[1] USC University of Southern California Information Sciences Institute

[2] Google AI

MixHop: Higher-Order Graph Convolutional Architectures via Sparsified Neighborhood Mixing

Sami Abu-El-Haija[1], Bryan Perozzi[2], Amol Kapoor[2], Nazanin Alipourfard[1], Kristina Lerman[1], Hrayr Harutyunyan[1], Greg Ver Steeg[1], Aram Galstyan[1]

Code: http://github.com/samihaija/mixhop

Slides: http://sami.haija.org/icml19

Agenda

- Review Graph Convolutional Networks (GCN)
  - Application Semi-Supervised Node Classification (SSNC)
  - Shortcoming of GCN
- MixHop: Higher-Order GCN
  - Sparsification
- MixHop Results on SSNC

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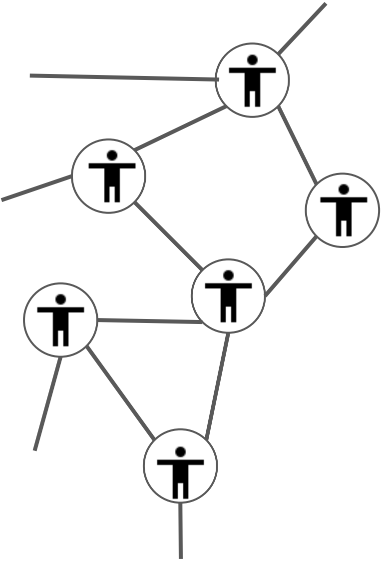

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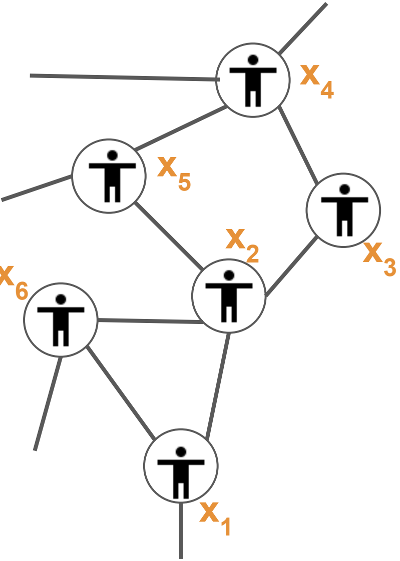

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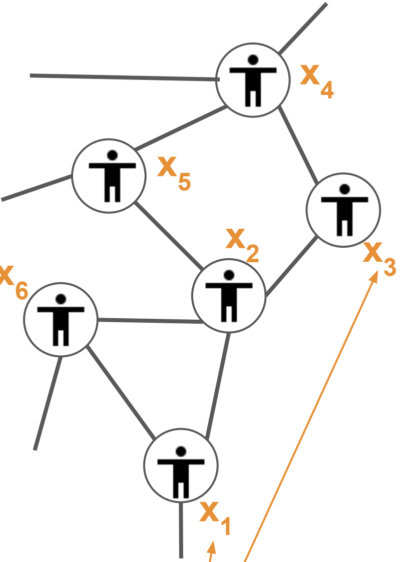

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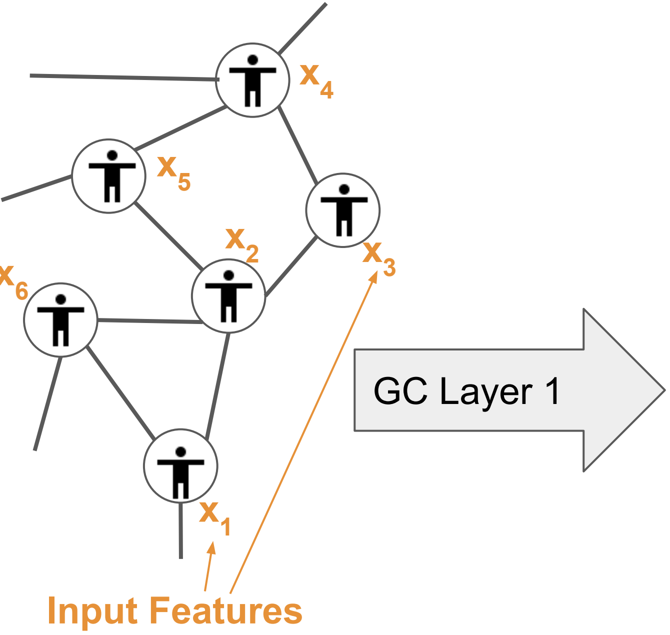

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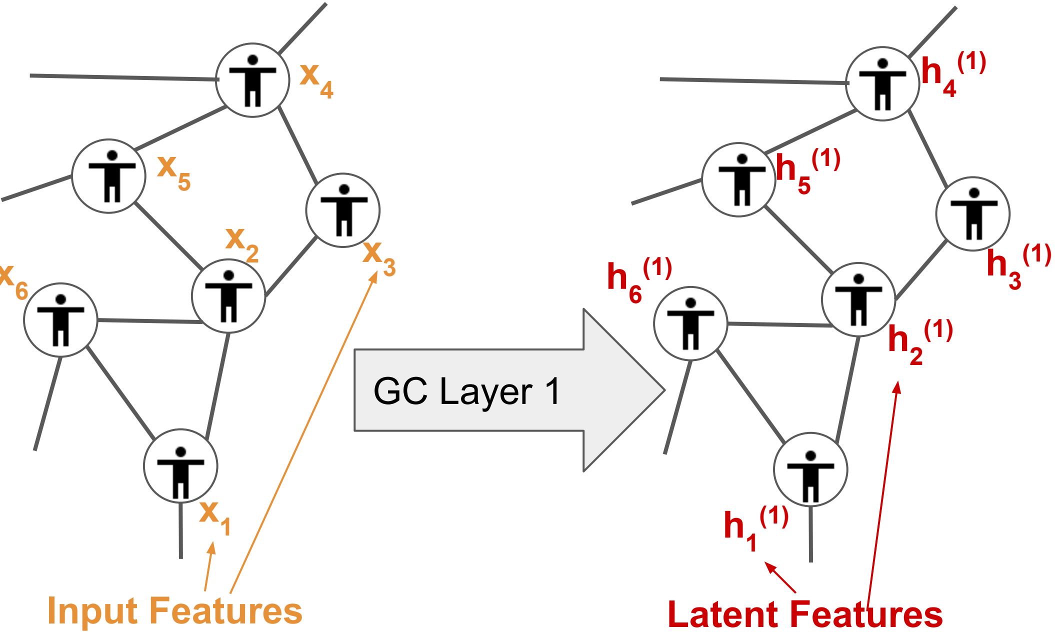

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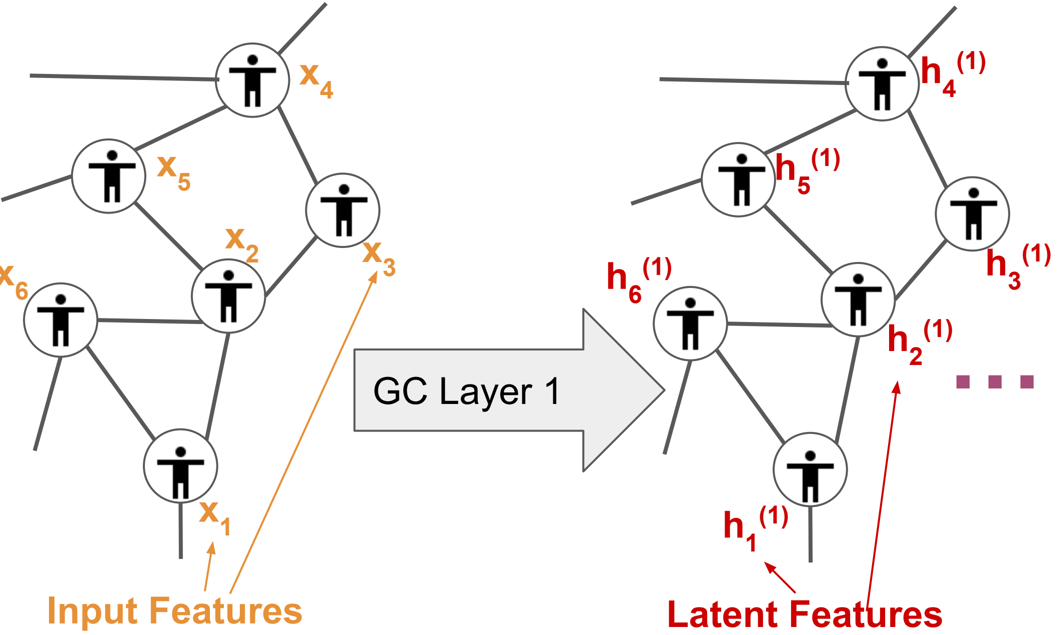

