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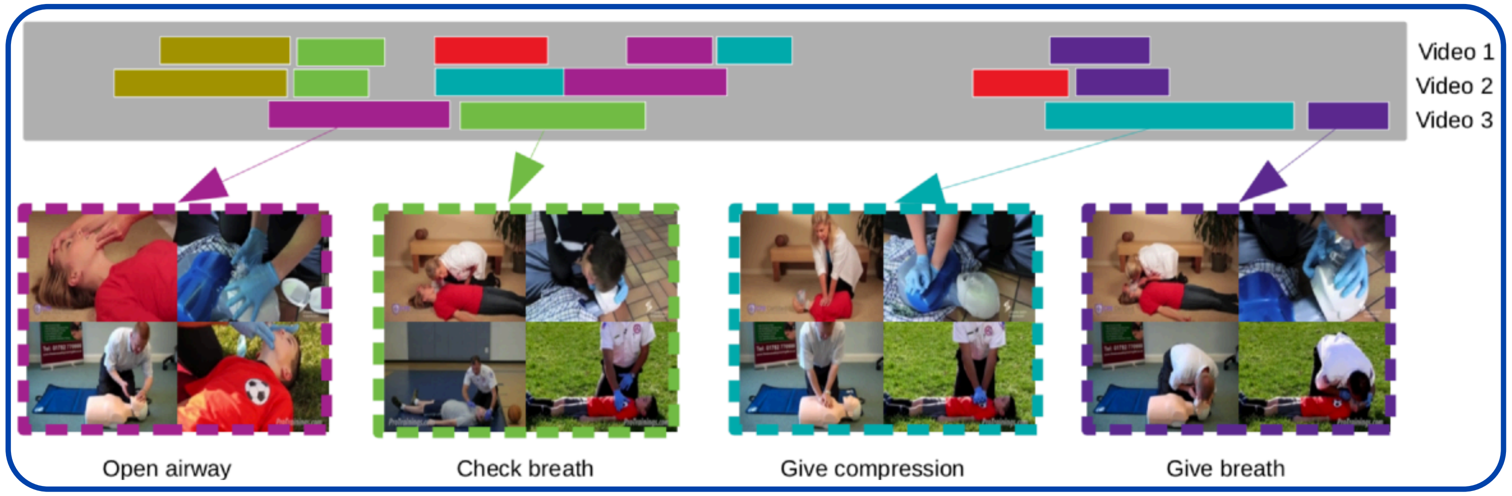
Sequential Facility Location: Approximate Submodularity and Greedy Algorithm

