An Optimal Private Stochastic-MAB Algorithm Based on an Optimal Private Stopping Rule

Touqir Sajed & Or Sheffet

K-Armed Stochastic Bandit Problem

- There are K arms
- The learner pulls an arm at rounds : 1, ... ,**T**
- Pulling an arm $\mathbf{i_t}$ at round **t** generates a reward: $r_t \sim \mathcal{D}_{i_t}(\mu_{i_t}) | r_t \in [0, 1]$
- Minimize Pseudo Regret:

$$\sum_{t=1}^{T} \max_{i \in \{1,\ldots,K\}} \mu_i - \sum_{t=1}^{T} \mu_{i_t}$$

- UCB family meets the lower bound by Lai and Robbins 1985 :

$$\Omega\left(\sum_{\Delta_i > 0} \frac{\log T}{\Delta_i}\right)$$

Differential Privacy

- Let D be a dataset of **m** datums and D' be its neighbour
  - They only differ in 1 reward sample
- An Algorithm M is epsilon-DP if for any output set O, the following holds:

$$\frac{P(M(D) \in O)}{P(M(D') \in O)} \leq e^{\epsilon}$$

- A function f has a sensitivity of Δf if for all neighbours D and D':

$$\max_{D,D'} |f(D) - f(D')| \leq \Delta f$$

Previous DP-MAB results

- DP-UCB algorithms by Mishra & Thakurta (2015), Tossou & Dimitrakakis (2016)
- Rely on tree-based binary mechanism by Chan et al (2011).
- Laplace noise of magnitude: $\frac{\text{polylog}(T)}{\epsilon}$
- Hence the extra pseudo regret bound of $\frac{\text{polylog}(T)\,K}{\epsilon}$
- Shariff & Sheffet (2018) showed a lower bound of $\Omega\left(\frac{K\log(T)}{\epsilon}\right)$
- We propose two algorithms that match the lower bound:

$$\Omega\left(\left(\sum_{a:\Delta_a>0}\frac{\log T}{\Delta_a}\right)+\frac{K\log T}{\epsilon}\right)$$

Our Contributions

- Proposed the first DP-MAB algorithm that meets the lower bound:

$$\Omega\left(\left(\sum_{a:\Delta_a>0}\frac{\log T}{\Delta_a}\right)+\frac{K\log T}{\epsilon}\right)$$

- Showed a lower bound for the private stopping rule problem: $\Omega\left(\frac{R\log(1/\beta)}{\alpha\epsilon|\mu|}\right)$

- Proposed an optimal DP-stopping rule that meets the lower bound:

$$\Omega\left(\left(\frac{R^2}{\alpha^2\mu^2}+\frac{R}{\alpha\epsilon|\mu|}\right)\log\left(\frac{1}{\beta}\right)\right)$$

Thank you!

Come visit our poster today from 6:30 - 9pm at Pacific Ballroom #173