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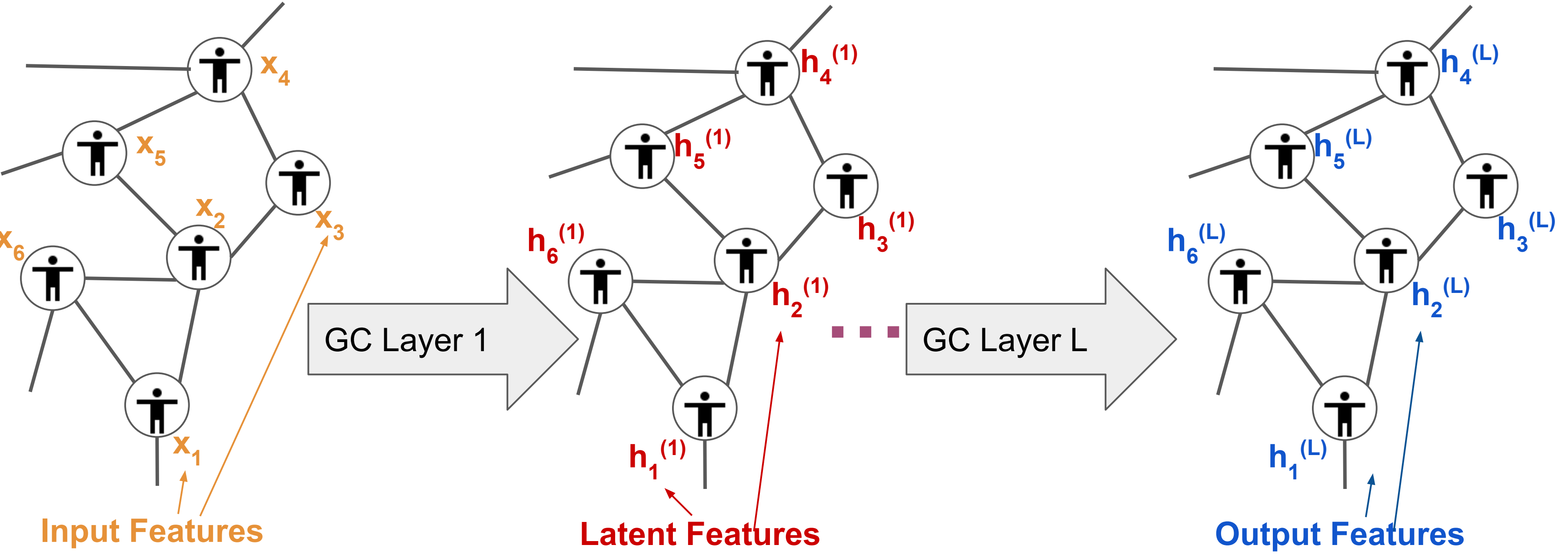

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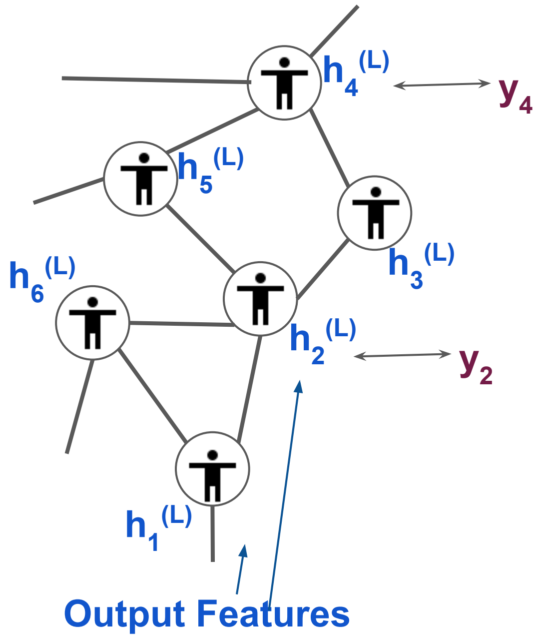
Graph Convolutional Network (GCN) [1]

Train on semi-supervised node classification:

- measure ***Loss*** on labeled nodes (**y_4, y_2**)



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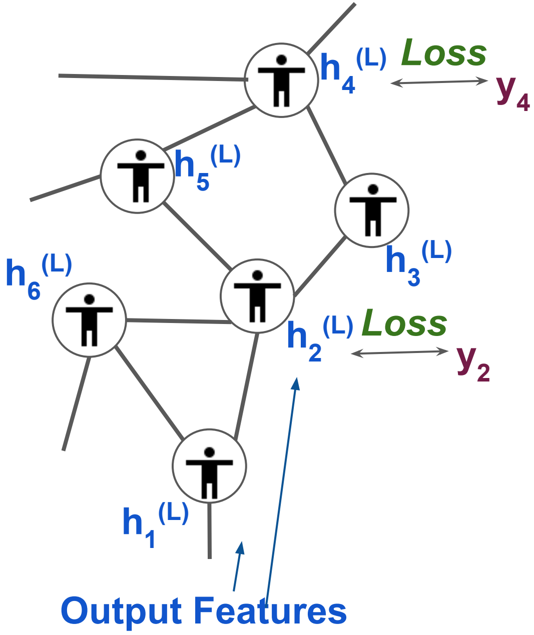
Graph Convolutional Network (GCN) [1]

Train on semi-supervised node classification:

- measure ***Loss*** on labeled nodes ($\mathbf{y}_4$, $\mathbf{y}_2$)
- Backprop to learn GC layers.



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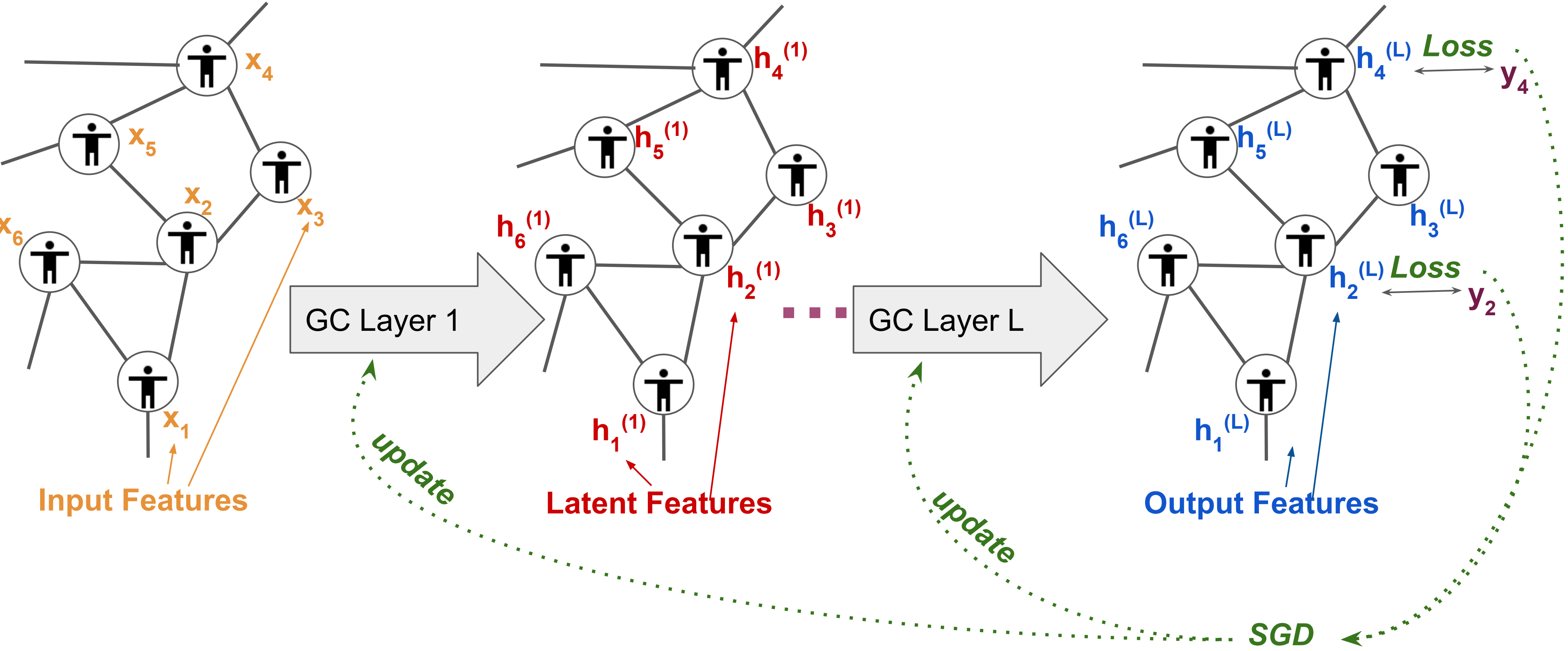