International Conference on Machine Learning 2019

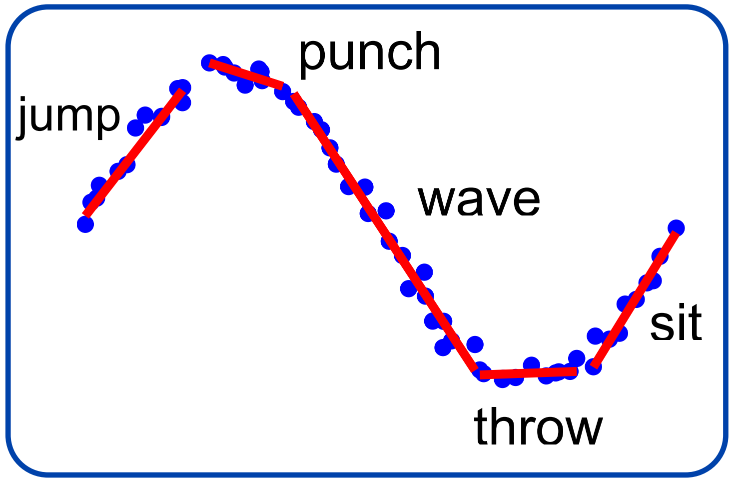
Subset Selection

- Find a **small subset** of representatives from a **large ground set**

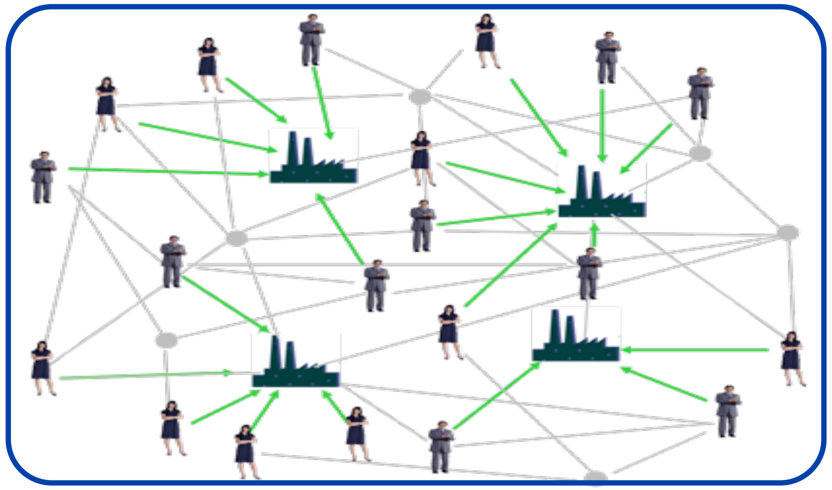
video/text summarization



procedure learning



clustering



sensor/facility placement

viral marketing

Subset Selection

- Find a **small subset** of representatives from a **large ground set**

- Sequential data have **structured dependencies**

Time series:
video, audio

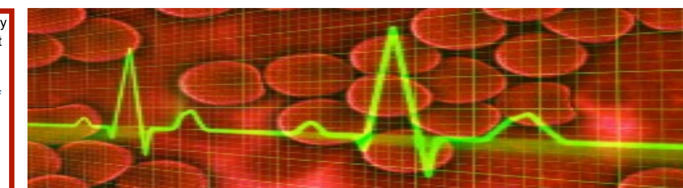
Ordered data:
text, genes

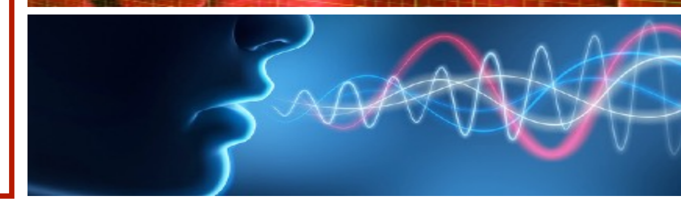
(CNN) -- A Japanese rocket roared into orbit early Friday (Thursday afternoon ET) carrying what NASA calls its most precise instrument yet for measuring rain and snowfall.

The Global Precipitation Measurement (GPM) satellite is the first of five earth science launches NASA has planned for 2014. The 4-ton spacecraft is the most sophisticated platform yet for measuring rainfall, capable of recording amounts as small as a hundredth of an inch an hour, said Gail Skofronick Jackson, GPM's deputy project scientist.

The $900 million satellite is a joint project with the Japanese space agency JAXA, and it lifted off from Tanegashima Space Center at 3:37 a.m. Friday (1:37 p.m. Thursday ET). In a little over a half hour, it had reached orbit, deployed its solar panels and began beaming signals back to its controllers, NASA said.

Also, once fully activated, GPM will use both radar and microwave instruments to detect falling snow for the first time. It will also combine data from other satellites with its own readings, beaming



