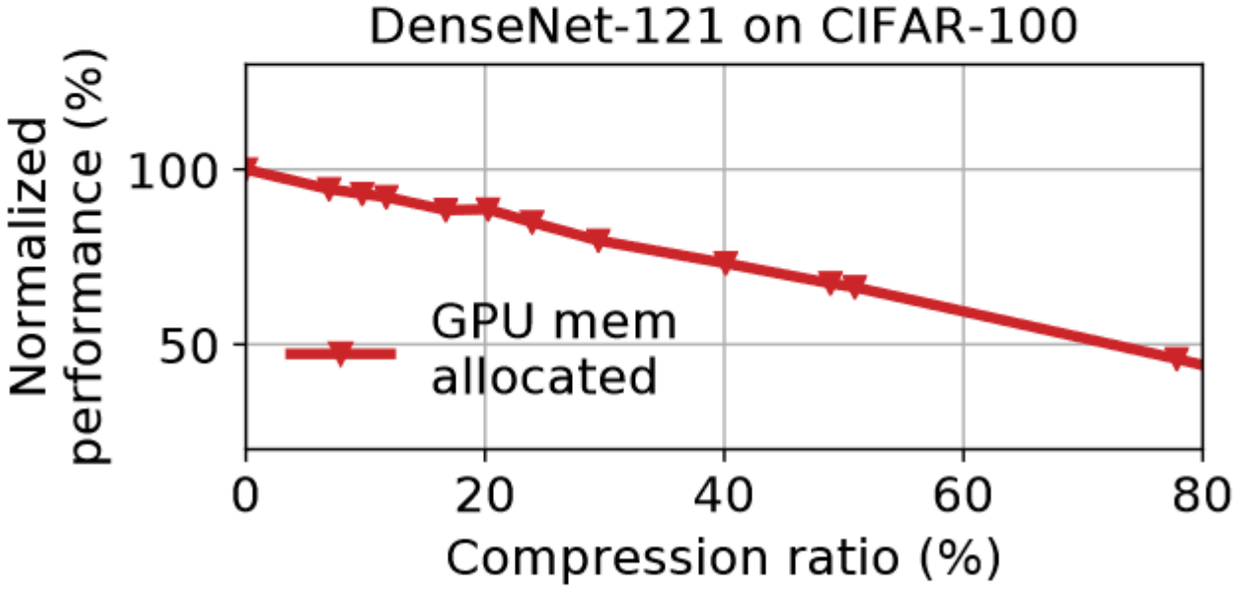

Compression ratio (%)
100
75
50
25
0
0
50
100
150
200
250
300
350
Epoch
E=80%
E=90%
E=95%

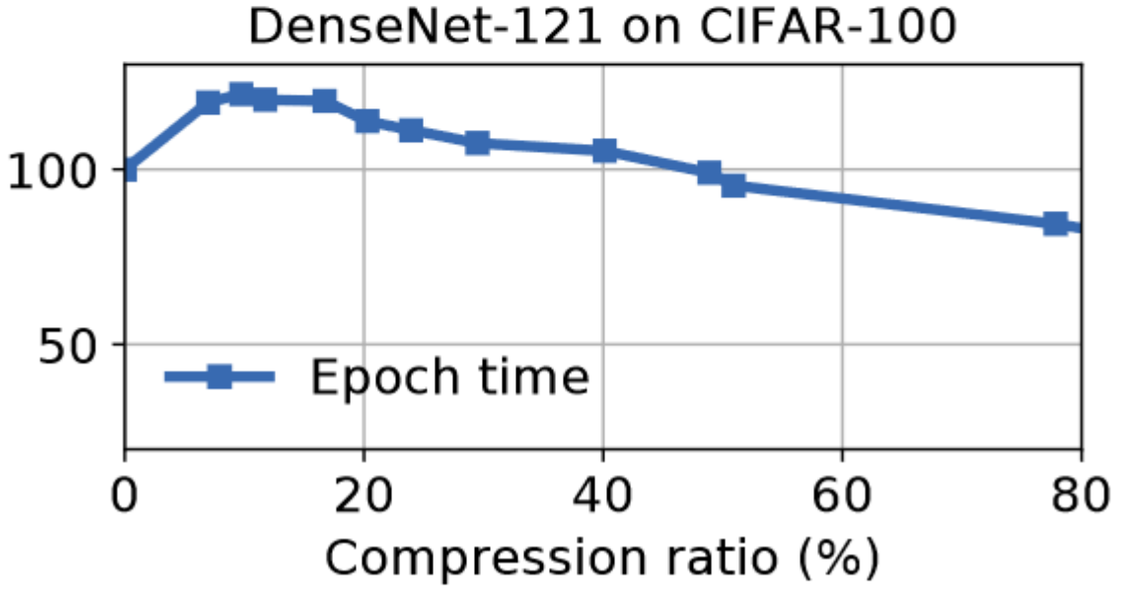

ResNet-18 on CIFAR-10
Test accuracy (%)
100
75
50
25
0
0
10
20
30
40
50
60
70
80
Test compression (%)
C=0%
C=30%
C=50%
C=85%


DenseNet-121 on CIFAR-100
Test accuracy (%)
100
75
50
25
0
0
10
20
30
40
50
60
70
80
Test compression (%)
C=0%
C=50%
C=75%
C=85%


ResNet-18 on CIFAR-10
Normalized performance (%)
100
50
GPU mem allocated
0
20
40
60
80
Compression ratio (%)


ResNet-18 on CIFAR-10
100
50
Epoch time
0
20
40
60
80
Compression ratio (%)


DenseNet-121 on CIFAR-100
Normalized performance (%)
100
50
GPU mem allocated
0
20
40
60
80
Compression ratio (%)


DenseNet-121 on CIFAR-100
100
50
Epoch time
0
20
40
60
80
Compression ratio (%)

"Speaking of longer term, it would be nice if the community migrated to a fully open sourced implementation for all of this [convolution operations, etc.]. This stuff is just too important to the progress of the field for it to be **locked away in proprietary implementations**. The more people working together on this the better for everyone. There's plenty of room to compete on the hardware implementation side."

Scott Gray

https://github.com/soumith/convnet-benchmarks/issues/93