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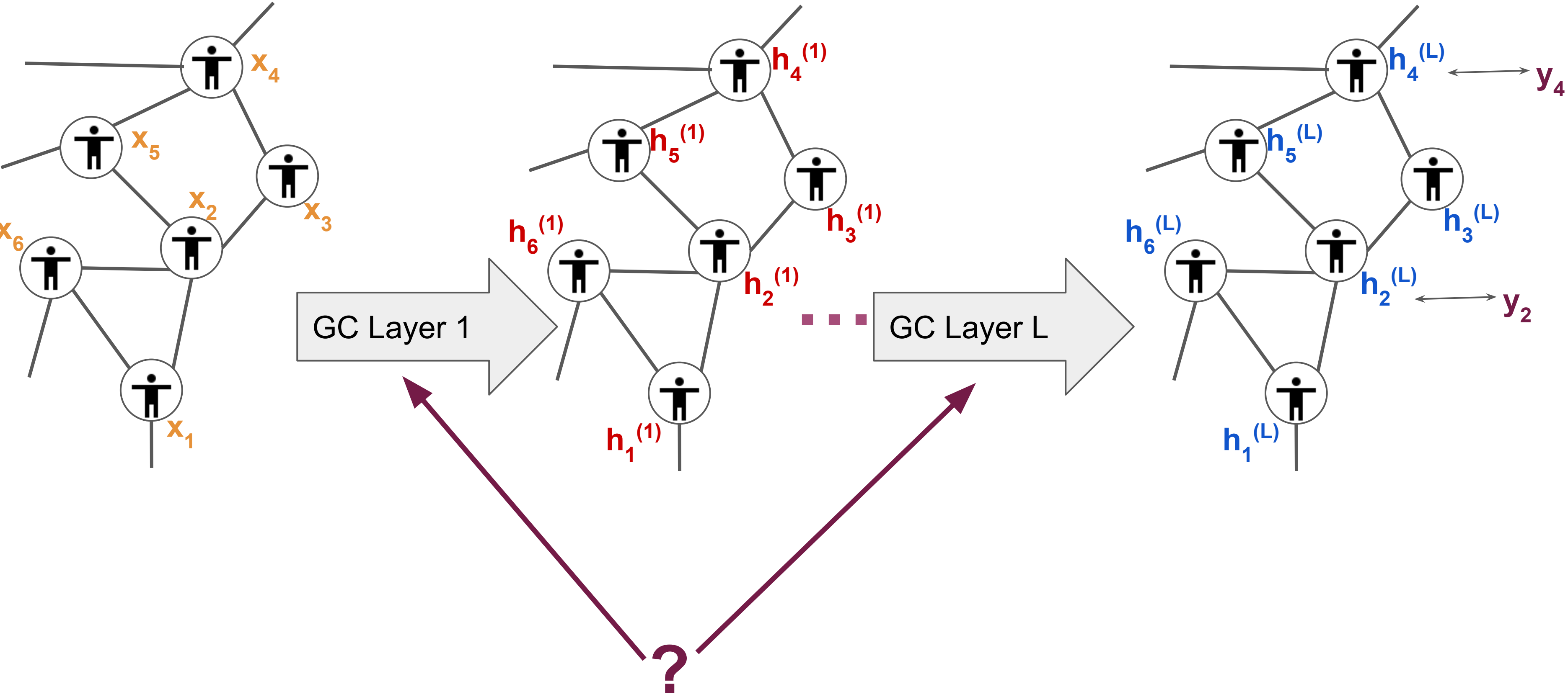

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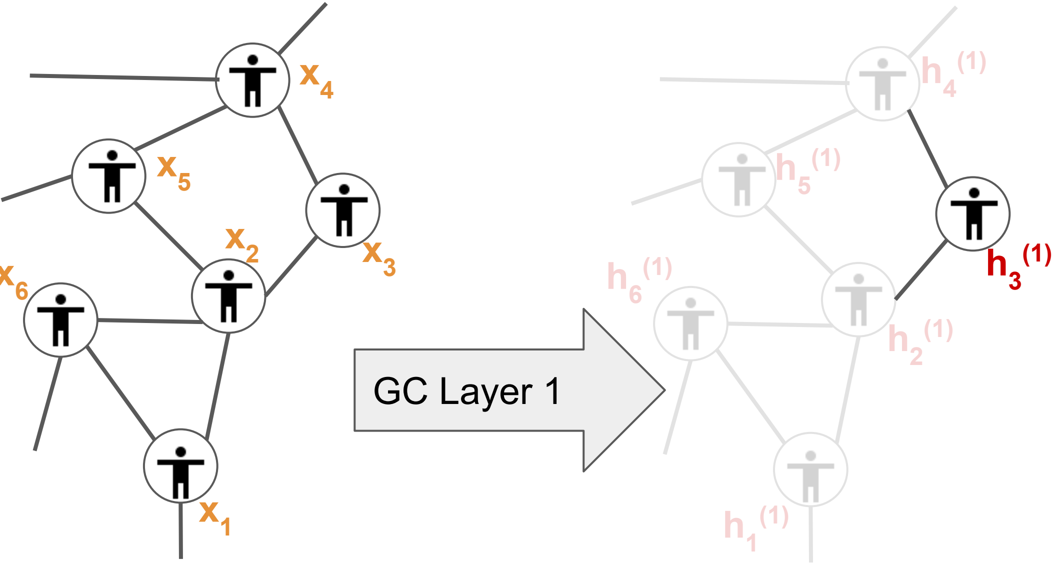

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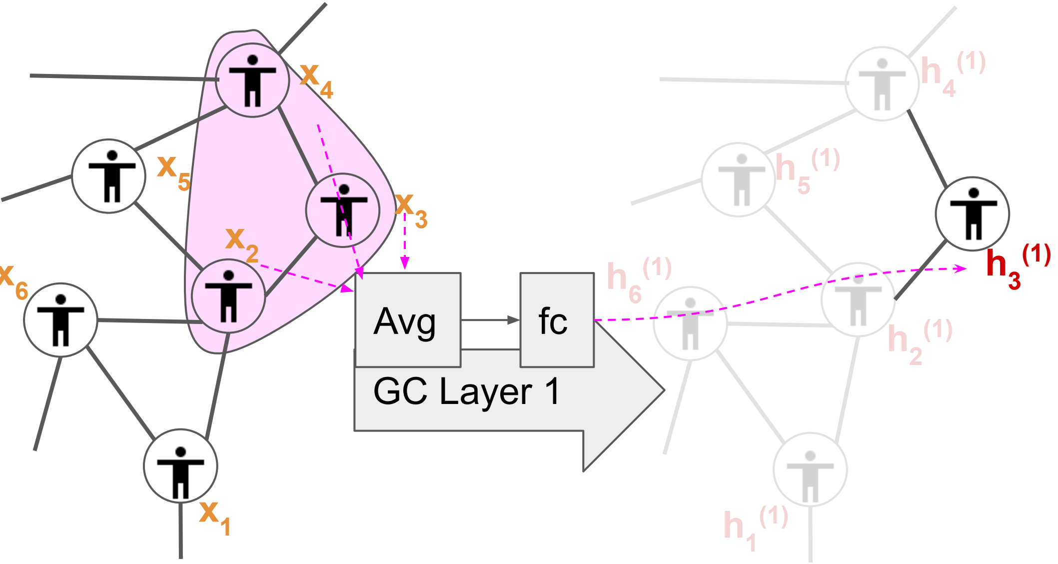