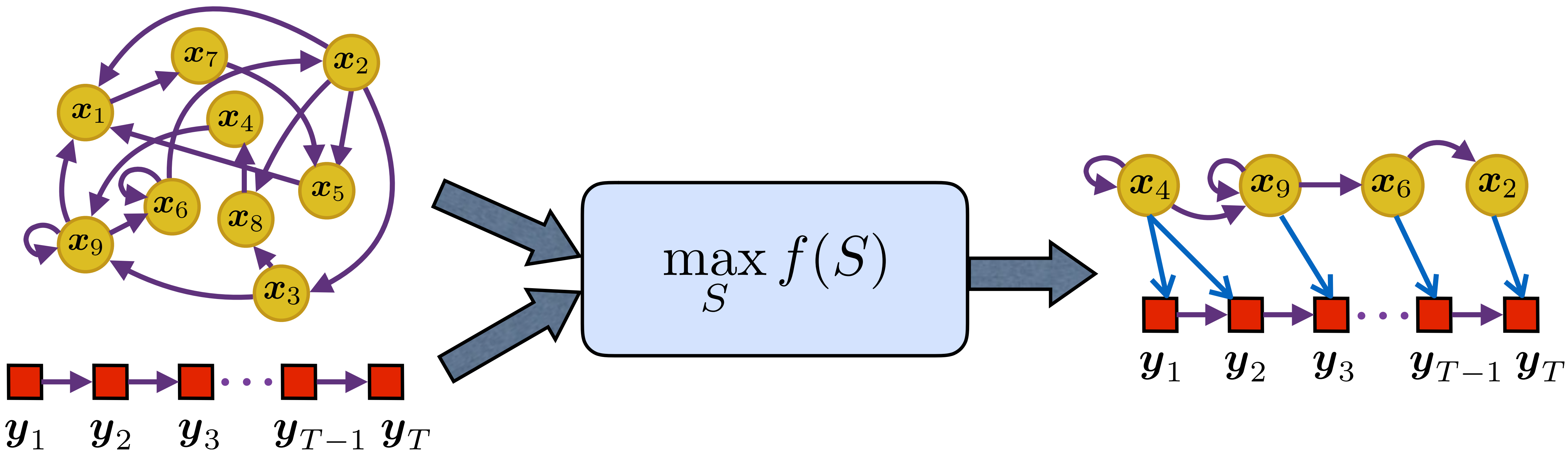


  - **Subactions** performed with a specific order in **instructional videos**
  - Logical way of connecting **sentences** in **speech**

- Most methods: data is a **bag of randomly permutable** items!

This Paper

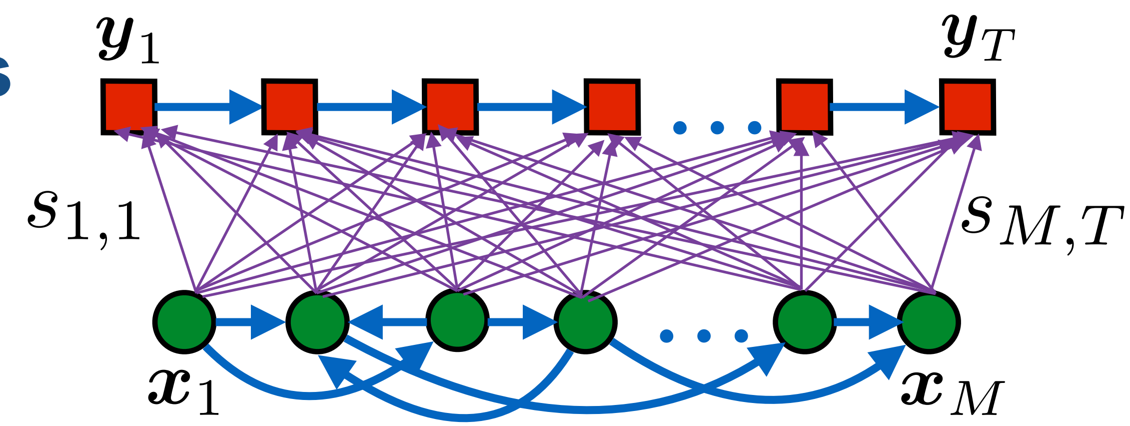
- Develop a framework for **sequential subset selection**



- Propose *cardinality-constrained* **sequential facility location**
- Develop a **fast greedy** algorithm to maximize SeqFL
- Theoretical conditions for **approximate submodularity**
- Address **procedure learning** from instructional videos

Sequential Facility Location

- $\mathbb{X}$ (Source): **items with transition scores**
- $\mathbb{Y}$ (Target): **sequential dataset**
- $s_{i,t}$: **pairwise similarity**
- $\boldsymbol{x}_{r_t}$ **(unknown) representative** of $\boldsymbol{y}_t$