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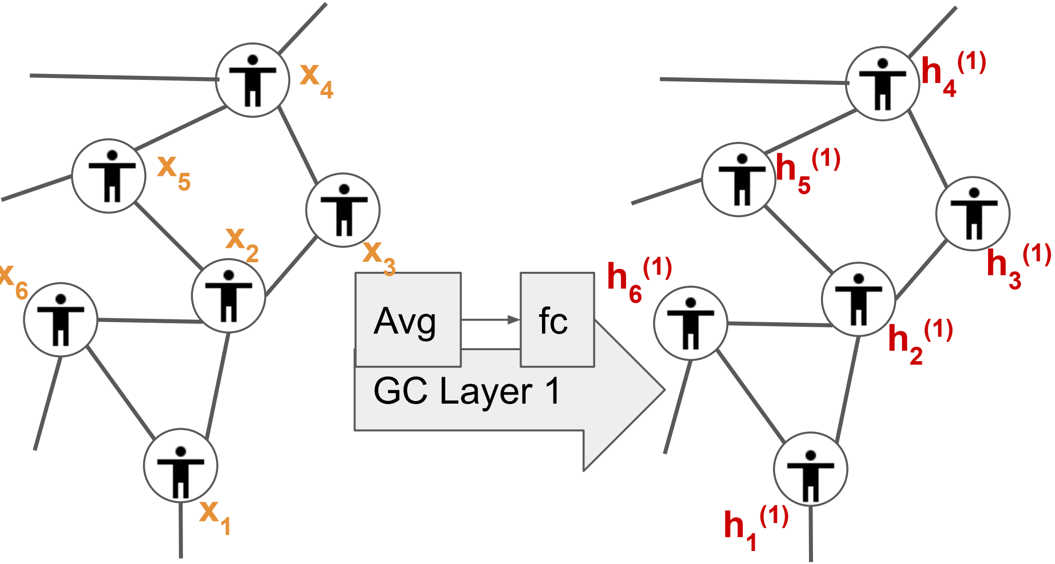
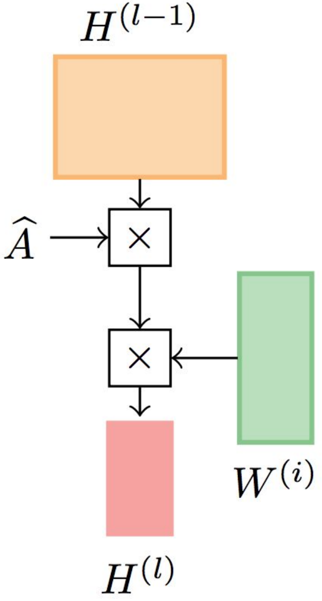


Tensor Graph

$$H^{(1)} = \sigma(\widehat{A} X W^{(1)})$$



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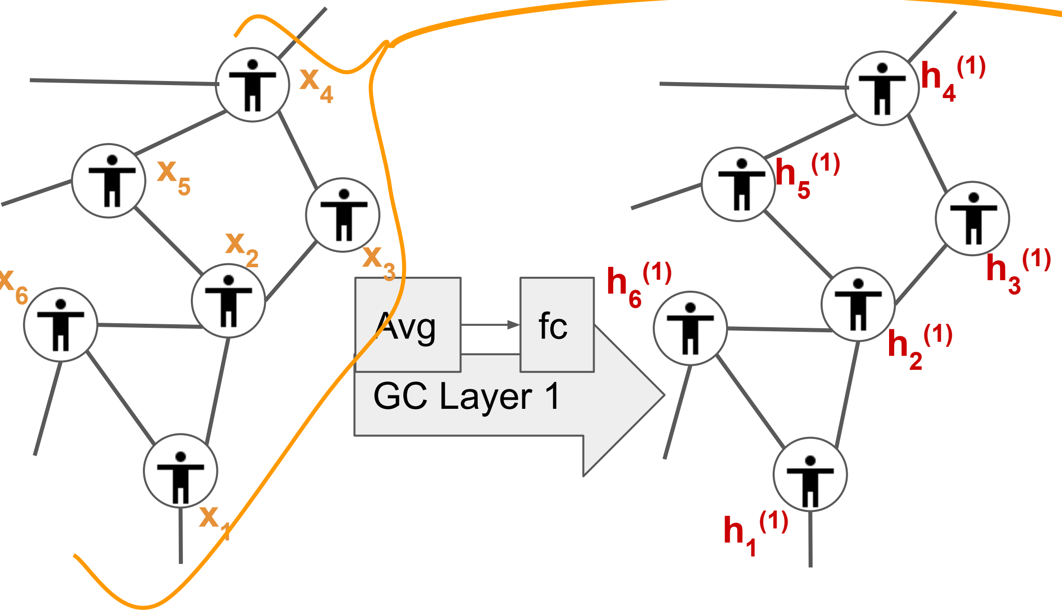

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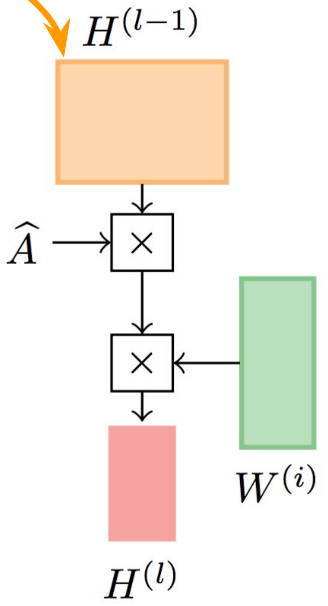

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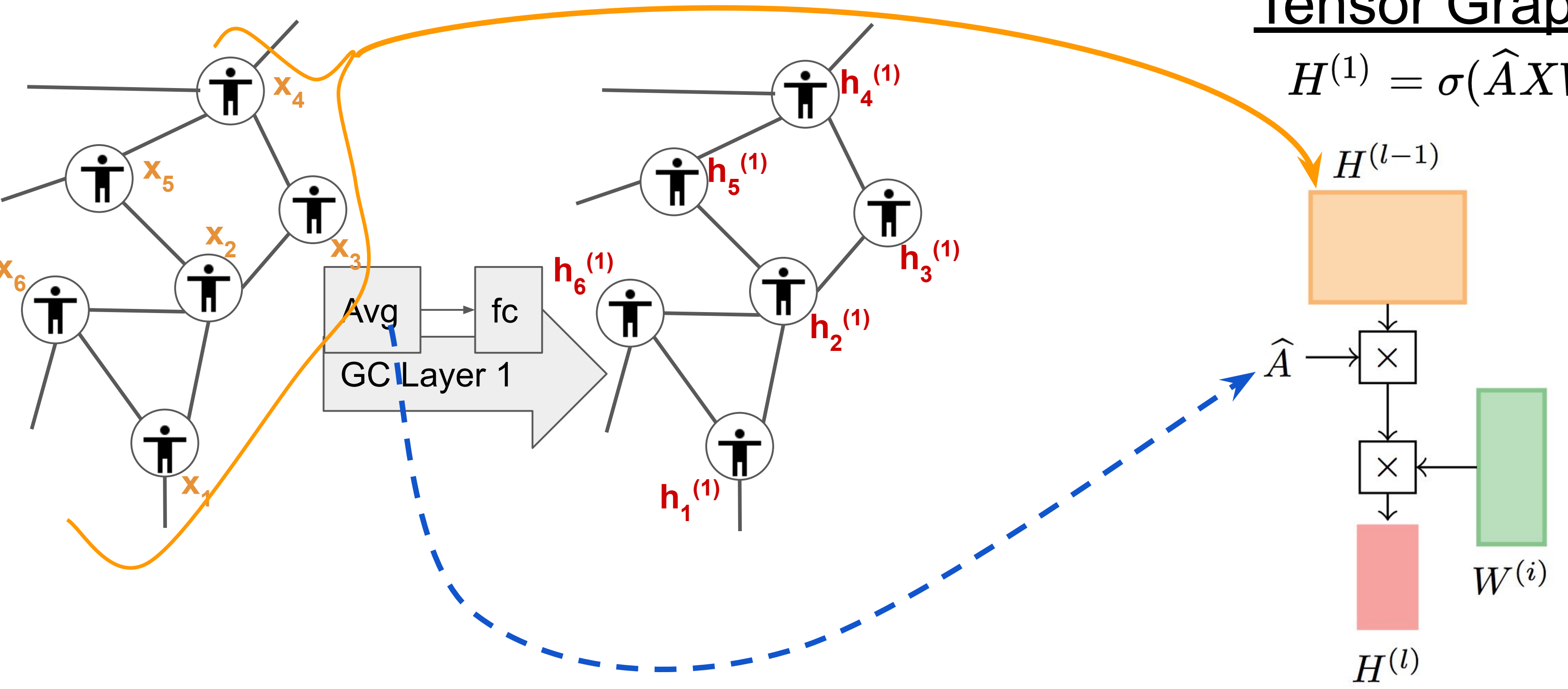

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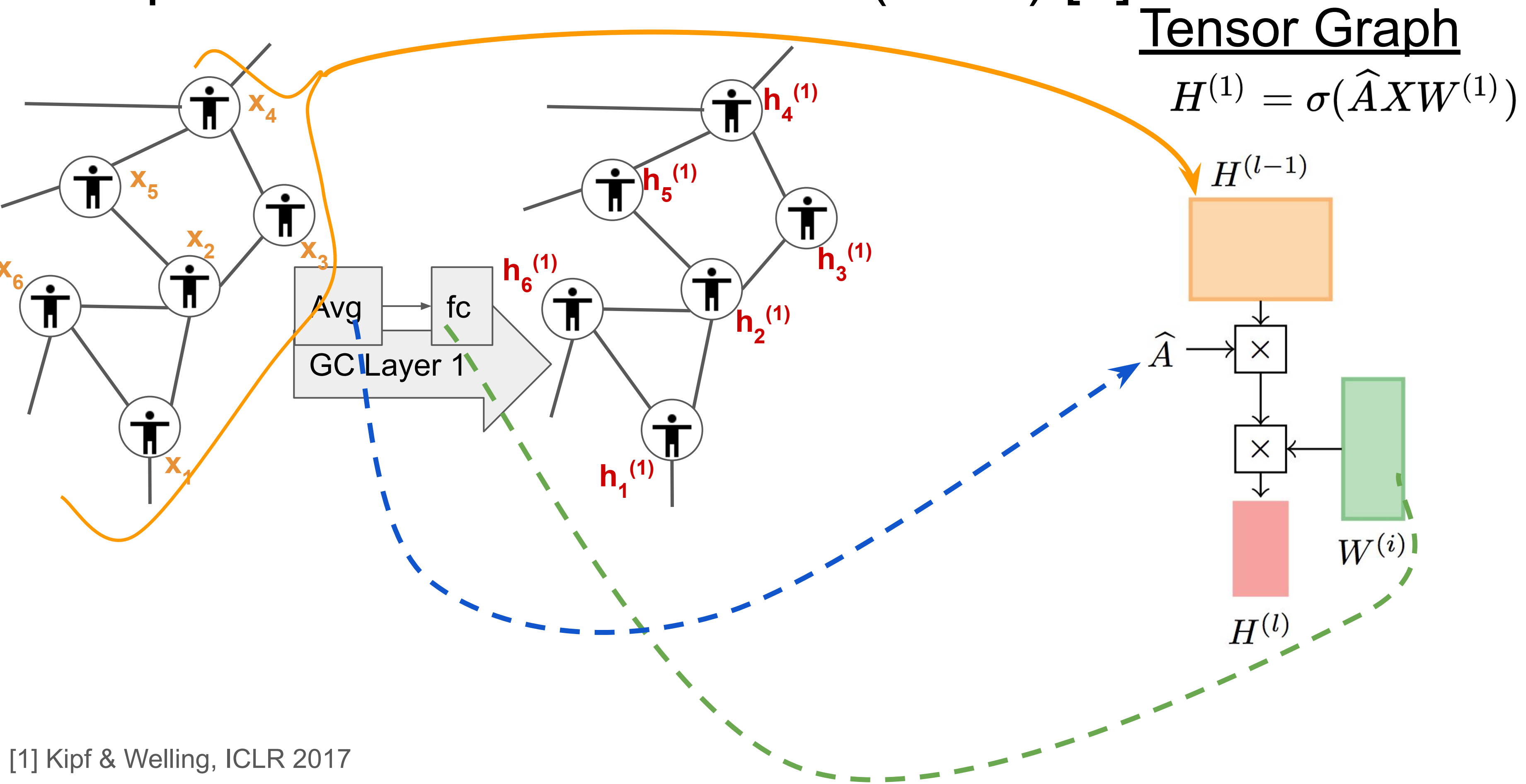

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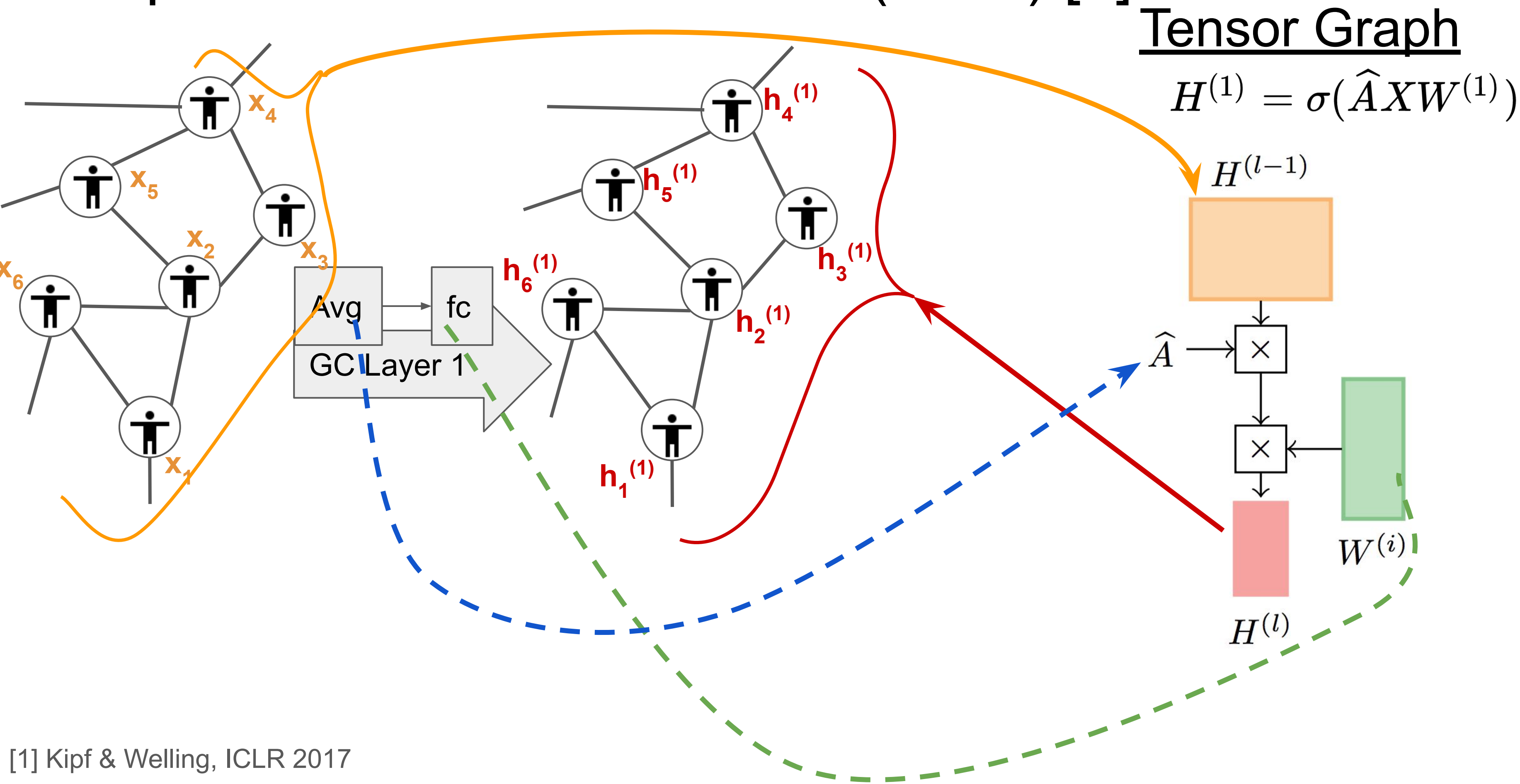

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Shortcoming of Vanilla GCN

Vanilla GC Layer

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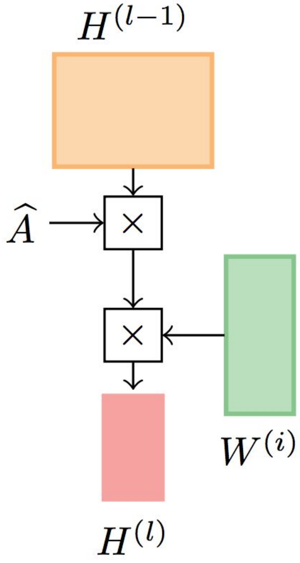

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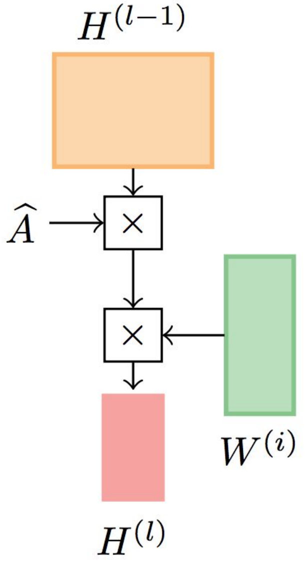

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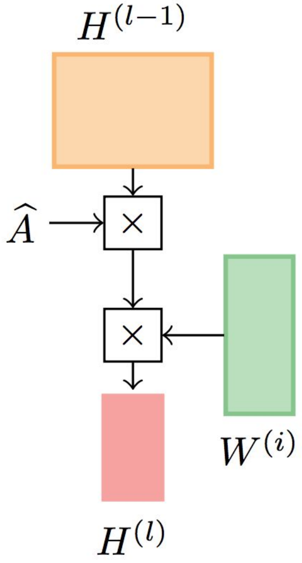

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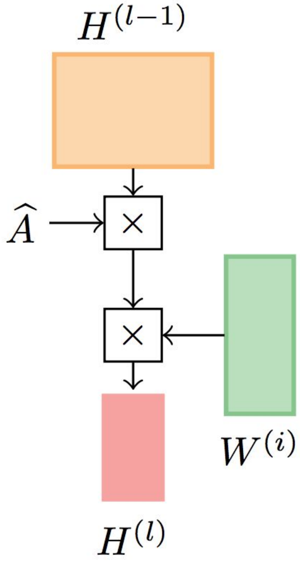