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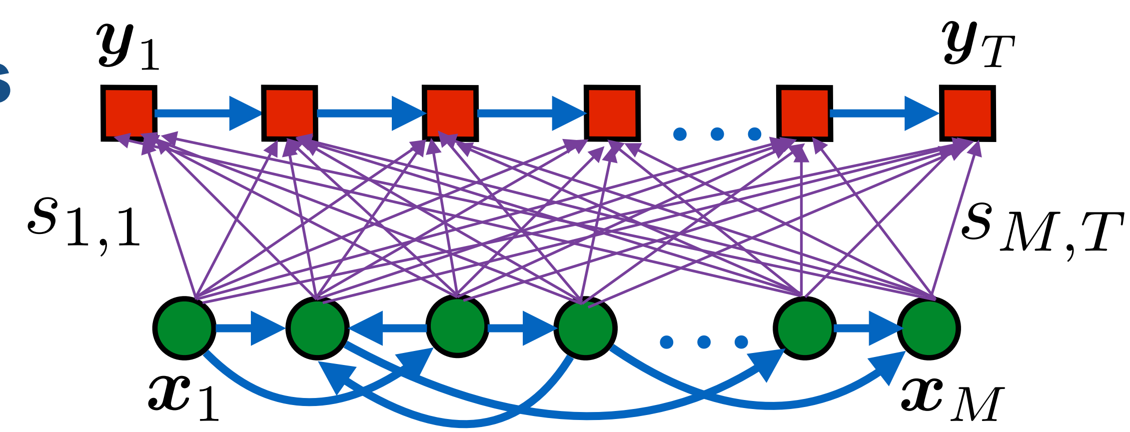
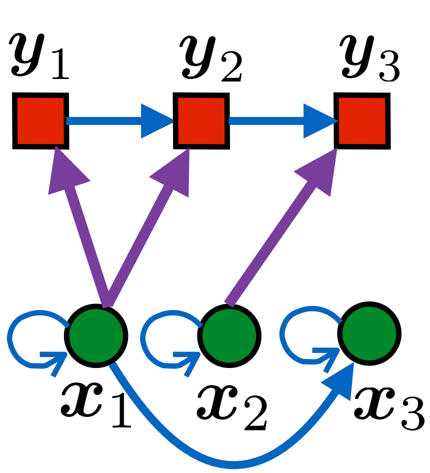
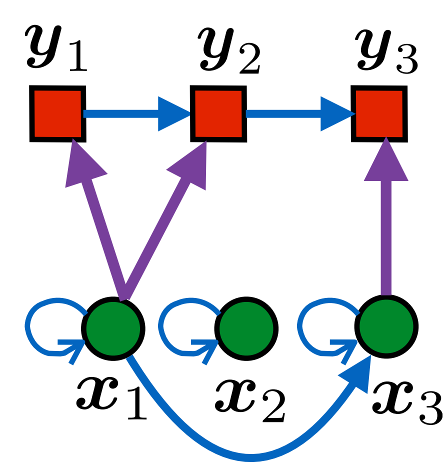


- Find a subset of $\mathbb{X}$ of size k that best encodes $\mathbb{Y}$ and **sequence of assignments** $(r_1, \ldots, r_T)$ obeys the transition dynamic on $\mathbb{X}$



$(r_1, r_2, r_3) = (1, 1, 2)$

$<$



$(r_1, r_2, r_3) = (1, 1, 3)$

Sequential Facility Location: Formulation

- Cardinality constrained sequential FL:

$$\max_{\mathcal{S}:|\mathcal{S}|\leq k} \max_{\{r_t \in \mathcal{S}\}} \boxed{\Phi_{\text{enc}}(r_1, \ldots, r_T)} \times \boxed{\Phi^{\beta}_{\text{dyn}}(r_1, \ldots, r_T)}$$

Encoding potential

Dynamic potential

Sequential Facility Location: Formulation

- Cardinality constrained sequential FL:

$$\max_{\mathcal{S}:|\mathcal{S}|\leq k} \max_{\{r_t \in \mathcal{S}\}} \Phi_{\text{enc}}(r_1, \ldots, r_T) \times \Phi^{\beta}_{\text{dyn}}(r_1, \ldots, r_T)$$

$$\max_{\mathcal{S}:|\mathcal{S}|\leq k} \max_{\boldsymbol{r} \in \mathcal{S}^T} \sum_{t=1}^{T} s_{r_t,t} + \beta \big(q^0_{r_1} + \sum_{t=2}^{T} q_{r_{t-1},r_t}\big)$$

encoding

initial representative score

representative transition score

- $\beta \geq 0$: setting effect of the dynamic term

Sequential Facility Location: Greedy Alg

- Cardinality constrained sequential FL:

$$\max_{\mathcal{S}:|\mathcal{S}|\leq k} \boxed{\max_{\boldsymbol{r}\in\mathcal{S}^T} \sum_{t=1}^{T} s_{r_t,t} + \beta\Big(q^0_{r_1} + \sum_{t=2}^{T} q_{r_{t-1},r_t}\Big)}$$

$$\triangleq f(\mathcal{S})$$

- To run standard greedy, need to compute marginal gain
  - Given $\mathcal{S}$, finding assignments **cannot** be done **independently** over t
  - Use **dynamic programming** to **exactly** compute marginal gain
  - $O(k^2MT)$ complexity! ⇒ reducing $O(M^2T^2 + M^3T)$ of [Elhamifar-Kaluza-NeurIPS'17]