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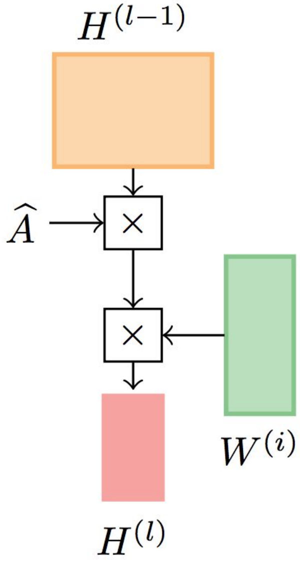

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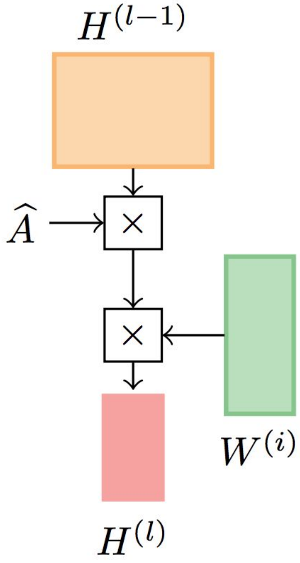

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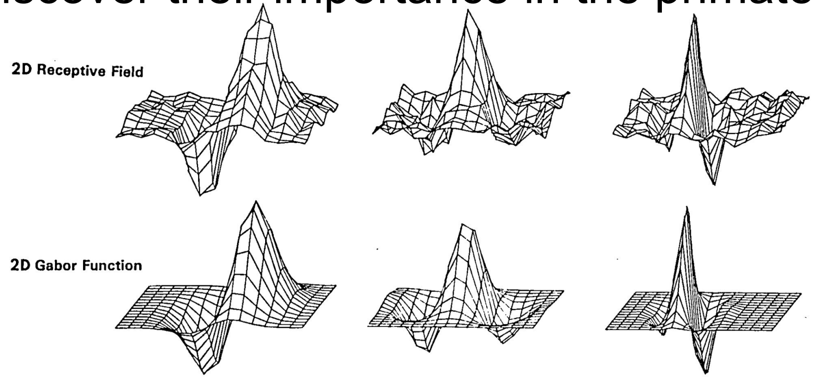
Detour: Review Gabor Filters

Neuroscientists discover their importance in the primate visual cortex [2, 3]:



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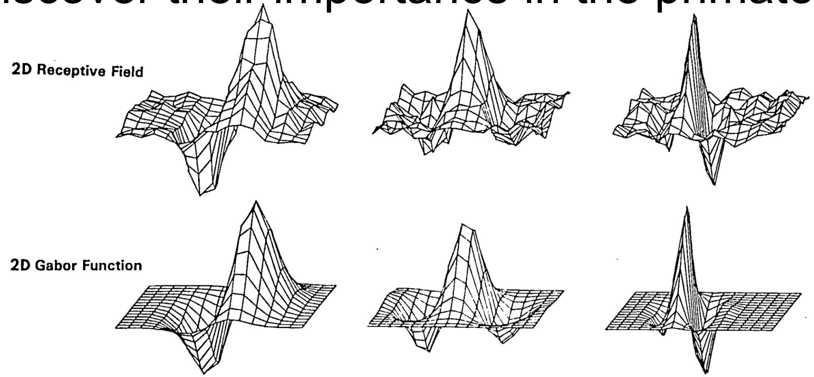
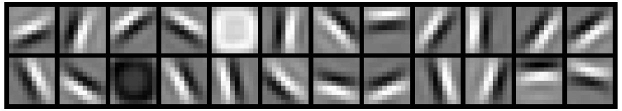
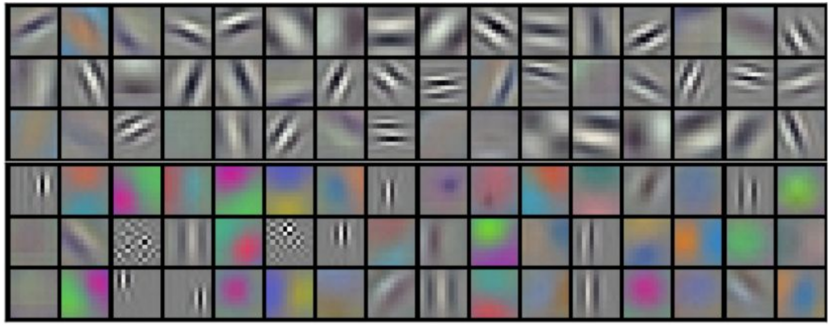


Further, they are automatically recovered by training CNNs on images [4, 5]





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Main Motivation

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Extend the class of representations realizable by GCNs
e.g. to learn Gabor Filters

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Our Model: MixHop

MixHop GC Layer

Vanilla GC Layer

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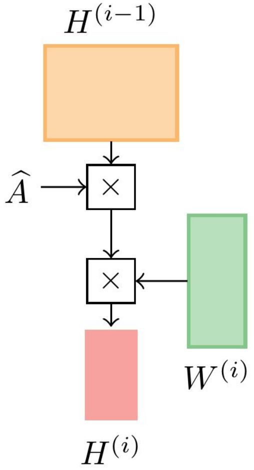

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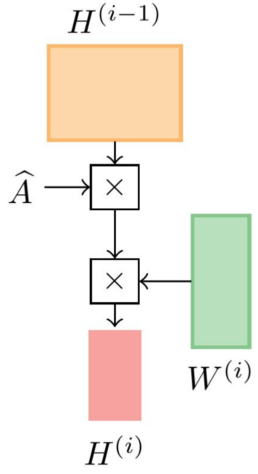
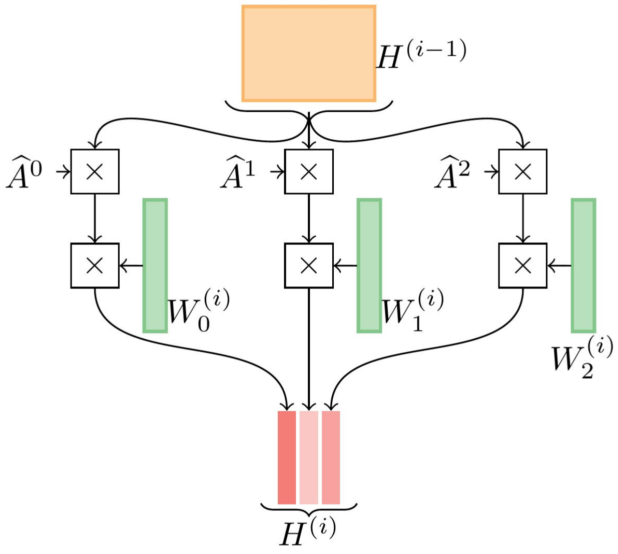
Couple of code lines
implements concatenation



Our Model: MixHop

MixHop GC Layer

$$H^{(1)} = \Big\Vert_{j \in P} \sigma\left(\widehat{A}^j X W_j^{(1)}\right)$$



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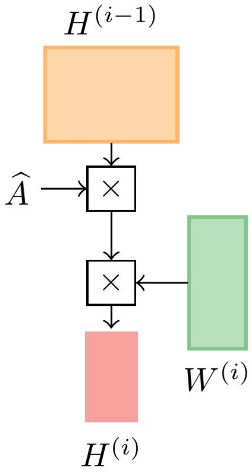

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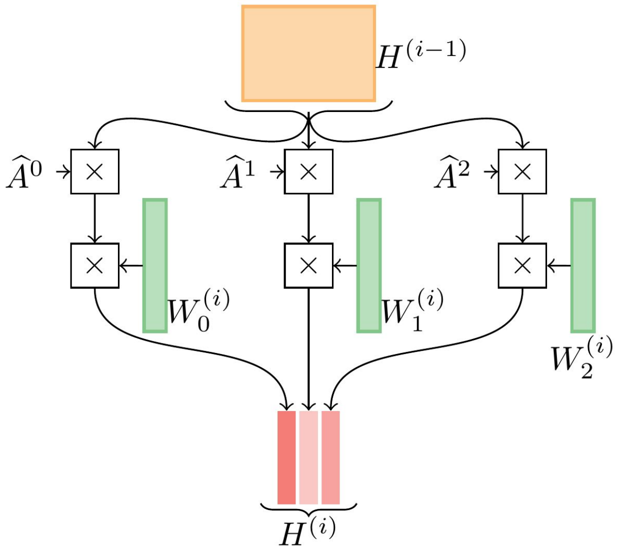