Sequential Facility Location: Theory

$$f(\mathcal{S}) \triangleq \max_{\boldsymbol{r} \in \mathcal{S}^T} \sum_{t=1}^{T} s_{r_t,t} + \beta\Big(q^0_{r_1} + \sum_{t=2}^{T} q_{r_{t-1},r_t}\Big)$$

- **Theorem**: Assume there exists $\varepsilon \in [0,1)$ so that

$$q_{i,i'} = \bar{q}_{i'}\psi_{i,i'} \qquad \psi_{i,i'} \in [1-\varepsilon, 1+\varepsilon] \qquad \forall i, i'$$

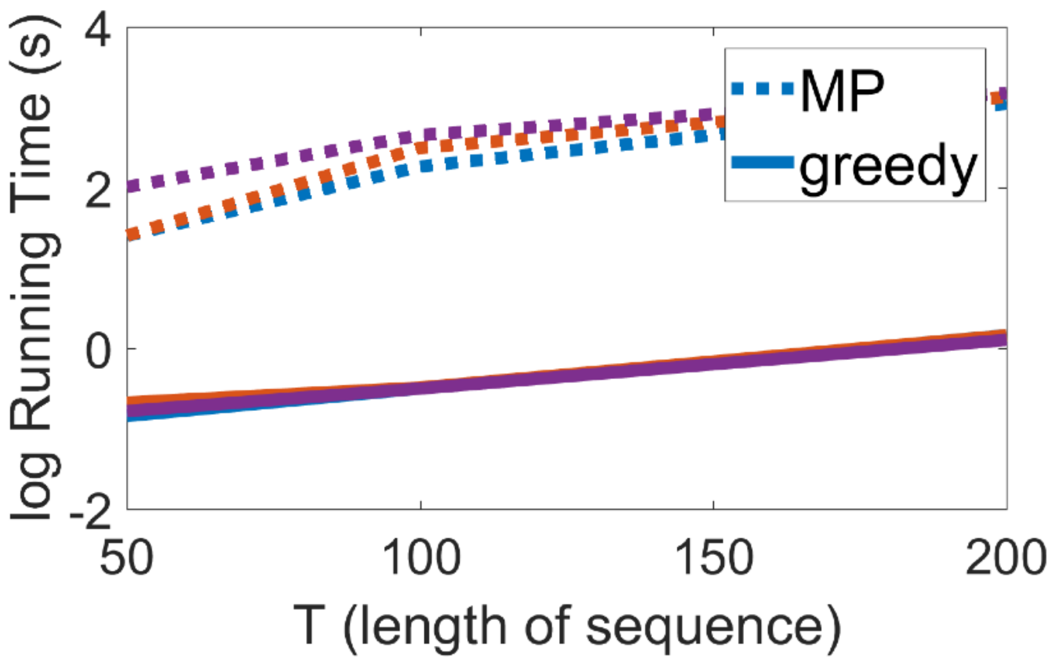
Then, sequential FL utility $f(\mathcal{S})$ is ε-approximately submodular.

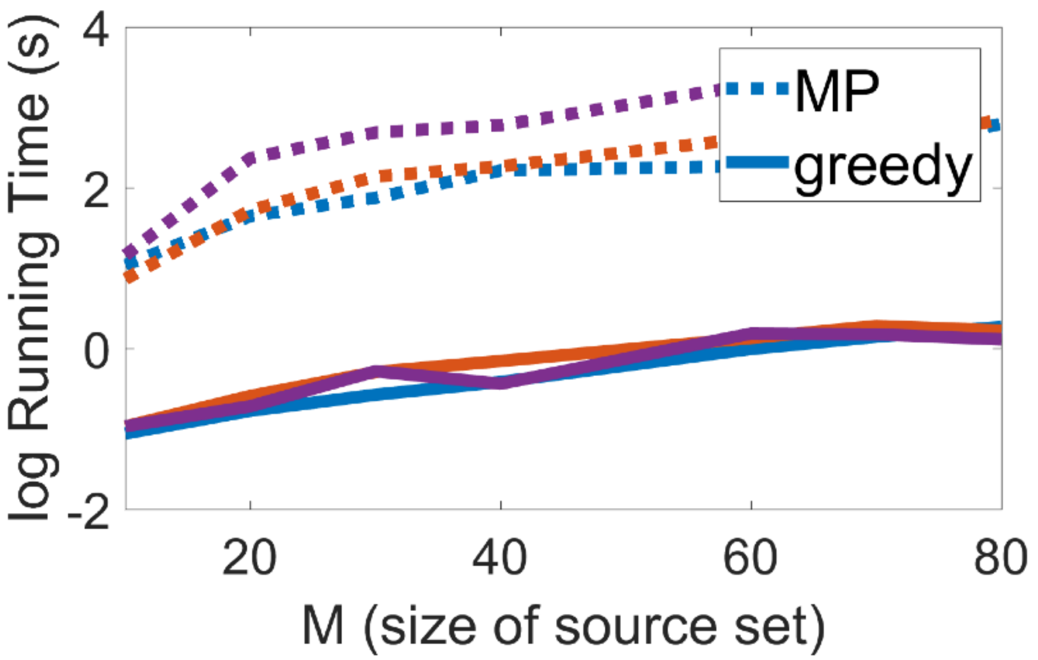
⇒ Greedy has $1 - 1/e - O(\varepsilon k)$ guarantees [Horel-Singer'16].

- **Corollary**: when $q_{1,i} = \cdots = q_{M,i}\,,\ \forall i\,,\ f(\mathcal{S})$ is submodular.