Our Model: MixHop

😀 Inductive

MixHop GC Layer

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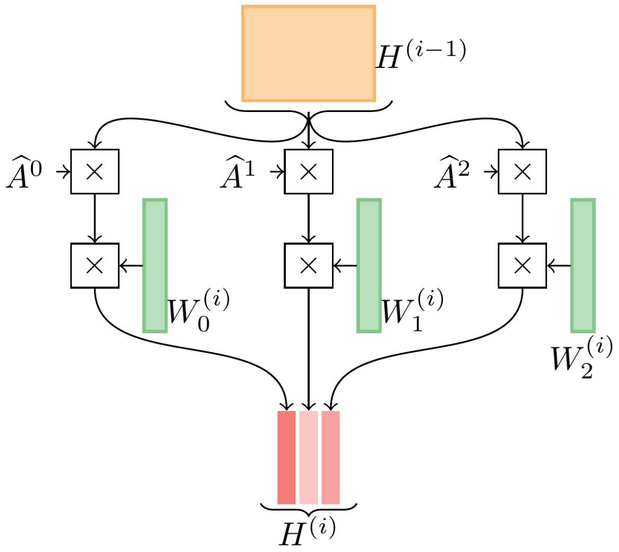

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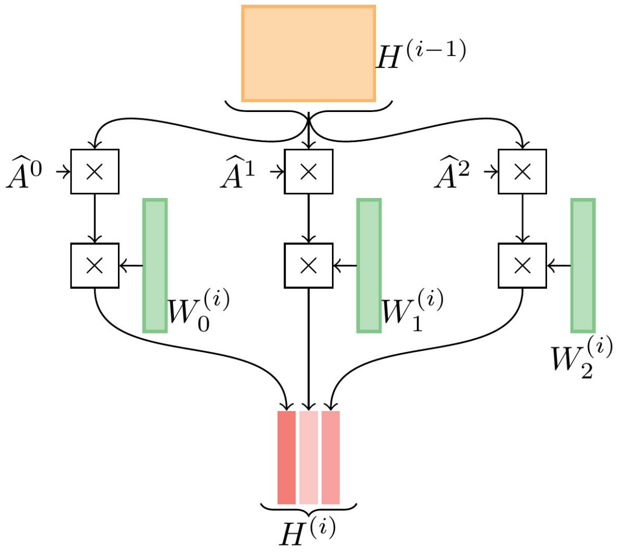

😀 Inductive

😀 Can incorporate distant nodes

Our Model: MixHop

<u>MixHop GC Layer</u>

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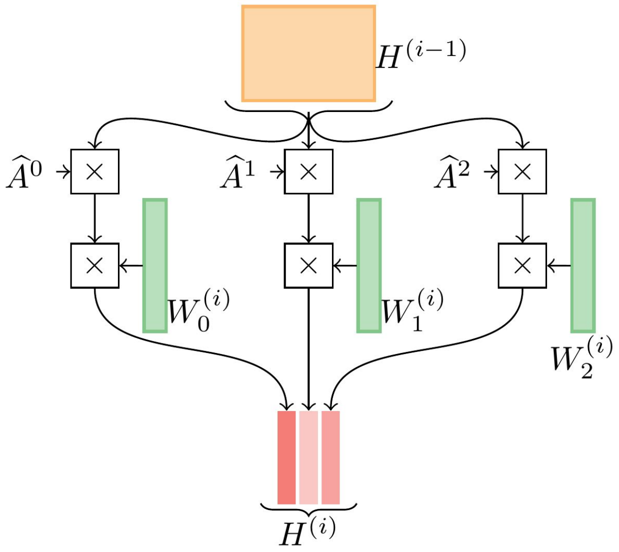

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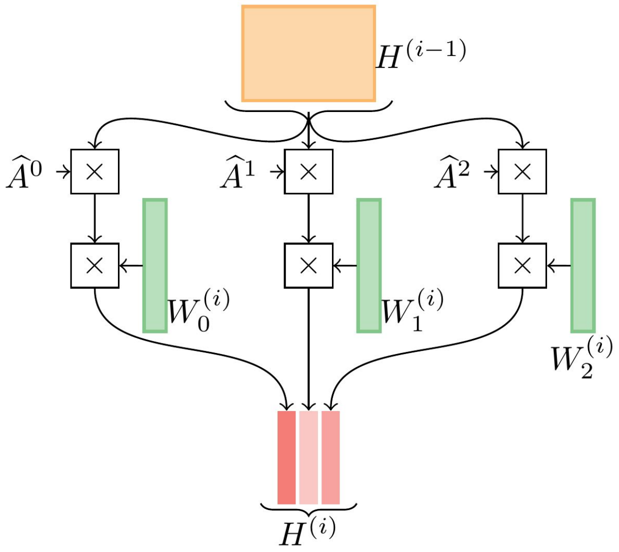

- 😀 Inductive
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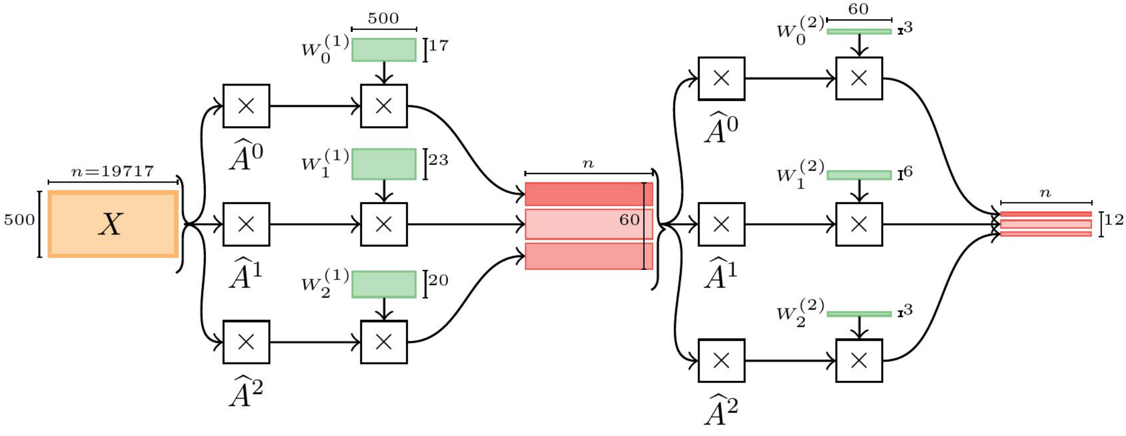
- 😀 Can mix neighbors across distances in arbitrary linear combinations i.e. **can learn <u>Gabor Filters</u>**!

Sparsification

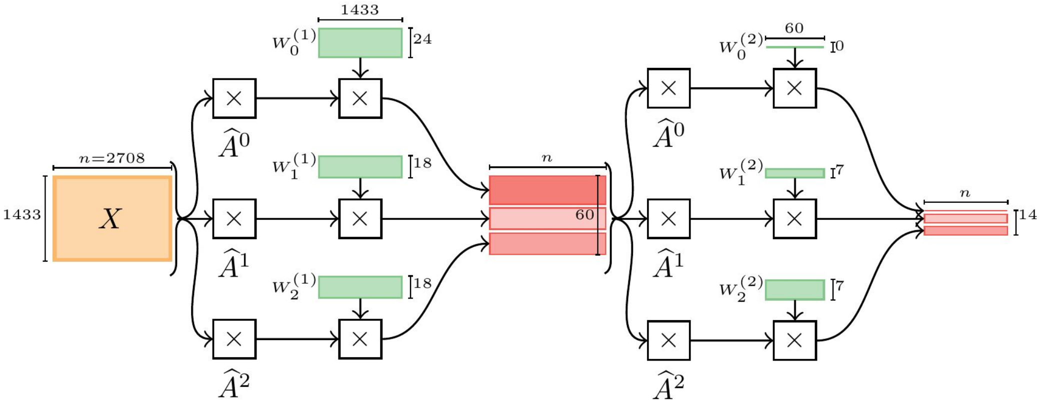
We add group L2-Lasso Regularization to drop-out columns feature matrices, similar to [6]

[images are rotated space]

[6] Gordon et al, CVPR, 2018



(a) Pubmed



(b) Cora

Sparsification

We add group L2-Lasso Regularization to drop-out columns feature matrices, similar to [6]

2nd layer of Cora drops-out zeroth-power completely.

[images are rotated space]

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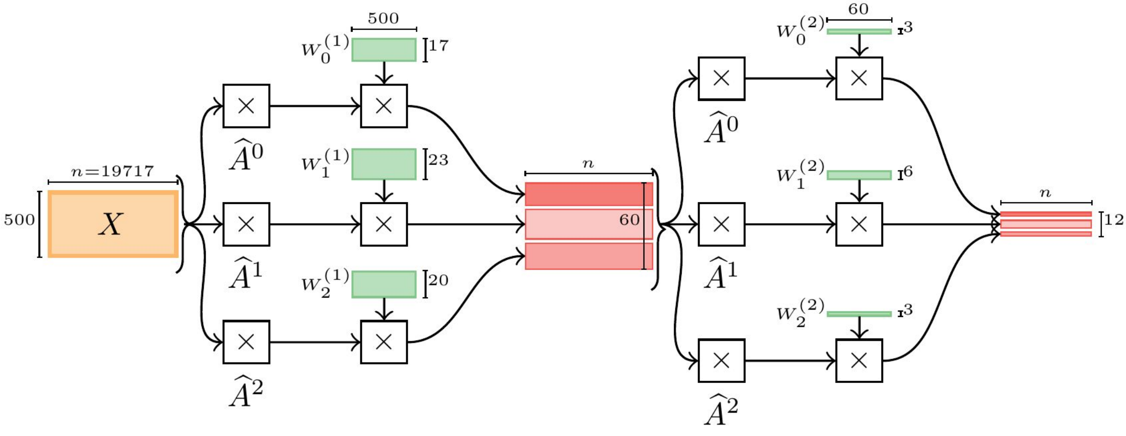

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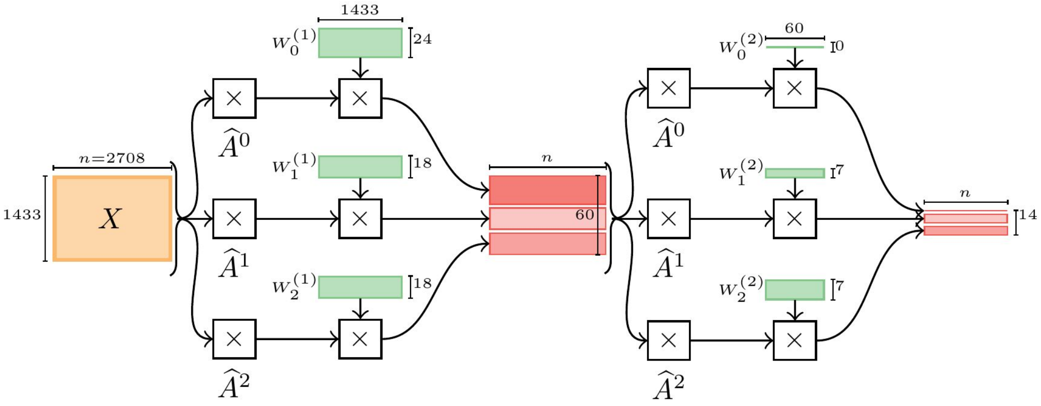

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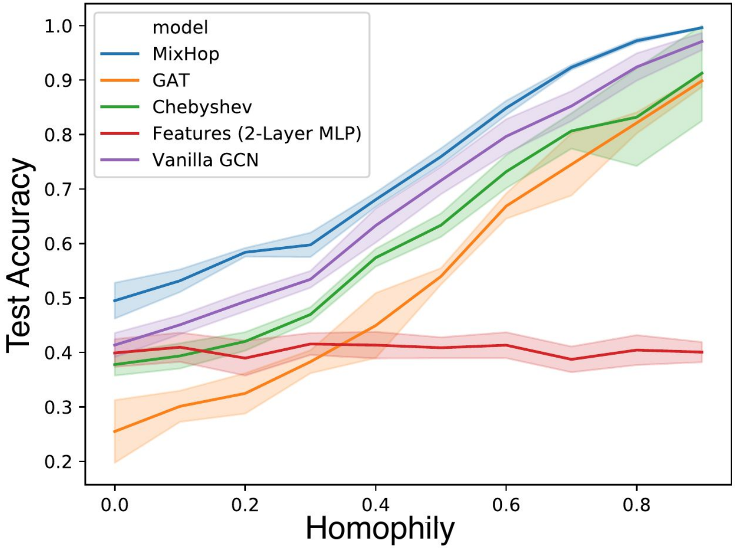
Results on Citation Datasets

Model	Citeseer	Cora	Pubmed
2-Layer MLP	70.6±1	69.0±1.1	78.3±0.54
Chebyshev (Defferrard et al., 2016)	74.2±0.5	85.5±0.4	81.8±0.5
Vanilla GCN (Kipf & Welling, 2017)	76.7±0.43	86.1±0.34	82.2±0.29
GAT (Velickovic et al., 2018)	74.8±0.42	83.0±1.1	81.8±0.18
MixHop: default architecture (ours)	76.3±0.41	87.0±0.51	83.6±0.68
MixHop: learned architecture (ours)	**77.0**±0.54	**87.2**±0.32	**83.8**±0.44

Table 3: Classification results on random partitions of (Yang et al., 2016) datasets.

Results on (Synthetic) Homophily Datasets

With less homophily, our performance gap increases



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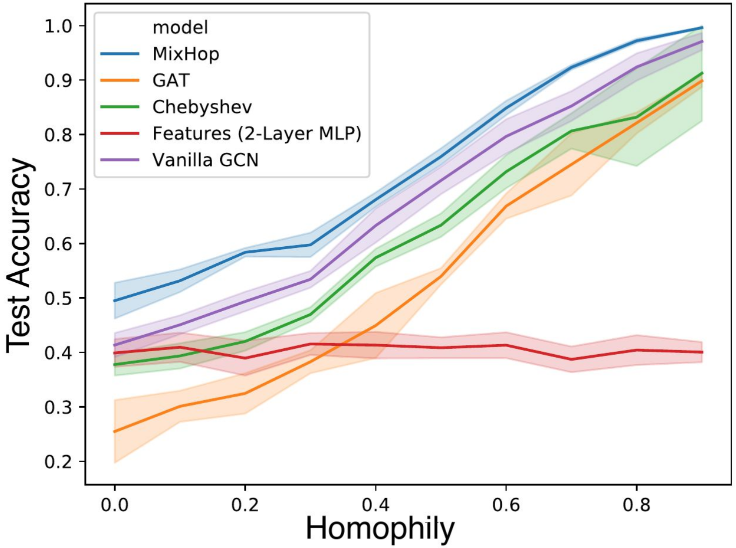
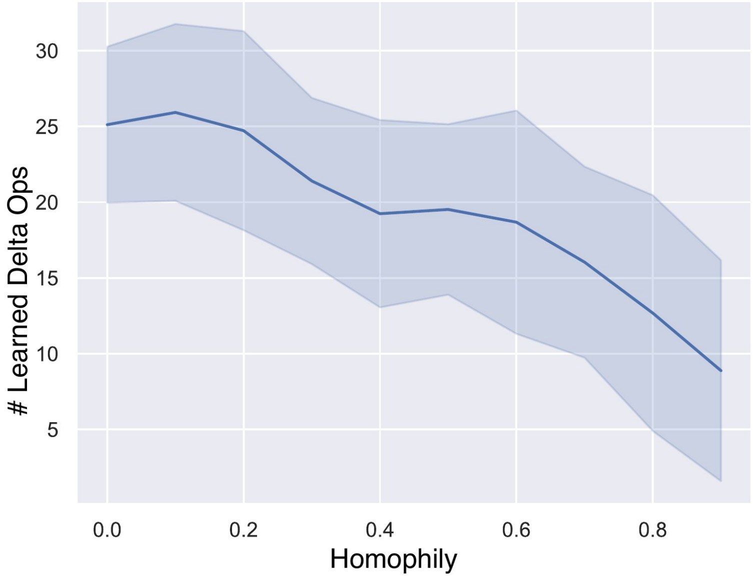


With less homophily, our method learns more feature differences (i.e. Gabor-like Filters)



References

[1] Kipf & Welling, “Semi-Supervised Classification with Graph Convolutional Networks, ICLR”, 2017
[2] Daugman, “Two-dimensional spectral analysis of cortical receptive field profiles”, Vision Research,1980
[3] Daugman, “Uncertainty relation for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters”, Journal of the Optical Society of America, 1985
[4] Honglak Lee et al, “Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations”, ICML, 2009
[5] Alex Krizhevsky et al, “ImageNet Classification with Deep Convolutional Neural Networks”, NeurIPS 2012
[6] Gordon et al, “Morphnet: Fast & simple resource-constrained structure learning of deep networks” CVPR 2018

Conclusion

- With just a couple of lines, Kipf's model can be extended to incorporate powers of (normalized) adjacency matrix
- Allowing it to learn general neighborhood mixing, and its special cases: Gabor-like Filter and Delta Ops
- Inspection shows Delta Ops are indeed learned with lower levels of homophily.

Thank you for listening!

Poster #88

Slides at: http://sami.haija.org/icml19