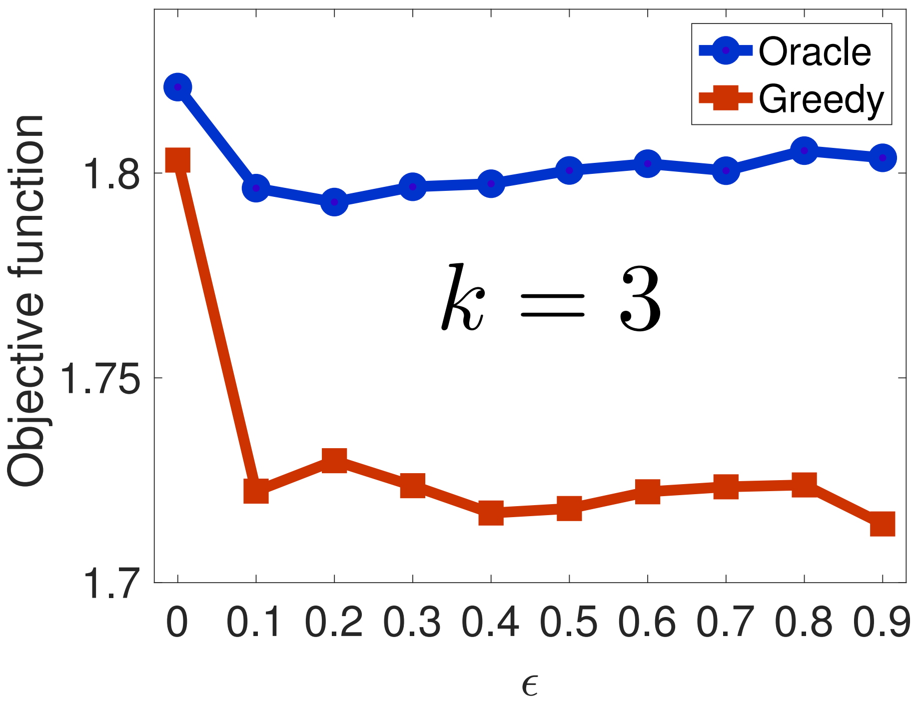
Synthetic Experiments

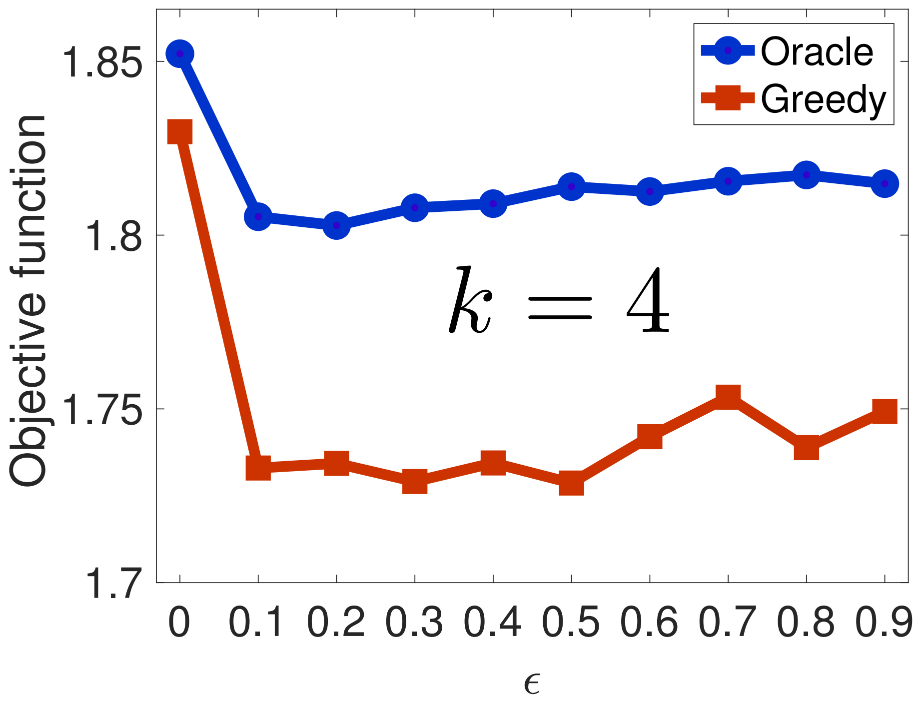
- Greedy ~**2,3 orders of magnitude faster** than message passing





- Greedy performance is insensitive to $\varepsilon > 0$

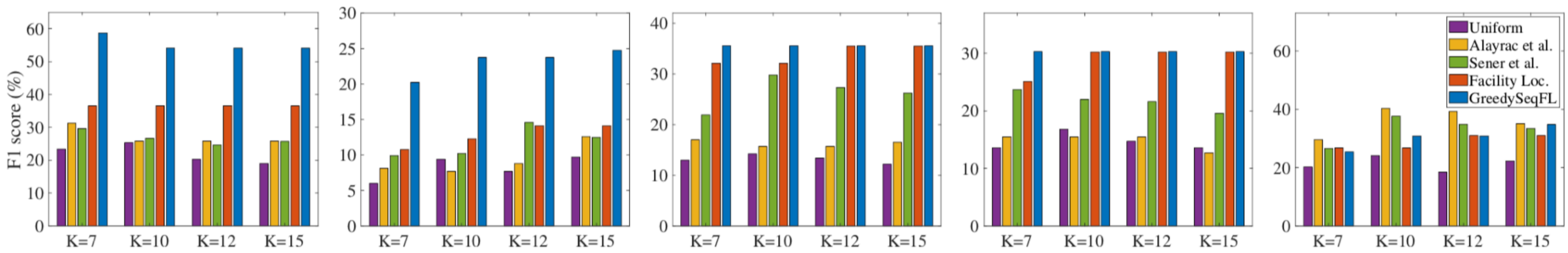




Real Experiments: Procedure Learning

- Recover **key-subactions** and **ordering** from instructional videos
- Inria instructional dataset [Alayrac et al'16]: 5 tasks, 30 videos/task

	Uniform	Alayrac et al.	Sener et al.	Facility Loc.	GreedySeqFL
$K = 7$	15.2	20.3	22.3	26.3	**34.1**
$K = 10$	18.0	21.0	25.3	27.6	**34.9**
$K = 12$	14.8	21.0	24.6	29.5	**34.9**
$K = 15$	15.4	20.5	23.5	29.5	**35.9**



(a) perform cpr (b) jump-start car (c) make coffee (d) repot plant (e) change tire

- Incorporating dynamics, **SeqFL significantly improves FL**
- Most improvement in videos with **repeated steps**

Poster #113

Acknowledgement: